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|c MIU
ano, Leonard M. |q (Leonard Magruder), |d 1866-1943.
ne and spherical trigonometry, |c by Leonard M. Passano.
York, |b The Macmillan company, |c 1923.
, 142 p. 1 L. |b diagrs. |c 21 cm.
rigonometry
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PLANE AND SPHERICAL TRIGONOMETRY

THE MACMILLAN COMPANY
NEW YORK · BOSTON · CHICAGO · DALLAS
ATLANTA · SAN FRANCISCO

MACMILLAN & CO., Limited
LONDON · BOMBAY · CALCUTTA
MELBOURNE

THE MACMILLAN CO. OF CANADA, Ltd.
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PLANE AND SPHERICAL TRIGONOMETRY

BY

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New York

THE MACMILLAN COMPANY

1923

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Set up and electrotyped. Published April, 1918.

FERRIS
PRINTING COMPANY
NEW YORK CITY

PREFACE

Of late years, in the writing of textbooks of trigonometry, a tendency to amplification has shown itself, doubtless with the idea that amplification means simplification. Unfortunately the amplification has spent itself upon details rather than upon principles, which latter have too often been inadequately treated. The result has been textbooks which overlook the comparative maturity of the boys and girls who study trigonometry and which cling almost with affection to the practices of the most elementary mathematics.

The present text aims to present the trigonometry in such a way as to make it interesting to students approaching some maturity, and so as to connect the subject, not only with the mathematics which the student has already had, but also with the mathematics which, in many cases at least, is to follow. A subject may be so burdened with detailed explanations as to become monotonous and lifeless, or, on the other hand, presented in so concise and difficult a manner as to be repellent. The present work endeavors to avoid both extremes. Full explanations are given of important principles, but many simple details are left to the work of the student.

The following points in the text may be noted:

1. Positive and negative angles of any magnitude and the trigonometric functions of such angles, defined by means of a system of rectangular coördinates, are taken up in the beginning of the book; acute angles, with their functions, being mentioned as a special case.

2. Thus the basic trigonometric identities are got at once for all angles.

3. The functions of 0°, 90°, etc., are carefully explained by the theory of limits.

4. The solution of right triangles and related problems are taken up early without the use of logarithms.

5. Logarithms are then very carefully explained and fully discussed, not so much as to their use in computation, but rather so as to clarify their meaning.

6. Right triangles are then solved by the use of logarithms, and the essentially approximate nature of all numerical results is emphasized.

7. The text next returns to trigonometric identities, giving a detailed and accurate proof of the addition formulæ for sines and cosines, with less detailed but sufficient explanation of other fundamental identities. The number of identities to be memorized is reduced to a minimum.

8. The circular measure of an angle and the inverse functions are then taken up, emphasis being laid upon the fact that the latter are angles.

9. There follows the solution of triangles in general. As each case is mentioned the theorems or formulæ needed for its solution are derived.

10. The last subject treated in the plane trigonometry is the solution of trigonometric equations, and the fact is emphasized that the operations are simply the solution of algebraic equations applied to a new class of quantities.

11. The lists of examples and problems are numerous and carefully chosen, many of them being taken from work in analytic geometry and calculus, though, of course, no knowledge of either of these subjects is assumed. Some of the problems are entirely new, being invented for this text, and all problems are chosen with a purpose to indicate the practical interest and value of trigonometry.

12. In the spherical trigonometry, as in the plane, the three chief aims are brevity, clarity, and simplicity; a chapter on the Earth treated as a sphere being given to enliven an otherwise somewhat formal and lifeless subject.

13. The author has *not* tried to revolutionize the teaching

of trigonometry, believing that much that has been done in the past is good though none the less open to improvement. Such improvement has been the aim of this work.

The author wishes to acknowledge the kindness of his colleagues Professor H. W. Tyler, Professor F. L. Hitchcock, and Professor J. Lipka in reading and criticizing the manuscript of his book, and to express his thanks to Professor E. R. Hedrick, editor of the tables appended, for permission to make use of them.

L. M. PASSANO.

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INTRODUCTION

Trigonometry is primarily the science concerned with the measurement of plane and spherical triangles, that is, with the determination of three of the parts of such triangles when the numerical values of the other three parts are given. This is done by means of the six trigonometric functions, defined in article 4 following. But these functions enter so intimately into many branches of mathematical and physical science not directly concerned with the measurement of angles, that their analytical properties are of fundamental importance. Analytical trigonometry, that is, the proof and use of various algebraic relations among the trigonometric functions of the same or related angles, is therefore, in modern times, of equal importance with the trigonometry which deals with triangular solutions.

The same functions which enable one to solve triangles constructed in a plane suffice also for the solution of spherical triangles. But the solution of triangles of which the sides are geodetic lines, that is, lines which are the shortest distances between pairs of points on the surface, on a spheroidal surface such as the Earth, requires the use of other functions than those needed for the solution of plane or spherical triangles. This spheroidal trigonometry is very complex, and becomes necessary only in the accurate survey of very large tracts of the Earth's surface. For ordinary purposes of surveying and for the solution of triangles on the Earth's surface over small areas, plane and spherical trigonometry are sufficient.

The study of trigonometry, as ancillary to astronomy, dates from very early times. Among the Greeks, who, however, were more famous as geometers than as investigators in other branches of mathematics, the names of Hipparchus (about 150 B.C.) and of Ptolemy (who lived in the second century of the Christian era), both astronomers, are prominent. Hipparchus left no mathematical writings, but we are told by an ancient writer that he created the science of trigonometry. Ptolemy, making use of the investigations and discoveries of Hipparchus, perfected the form of the science. The theorems of these two astronomers are still the basis of trigonometry.

Ptolemy calculated a table of chords, which were used in those earliest days of the science, as we now use the sines of angles. The radius of a circle he divided into sixty equal parts. Each of these he divided again into sixty equal parts, called, in the Latin translation of his work the Almagest, "**partes minutae primae**"; and each of these in turn into sixty, called "**partes minutae secundae**"; whence have come the names "minutes" and "seconds" for the subdivisions of the angular degree. Ptolemy, however, was not the first to calculate a table of chords, Hipparchus, among others, having done so previously, but he invented theorems by means of which the calculations could be more readily made.

The Hindus, more skillful calculators than the Greeks, acquired the knowledge of the latter and improved upon it, notably in that they calculated tables of the half-chord, or sine, instead of the whole chord of the angle. The Arabs also were acquainted with the Almagest, and with the investigations of the Hindus. It was an Arab, Al Battani or Albategnius, who first calculated a table of what may be called cotangents, by computing the lengths of shadows of a vertical object cast by the sun at different altitudes. Another Arab invented, as a separate function, the tangent, which had previously been used only as an abbreviation of the ratio sine to cosine. Curiously enough this invention was afterwards forgotten until the tangent was re-invented in England in the fourteenth century by Bradwardine, and in the fifteenth century by the German, Johannes Müller, called Regiomontanus, who wrote the first complete European treatise on trigonometry.

When Napier * invented logarithms, in 1614, they were at once adopted in trigonometric calculations, and the first tables of logarithmic sines and tangents were made by Edmund Gunter, an English astronomer (1581–1626). He it was who first used the names cosine, cotangent, and cosecant. During the following century the science of trigonometry progressed slowly, becoming more analytical in form, until, in the hands of Euler (1707–1783), it became essentially what it is at the present day.

With this brief introduction to the history of trigonometry let us now proceed to become acquainted with that homely, perhaps, but most serviceable handmaid to so many of the arts and sciences,

> " . . . being just as great, no doubt,
> Useful to men, and dear to God, as they ! "

* John Napier, 1550–1617.

PLANE TRIGONOMETRY

CHAPTER I

THE TRIGONOMETRIC FUNCTIONS OF ANY ANGLE, AND IDENTICAL RELATIONS AMONG THEM

1. Rectangular Coördinates. Two lines, $x'x$ and $y'y$, drawn in a plane at right angles to each other, as in Fig. 1, form a system of rectangular, Cartesian coördinates. The point O in which the lines intersect is called the origin; the two lines are called the axes of coördinates. One of these, usually the horizontal line, is called the axis of abscissæ, or the axis of x. The other is called the axis of ordinates, or the axis of y. We shall speak of XOY, YOX', $X'OY'$, and $Y'OX$ as the first, second, third, and fourth quadrants respectively.

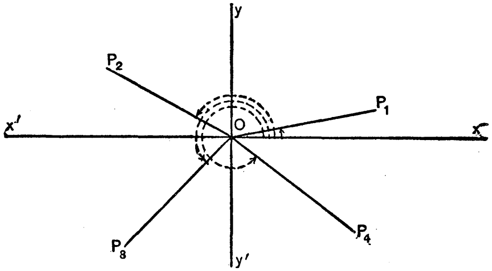

FIG. 1.

2. Angles of any Magnitude. There are many ways in which a system of coördinates is used in mathematics. In trigonometry such a system is used primarily in defining the trigonometric functions, but before we proceed to do so we shall extend our ideas of angles beyond the knowledge we obtained of them in the elementary geometry. There an angle is defined by some such definition as the following: the plane figure formed by two straight lines drawn from the same point. The unit of angles is either the right angle, or the degree, and the largest angle usually dealt with is equivalent to two right angles and is often called a straight angle. In trigonometry, on the other hand, we deal with angles of any magnitude whatever. To do so we introduce the idea of motion, of revolution. Starting from the initial position OX, Fig. 1, we may revolve the line about O in the direction indicated by the arrows, stopping in any desired terminal position OP_1, OP_2, OP_3, OP_4, etc. In this way angles of any number of degrees whatever may be generated. Thus, if we stop in the position OY, we have an angle of 90°; in the position OX', 180°; in the position OP_3, 225°; in the position OY', 270°, and so on. By making one whole revolution we should arrive at an angle of 360°; two and one half revolutions, 900°; etc.

Not only so, but we might revolve from the initial position OX in the opposite direction. Now oppositeness is indicated algebraically by the use of the signs *plus* (+) and *minus* (−). So that if we agree to take the positive direction of revolution counterclockwise, then clockwise will be the negative direction and we can thus generate negative angles of any magnitude whatever. Thus, Fig. 1, the angle XOP_3 is 225° if we have revolved in the positive direction, but is $-135°$ if we have revolved in the negative direction. When an angle lies in value between 0° and 90° it is said to be an angle in the first quadrant since its terminal side lies in the first quadrant. An angle lying in value between 90° and 180° is said to be in the second quadrant; between 180° and 270°, in the third quadrant; between 270° and 360°, in the fourth quadrant.

EXAMPLES

Construct the angles

1. 300°.
2. $-210°$.
3. 750°.
4. $-495°$.
5. $-1215°$.
6. 420°.

Add the following angles graphically:

7. 720° and 30°.
8. $-180°$ and 60°.
9. $-90°$ and $-45°$.
10. 990° and $-60°$.
11. $-45°$ and 120°.
12. 135° and $-450°$.

If A is a positive angle in the first, second, third, or fourth quadrant respectively, add graphically

13. 450° and A.
14. $-270°$ and A.
15. 180° and $-A$.
16. $-540°$ and $-A$.

3. Abscissa, Ordinate, and Distance. Consider an angle, positive or negative, of any magnitude whatever*, XOP, of Fig. 2. From P, any point in the terminal side of this angle, drop a perpendicular upon the axis of x. The lines of the figure are named as follows: OM is called the abscissa of the point P, MP the ordinate, and OP the distance. The abscissa OM and the ordinate MP are together called the coördinates of the point P. Note very carefully that the abscissa is always read from O to M, the ordinate from M to P; that is, in each case *from* the axis *to* the

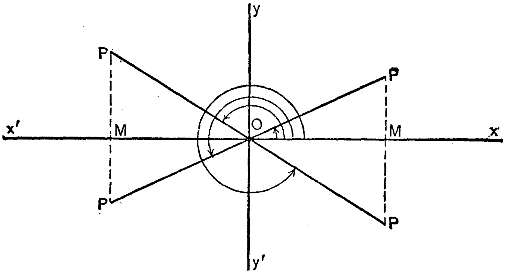

Fig. 2.

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* As a matter of convenience we do not consider angles numerically greater than 360°. It is obvious that the discussion applies equally well to such angles.

point. The distance is read from O to P. Thus, for an angle in the second or third quadrant the direction of the abscissa is opposite to that of an angle in the first or fourth quadrant. For an angle in the third or fourth quadrant the direction of the ordinate is opposite to that of the ordinate of an angle in the first or second quadrant. Oppositeness in direction being distinguished as usual by difference in algebraic sign we have the following conventions:

The abscissa measured to the right of the axis of y is positive; to the left, negative. The ordinate measured upward from the axis of x is positive; downward, negative. The distance is measured from the origin outward and is taken positive.

EXAMPLES

1. The abscissa of a point is 3, its ordinate 4; find the distance.

2. The distance of a point is 5, its ordinate 4; find the abscissa

3. The ordinate of a point is -2, its distance 3; find the abscissa.

4. The ordinate of a point is -5, its abscissa -4; find the distance.

5. Prove that the square of the distance of any point is equal to the sum of the squares of the abscissa and ordinate.

6. Prove that for all points on a straight line through the origin the ratio of the ordinate to the abscissa is constant.

4. The Trigonometric Functions Defined. Let us now proceed to define the six trigonometric functions of an angle; six quantities which depend upon the angle for their values. They are the possible ratios between the various pairs of the three lines named in Art. 3. Thus, Fig. 2, the

$$\text{sine } XOP = \frac{\text{ordinate of } P}{\text{distance of } P} = \frac{MP}{OP},$$

$$\text{cosine } XOP = \frac{\text{abscissa of } P}{\text{distance of } P} = \frac{OM}{OP},$$

$$\text{tangent } XOP = \frac{\text{ordinate of } P}{\text{abscissa of } P} = \frac{MP}{OM},$$

$$\textbf{cotangent } XOP = \frac{\textbf{abscissa of } P}{\textbf{ordinate of } P} = \frac{OM}{MP},$$

$$\textbf{secant } XOP = \frac{\textbf{distance of } P}{\textbf{abscissa of } P} = \frac{OP}{OM},$$

$$\textbf{cosecant } XOP = \frac{\textbf{distance of } P}{\textbf{ordinate of } P} = \frac{OP}{MP}.$$

Three other functions are sometimes used: The versed sine, which is unity minus the cosine; the coversed sine, which is unity minus the sine; the suversed sine, which is unity plus the cosine. They are relatively unimportant.

5. Trigonometric Functions are Ratios. The first thing to be noted about these functions is that, being ratios, they are independent of the actual lengths of the abscissa, ordinate, and distance. Thus, Fig. 3, the triangles OM_1P_1, OM_2P_2, and OM_3P_3 being similar, their homologous sides are proportional, so that

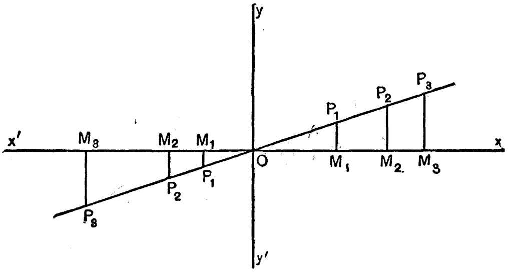

FIG. 3.

$$\sin XOP = \frac{M_1P_1}{OP_1} = \frac{M_2P_2}{OP_2} = \frac{M_3P_3}{OP_3},$$

$$\tan XOP = \frac{M_1P_1}{OM_1} = \frac{M_2P_2}{OM_2} = \frac{M_3P_3}{OM_3}.$$

Similarly the truth of the statement may be shown for the remaining functions.

6. Signs of the Functions. The second point to be noted is that the signs of the functions vary according to the quadrant in which the angle lies. Thus, Fig. 2, for the angle XOP in the first quadrant the abscissa, ordinate and distance are all positive so that all the functions are positive. For the angle XOP in the second quadrant the ordinate and distance are positive, the abscissa negative. Thus we have for the angle in the second quadrant

$$\sin XOP = \frac{MP}{OP} = \frac{+}{+} = +,$$

$$\cos XOP = \frac{OM}{OP} = \frac{-}{+} = -,$$

$$\tan XOP = \frac{MP}{OM} = \frac{+}{-} = -,$$

$$\cot XOP = \frac{OM}{MP} = \frac{-}{+} = -,$$

$$\sec XOP = \frac{OP}{OM} = \frac{+}{-} = -,$$

$$\csc XOP = \frac{OP}{MP} = \frac{+}{+} = +.$$

The following table gives the signs of the functions in the four quadrants.

QUAD.	I	II	III	IV	QUAD.
sine	+	+	−	−	cosecant
cosine	+	−	−	+	secant
tangent	+	−	+	−	cotangent

EXAMPLES

Determine the algebraic signs of

1. cos 218°.
2. tan (− 460°).
3. sin 1100°.
4. cot (− 99°).
5. sec 315°.
6. csc (− 210°).
7. Let the student determine, as above, the signs of the trigonometric functions of angles in the third and fourth quadrants.

7. Functions of Acute Angles. A special set of definitions for the functions of acute angles, which are sometimes useful and should be known, follows directly as a special case of the general definitions given above. Thus, Fig. 4, in which the angle XOP lies in a right triangle,

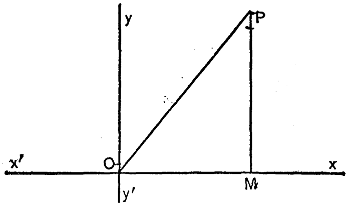

Fig. 4.

$$\sin XOP = \frac{\text{ordinate}}{\text{distance}} = \frac{\text{opposite side}}{\text{hypotenuse}},$$

$$\cos XOP = \frac{\text{abscissa}}{\text{distance}} = \frac{\text{adjacent side}}{\text{hypotenuse}},$$

$$\tan XOP = \frac{\text{ord.}}{\text{abs.}} = \frac{\text{opp. side}}{\text{adj. side}},$$

$$\cot XOP = \frac{\text{abs.}}{\text{ord.}} = \frac{\text{adj. side}}{\text{opp. side}},$$

$$\sec XOP = \frac{\text{dist.}}{\text{abs.}} = \frac{\text{hypot.}}{\text{adj. side}},$$

$$\csc XOP = \frac{\text{dist.}}{\text{ord.}} = \frac{\text{hypot.}}{\text{opp. side}}.$$

These definitions, it must be noted, completely agree with the more general definitions, but are applicable only to angles less than ninety degrees, since angles greater than ninety degrees cannot occur in right triangles.

8. Reciprocal Functions. Two questions would naturally suggest themselves at this point: Are the trigonometric functions of an angle related to each other in any particular way? and, second, if there be a definite relation between two given angles will the functions of those angles bear some special relation to each other? We shall proceed to answer the first of these questions affirmatively, but shall leave the discussion of the second question to a later chapter (Chap. II). Thus, if α be any angle, it follows by the definitions of the trigonometric functions that

$$\sin\alpha = \frac{\text{ordinate}}{\text{distance}} = \frac{1}{\dfrac{\text{dist.}}{\text{ord.}}} = \frac{1}{\csc\alpha},$$

$$\cos\alpha = \frac{\text{abscissa}}{\text{distance}} = \frac{1}{\dfrac{\text{dist.}}{\text{abs.}}} = \frac{1}{\sec\alpha},$$

$$\tan\alpha = \frac{\text{ordinate}}{\text{abscissa}} = \frac{1}{\dfrac{\text{abs.}}{\text{ord.}}} = \frac{1}{\cot\alpha},$$

or, the *sine* and *cosecant*, the *cosine* and *secant*, the *tangent* and *cotangent* respectively of the same angle *are reciprocals of each other.*

9. Tangent, Sine and Cosine. Again, by definition, and by Art. 8,

$$\tan\alpha = \frac{\text{ordinate}}{\text{abscissa}} = \frac{\dfrac{\text{ordinate}}{\text{distance}}}{\dfrac{\text{abscissa}}{\text{distance}}} = \frac{\sin\alpha}{\cos\alpha},$$

$$\cot\alpha = \frac{1}{\tan\alpha} = \frac{1}{\dfrac{\sin\alpha}{\cos\alpha}} = \frac{\cos\alpha}{\sin\alpha}.$$

These relations may be proved otherwise, thus, Fig. 5:

$$\tan XOP = \frac{MP}{OM} = \frac{\dfrac{MP}{OP}}{\dfrac{OM}{OP}} = \frac{\sin XOP}{\cos XOP},$$

$$\cot XOP = \frac{OM}{MP} = \frac{\frac{OM}{OP}}{\frac{MP}{OP}} = \frac{\cos XOP}{\sin XOP}.$$

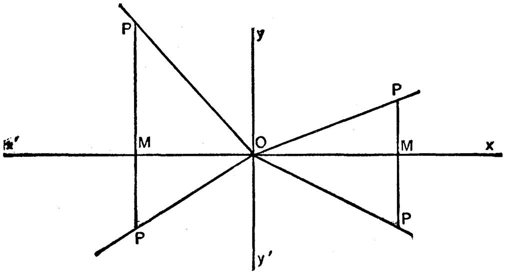

FIG. 5.

10. Sine and Cosine. Also, Fig. 5, it is obvious that

$$\overline{MP}^2 + \overline{OM}^2 = \overline{OP}^2.$$

Dividing each term by $\overline{OP}^2$ gives

$$\left(\frac{MP}{OP}\right)^2 + \left(\frac{OM}{OP}\right)^2 = 1.$$

Whence, by definition,

$$(\sin XOP)^2 + (\cos XOP)^2 = 1$$

or as it is usually written, letting $\alpha = \angle XOP$,

$$\sin^2 \alpha + \cos^2 \alpha = 1.$$

11. Tangent and Secant. Similarly, writing the first equation of Art. 10 in the form

$$\overline{OP}^2 = \overline{MP}^2 + \overline{OM}^2$$

and dividing each term by $\overline{OM}^2$, we have

$$\left(\frac{OP}{OM}\right)^2 = \left(\frac{MP}{OM}\right)^2 + 1.$$

That is $$(\sec XOP)^2 = (\tan XOP)^2 + 1$$

or, $$\sec^2 \alpha = \tan^2 \alpha + 1.$$

In the same way we obtain the relation

$$\csc^2 \alpha = \cot^2 \alpha + 1.$$

12. Fundamental Relations. These relations, summarized below, are of great importance and must be memorized.

$$\sin \alpha = \frac{1}{\csc \alpha}, \quad \cos \alpha = \frac{1}{\sec \alpha}, \quad \tan \alpha = \frac{1}{\cot \alpha}, \tag{1}$$

$$\csc \alpha = \frac{1}{\sin \alpha}, \quad \sec \alpha = \frac{1}{\cos \alpha}, \quad \cot \alpha = \frac{1}{\tan \alpha}.$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}, \quad \cot \alpha = \frac{\cos \alpha}{\sin \alpha}. \tag{2}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1. \tag{3}$$

$$\sec^2 \alpha = 1 + \tan^2 \alpha, \qquad \csc^2 \alpha = 1 + \cot^2 \alpha. \tag{4}$$

13. By means of the identities of Art. 12 the value of any one of the trigonometric functions may be expressed in terms of each of the other five. Thus, by (3)

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha}.$$

By (2), (1), and (4),

$$\sin \alpha = \tan \alpha \cdot \cos \alpha = \frac{\tan \alpha}{\sec \alpha} = \frac{\tan \alpha}{\sqrt{1 + \tan^2 \alpha}},$$

where the radical may be either plus or minus.

14. To Compute the Values of the Other Functions when One Function of an Angle is Given. By means of the relations of the preceding article if the value of any one function of an angle be given, the values of the remaining functions may be found, but a simpler method of obtaining them is illustrated by the following examples.

Example 1. Given $\sin A = -\frac{2}{3}$, find the values of the remaining functions.

$$\sin A = \frac{\text{ord.}}{\text{dist.}} = -\frac{m \cdot 2}{m \cdot 3} = -\frac{2}{3} = \frac{-2}{3}.$$

The distance being always positive, the minus sign necessarily is taken with the ordinate. Therefore, Fig. 6, construct an angle whose ordinate is -2 and whose distance is 3, or any multiple (m) of -2 and 3. The third side of the right triangle is $\pm\sqrt{9-4} = \pm\sqrt{5}$. This is the value of the abscissa and we may write the values of the six functions from the definitions.

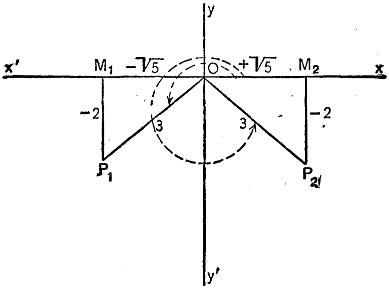

FIG. 6.

$$\sin XOP_1 = -\frac{2}{3}, \qquad \sin XOP_2 = -\frac{2}{3},$$

$$\cos XOP_1 = -\frac{\sqrt{5}}{3}, \qquad \cos XOP_2 = \frac{\sqrt{5}}{3},$$

$$\tan XOP_1 = \frac{2}{\sqrt{5}}, \qquad \tan XOP_2 = -\frac{2}{\sqrt{5}},$$

$$\cot XOP_1 = \frac{\sqrt{5}}{2}, \qquad \cot XOP_2 = -\frac{\sqrt{5}}{2},$$

$$\sec XOP_1 = -\frac{3}{\sqrt{5}}, \qquad \sec XOP_2 = \frac{3}{\sqrt{5}},$$

$$\csc XOP_1 = -\frac{3}{2}. \qquad \csc XOP_2 = -\frac{3}{2}.$$

Example 2. Given $\cot A = -\frac{5}{4}$, find the remaining functions.

$$\cot A = \frac{\text{abs.}}{\text{ord.}} = -\frac{m \cdot 5}{m \cdot 4} = -\frac{5}{4} = \frac{-5}{4} \text{ or } \frac{5}{-4},$$

since either the abscissa or the ordinate may be negative.

Construct an angle XOP_1, Fig. 7, having an abscissa -5 and an ordinate $+4$, and an angle XOP_2 having an abscissa $+5$ and an ordinate -4. In each case the distance is found to be $\sqrt{25+16} = \sqrt{41}$, and we may write

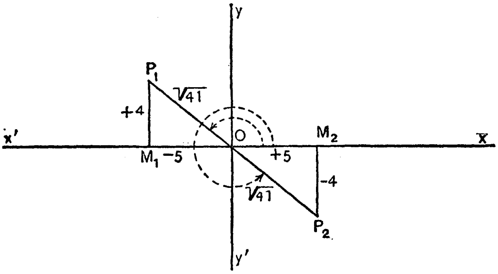

FIG. 7.

$$\sin XOP_1 = \frac{4}{\sqrt{41}}, \qquad \sin XOP_2 = -\frac{4}{\sqrt{41}},$$

$$\cos XOP_1 = -\frac{5}{\sqrt{41}}, \qquad \cos XOP_2 = \frac{5}{\sqrt{41}},$$

$$\tan XOP_1 = -\frac{4}{5}, \qquad \tan XOP_2 = -\frac{4}{5},$$

$$\cot XOP_1 = -\frac{5}{4}, \qquad \cot XOP_2 = -\frac{5}{4},$$

$$\sec XOP_1 = -\frac{\sqrt{41}}{5}, \qquad \sec XOP_2 = \frac{\sqrt{41}}{5},$$

$$\csc XOP_1 = \frac{\sqrt{41}}{4}. \qquad \csc XOP_2 = -\frac{\sqrt{41}}{4}.$$

It will be seen that the ambiguity of the two sets of values will occur in every case, no matter what function be given and no matter whether the sign of the given function be plus or minus.

EXAMPLES

Find the values of the remaining functions, given that

1. $\sin\alpha = \frac{1}{2}$.
2. $\cos\alpha = -\frac{3}{8}$.
3. $\cot\alpha = -3$.
4. $\tan\alpha = \frac{9}{4}$.
5. $\sec\alpha = 4$.
6. $\csc\alpha = -\frac{25}{7}$.

7. If $\sin x = 5$, can the values of the remaining functions be found? Why?

8. If $\sec x = \frac{1}{2}$, can the values of the remaining functions be found? Why?

9. If $\tan x = -4$, can the values of the remaining functions be found? Why?

10. Given $\sec\alpha = \frac{5}{4}$, find the functions of $90° - \alpha$.

11. Given $\cot\alpha = x$, find the functions of $90° - \alpha$.

Prove the following relations:

12. $\cos\alpha = \dfrac{\cot\alpha}{\sqrt{1+\cot^2\alpha}}$.

13. $\cot\alpha = \dfrac{1}{\sqrt{\sec^2\alpha - 1}}$.

14. $\sec\alpha = \dfrac{\csc\alpha}{\sqrt{\csc^2\alpha - 1}}$.

15. $\sin\alpha = \dfrac{1}{\sqrt{1+\cot^2\alpha}}$.

16. $\tan\alpha = \dfrac{\sqrt{1-\cos^2\alpha}}{\cos\alpha}$.

17. $\tan\alpha = \dfrac{\sin\alpha}{\sqrt{1-\sin^2\alpha}}$.

18. $\cot\alpha = \dfrac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}$.

19. $\csc\alpha = \dfrac{\sqrt{1+\tan^2\alpha}}{\tan\alpha}$.

20. $\dfrac{\cos\alpha}{1-\sin\alpha} = \dfrac{1+\sin\alpha}{\cos\alpha}$.

21. $\dfrac{\tan\alpha}{\sec\alpha} = \sin\alpha$.

22. $\dfrac{\sec^2\alpha}{\sin^2\alpha} = \csc^2\alpha + \sec^2\alpha$.

23. $\dfrac{\cot\alpha}{\csc\alpha} = \cos\alpha$.

24. $\tan\alpha \cdot \csc\alpha = \sec\alpha$.

25. $\cos\alpha \cdot \sec\alpha = \dfrac{1}{\cos\alpha \cdot \sec\alpha}$.

26. $\sec^2\alpha - \csc^2\alpha = \tan^2\alpha - \cot^2\alpha$.

CHAPTER II

IDENTICAL RELATIONS AMONG THE FUNCTIONS OF RELATED ANGLES. THE VALUES OF THE FUNCTIONS OF CERTAIN ANGLES

15. Functions of Negative Angles. We shall now proceed to determine the relations which exist among the functions of two angles when those angles are related in some particular way. Let us consider first two angles one of which is the negative of the other, Fig. 8. Let the value of the positive angle XOP be α, and of the numerically equal negative angle XOP' be $-\alpha$. On the terminal sides of these angles lay off the equal distances OP and OP', and drop perpendiculars from P and P' upon the axis of x. These perpendiculars will obviously cut the axis of x in

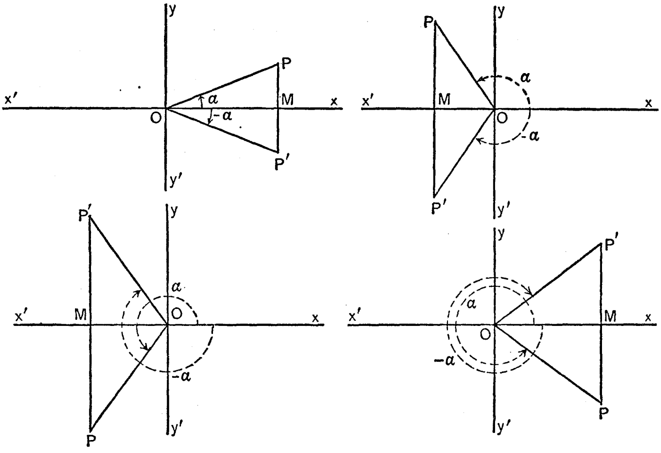

Fig. 8.

the same point, M, and the two right triangles MOP and MOP' will be congruent. Therefore, $OP' = OP$, $OM = OM$, and $MP' = -MP$. We then have

$$
\begin{aligned}
\sin(-\alpha) &= \frac{MP'}{OP'} = \frac{-MP}{OP} = -\sin\alpha, \\
\cos(-\alpha) &= \frac{OM}{OP'} = \frac{OM}{OP} = \cos\alpha, \\
\tan(-\alpha) &= \frac{MP'}{OM} = \frac{-MP}{OM} = -\tan\alpha, \\
\cot(-\alpha) &= \frac{OM}{MP'} = \frac{OM}{-MP} = -\cot\alpha, \\
\sec(-\alpha) &= \frac{OP'}{OM} = \frac{OP}{OM} = \sec\alpha, \\
\csc(-\alpha) &= \frac{OP'}{MP'} = \frac{OP}{-MP} = -\csc\alpha.^*
\end{aligned}
\tag{5}
$$

Thus any function of a negative angle is equal, numerically, to the same function of an equal positive angle. The algebraic sign is determined by the quadrant which $-\alpha$ lies in when α is acute.

16. Functions of 90° − α. Consider next two angles, α and $90° - \alpha$, Fig. 9. Let XOP be the angle α and XOP' be $90° - \alpha$. Lay off on the terminal sides of these angles the equal distances OP and OP', and from P and P' drop perpendiculars PM and $P'M'$ upon the axis of x. Then obviously the right triangles MOP and $M'OP'$ are congruent, and $OP' = OP$, $M'P' = OM$, and $OM' = MP$. Therefore,

$$
\begin{aligned}
\sin(90° - \alpha) &= \frac{M'P'}{OP'} = \frac{OM}{OP} = \cos\alpha, \\
\cos(90° - \alpha) &= \frac{OM'}{OP'} = \frac{MP}{OP} = \sin\alpha,
\end{aligned}
$$

---

* By virtue of Art. 12, it is necessary to memorize only sin $(-\alpha)$ and cos $(-\alpha)$.

$$\tan(90° - \alpha) = \frac{M'P'}{OM'} = \frac{OM}{MP} = \cot\alpha,$$

$$\cot(90° - \alpha) = \frac{OM'}{M'P'} = \frac{MP}{OM} = \tan\alpha,$$

$$\sec(90° - \alpha) = \frac{OP'}{OM'} = \frac{OP}{MP} = \csc\alpha,$$

$$\csc(90° - \alpha) = \frac{OP'}{M'P'} = \frac{OP}{OM} = \sec\alpha.$$

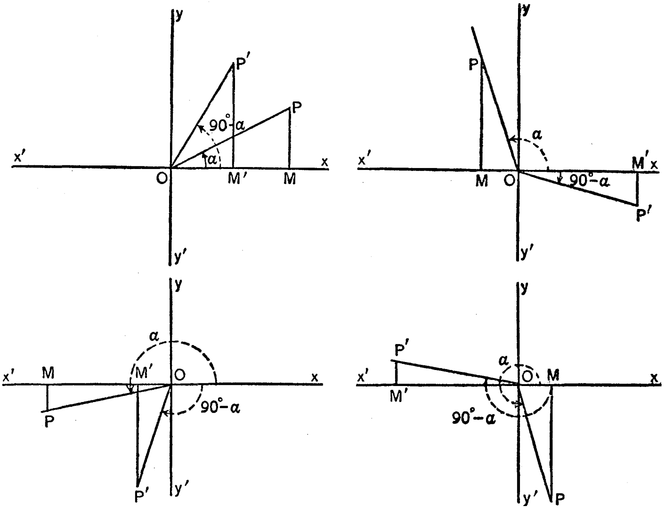

FIG. 9.

Thus we see that each of the functions of 90° − α is equal, numerically, to the co-function of the angle α. The important case is when α is acute, and α and 90° − α are complementary angles. Indeed it was because of this relation that the cosine, cotangent, and cosecant received their names. They are the sine, tangent, and secant of the complementary angle.

17. Functions of 90° + α. We shall consider two more cases, limiting the discussion to acute values of α, although the results will be equally true for any value of α whatever. In Fig. 10 let the angle XOP be α and XOP' be $90° + \alpha$. On the terminal sides of these angles lay off the equal distances OP and OP', and from P and P' drop perpendiculars PM and $P'M'$ upon the axis of x. It follows that the two triangles MOP and $M'OP'$ are congruent and that $OP' = OP$, $M'P' = OM$, $OM' = -MP$. Therefore,

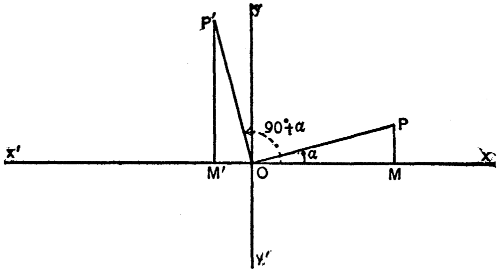

FIG. 10.

$$\sin (90° + \alpha) = \frac{M'P'}{OP'} = \frac{OM}{OP} = \cos \alpha,$$

$$\cos (90° + \alpha) = \frac{OM'}{OP'} = \frac{-MP}{OP} = -\sin \alpha,$$

$$\tan (90° + \alpha) = \frac{M'P'}{OM'} = \frac{OM}{-MP} = -\cot \alpha,$$

$$\cot (90° + \alpha) = \frac{OM'}{M'P'} = \frac{-MP}{OM} = -\tan \alpha,$$

$$\sec (90° + \alpha) = \frac{OP'}{OM'} = \frac{OP}{-MP} = -\csc \alpha,$$

$$\csc (90° + \alpha) = \frac{OP'}{M'P'} = \frac{OP}{OM} = \sec \alpha.$$

18. Functions of 180° + α. In Fig. 11 let the angle XOP be α and XOP' be $180° + \alpha$. On the terminal sides of these angles lay off the equal distances OP and OP', and from P and P' drop perpendiculars PM and $P'M'$ upon the axis

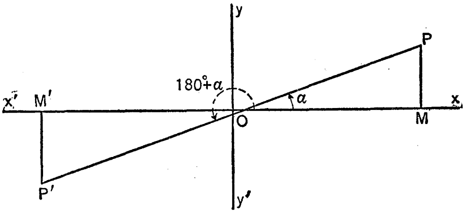

FIG. 11.

of X. Then the triangles MOP and $M'OP'$ are congruent and $OP' = OP$, $M'P' = -MP$, $OM' = -OM$. Therefore,

$$\sin(180° + \alpha) = \frac{M'P'}{OP'} = \frac{-MP}{OP} = -\sin\alpha,$$

$$\cos(180° + \alpha) = \frac{OM'}{OP'} = \frac{-OM}{OP} = -\cos\alpha,$$

$$\tan(180° + \alpha) = \frac{M'P'}{OM'} = \frac{-MP}{-OM} = \tan\alpha,$$

$$\cot(180° + \alpha) = \frac{OM'}{M'P'} = \frac{-OM}{-MP} = \cot\alpha,$$

$$\sec(180° + \alpha) = \frac{OP'}{OM'} = \frac{OP}{-OM} = -\sec\alpha,$$

$$\csc(180° + \alpha) = \frac{OP'}{M'P'} = \frac{OP}{-MP} = -\csc\alpha.$$

19. Generalization. In a similar manner may be found analogous relations connecting the functions of an angle α with the functions of any integral multiple of 90° plus or minus α. Upon examining these relations we are led, by induction, to express them in the following general rule.

Any function of an **even** * *multiple of 90° plus or minus* α *is the* **same** *function of the angle* α.

---

* Zero is taken as an even number, so that the rule includes the case of Art. 15.

Any function of an **odd** *multiple of 90° plus or minus α is the* **co-function** *of the angle α.*

The algebraic sign of the value is determined by the quadrant (counting in the positive direction) in which the terminal side of the angle lies when α is acute.

Examples.

1. $\sin(720° - \alpha) = -\sin\alpha$, since 720° is an even multiple of 90° and the terminal side of $720° - \alpha$, when α is acute, lies in the fourth quadrant.

2. $\cot(-90° - \alpha) = \tan\alpha$, since $-90°$ is an odd multiple of 90° and the terminal side of $-90° - \alpha$, when α is acute, lies in the third quadrant.

3. $\sec(-180° + \alpha) = -\sec\alpha$, since $-180°$ is an even multiple of 90° and the terminal side of $-180° + \alpha$, α acute, lies in the third quadrant.

4. $\tan 281° = \tan(270° + 11°) = -\cot 11°$, or
$\tan 281° = \tan(360° - 79°) = -\tan 79°$,

since $\begin{cases} 270° \\ 360° \end{cases}$ is an $\begin{cases} \text{odd} \\ \text{even} \end{cases}$ multiple of 90° and the terminal side of 281° lies in the fourth quadrant.

EXAMPLES

By means of a geometrical construction express each of the following as a function of α, where α is an acute angle. Check your results by the rule given above.

1. $\cos(270° + \alpha)$.
2. $\sin(180° - \alpha)$.
3. $\csc(-90° + \alpha)$.
4. $\tan(540° + \alpha)$.
5. $\cot(270° - \alpha)$.
6. $\sec(270° - \alpha)$.
7. $\sin(-180° - \alpha)$.
8. $\cos(-270° + \alpha)$.

Express as a function of an acute angle

9. $\sin 324°$.
10. $\cos(-375°)$.
11. $\tan 457°$.
12. $\csc(-801° 32')$.
13. $\sec(-537°)$.
14. $\cot 1140°$.
15. $\tan 495°$.
16. $\cos(-480°)$.

20. Functions of Certain Angles. We see by the preceding article that the functions of angles greater than 90°, and of negative angles, can be expressed in terms of the functions of angles lying between 0° and 90°. It follows that if we wish to use the trigonometric functions for computation or for other purposes we need find their values only for all positive acute angles. We shall not discuss the methods by means of which these values are computed in general, but shall proceed to find the values of the functions of certain angles which frequently occur. We shall then, in the following chapter, show how we may find the values of the functions of any angle from tables with which we are provided. We shall see, also, how the values thus found may be used in the solution of triangles; that is, in finding the unknown parts, angles or sides, of a triangle from parts which are given.

21. Functions of 30° and 60°. Let the angle *XOP*, Fig. 12, be an angle of 30°, and from *P* drop a perpendicular, *PM*, upon the axis of x. Then, as we know, the angle *OPM* is 60°, and if *OP* have the value a, *MP* must be equal to $\frac{a}{2}$ and *OM* equal to $\frac{a\sqrt{3}}{2}$. Therefore, by definition,

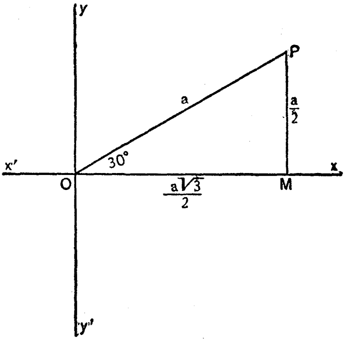

FIG. 12.

$$\sin 30^\circ = \frac{MP}{OP} = \frac{\frac{a}{2}}{a} = \frac{1}{2}, \qquad \cos 30^\circ = \frac{OM}{OP} = \frac{\frac{a\sqrt{3}}{2}}{a} = \frac{\sqrt{3}}{2},$$

$$\tan 30^\circ = \frac{MP}{OM} = \frac{\frac{a}{2}}{\frac{a\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}, \qquad \cot 30^\circ = \frac{OM}{MP} = \frac{\frac{a\sqrt{3}}{2}}{\frac{a}{2}} = \sqrt{3},$$

$$\sec 30^\circ = \frac{OP}{OM} = \frac{a}{\frac{a\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}, \qquad \csc 30^\circ = \frac{OP}{MP} = \frac{a}{\frac{a}{2}} = 2.$$

By a similar construction, or by the relations of Art. 16, the following values may be derived:

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \qquad \tan 60^\circ = \sqrt{3}, \qquad \sec 60^\circ = 2,$$

$$\cos 60^\circ = \frac{1}{2}, \qquad \cot 60^\circ = \frac{1}{\sqrt{3}}, \qquad \csc 60^\circ = \frac{2}{\sqrt{3}}.$$

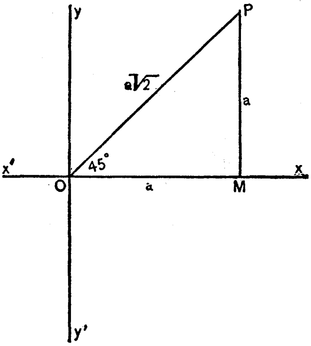

Fig. 13.

22. Functions of 45°. Let the angle XOP, Fig. 13, be an angle of 45°, and from P drop a perpendicular, PM, upon the axis of x. Then the angle OPM is an angle of 45°, and if OM have the value a, MP also will be equal to a and OP will be $a\sqrt{2}$. Therefore, by definition,

$$\sin 45° = \frac{MP}{OP} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}, \qquad \cos 45° = \frac{OM}{OP} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}},$$

$$\tan 45° = \frac{MP}{OM} = \frac{a}{a} = 1, \qquad \cot 45° = \frac{OM}{MP} = \frac{a}{a} = 1,$$

$$\sec 45° = \frac{OP}{OM} = \frac{a\sqrt{2}}{a} = \sqrt{2}, \qquad \csc 45° = \frac{OP}{MP} = \frac{a\sqrt{2}}{a} = \sqrt{2}.$$

23. Functions of Other Angles Readily Found. By similar constructions the functions of 120°, 150°, 135°, etc., or, in general, any integral multiple of 90° plus or minus 30°, 60°, or 45°, may be found. They may be found more readily, however, by using the rule given in Art. 19. Thus,

$$\sin 120° = \sin\,(90° + 30°) = \cos 30° = \frac{\sqrt{3}}{2}$$

or

$$\sin 120° = \sin\,(180° - 60°) = \sin 60° = \frac{\sqrt{3}}{2}.$$

EXAMPLES

Find the values of the functions of

1. 120°.
2. 135°.
3. 150°.
4. 210°.
5. 225°.
6. 240°.
7. 300°.
8. 315°.
9. 330°.

Prove that

10. $\sin 210° \tan 300° = \sin 120°$.
11. $\sec 315° \sec 300° = \sec 240° \sec 225°$.
12. $\tan 210° : \cos 150° = \tan 150° : \cos 330°$.
13. $\csc 330° \sec 315° \sin 225° = -\sec 120°$.

24. Functions of Zero. Let the value of the angle *XOP*, Fig. 14, be represented by α, and from *P*, any point in the terminal side of the angle, drop a perpendicular, *PM*, upon the axis of x. By definition,

$$\sin \alpha = \frac{MP}{OP} \text{ and } \cos \alpha = \frac{OM}{OP}.$$

Now, for the sake of convenience keeping the distance OP constant in length, let the line OP approach nearer and nearer to the position OX. Then the angle α can be made,* smaller than any angle that may be assigned, however small, or, as it is otherwise expressed, α will approach the limit zero. At the same time MP will approach zero as a limit, and OM will approach OP as a limit. Then $\frac{MP}{OP}$ will approach the limit zero and $\frac{OM}{OP}$ will approach the limit unity. Thus, as the angle approaches the limit zero (or, becomes smaller than any value that may be assigned, however small) its sine approaches the limit zero (or, becomes smaller than any value that may be assigned, however small) and its cosine approaches the limit unity (or, differs from unity by a number smaller than any number that may be assigned, however small). This may be written

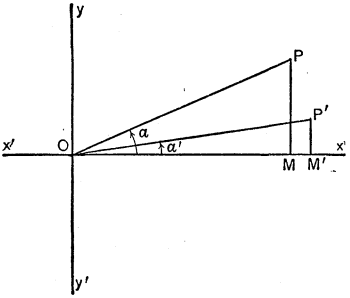

Fig. 14.

$$\underset{\alpha \doteq 0}{\text{limit}} \sin \alpha = 0, \qquad \underset{\alpha \doteq 0}{\text{limit}} \cos \alpha = 1.$$

Again, by definition, $\csc \alpha = \frac{OP}{MP}$, and as α grows smaller OP remains constant and MP grows smaller, so that $\frac{OP}{MP}$ becomes continually greater. Finally, when α approaches zero as a limit, MP becomes smaller than any number that

* And will remain.

may be assigned, however small, and $\frac{OP}{MP}$ becomes greater than any number that may be assigned, however great. This we express by saying that $\frac{OP}{MP}$ approaches the limit infinity, or increases without limit. We may then write

$$\underset{\alpha \doteq 0}{\text{limit}} \csc \alpha = \infty.$$

Similarly it may be shown that

$$\underset{\alpha \doteq 0}{\text{limit}} \tan \alpha = 0, \qquad \underset{\alpha \doteq 0}{\text{limit}} \cot \alpha = \infty, \qquad \underset{\alpha \doteq 0}{\text{limit}} \sec \alpha = 1.$$

These relations are often briefly expressed,

$$\begin{array}{lll} \sin 0^\circ = 0, & \tan 0^\circ = 0, & \sec 0^\circ = 1, \\ \cos 0^\circ = 1, & \cot 0^\circ = \infty, & \csc 0^\circ = \infty. \end{array} \qquad (6)$$

to which there is no objection if we remember that these are merely abbreviations of the preceding statements, and that 0° means, not that we have no angle, but that we are dealing with an angle which becomes smaller than any value that may be assigned, however small; and that when this happens the sine of the angle also becomes smaller than any value that may be assigned, however small, the cotangent

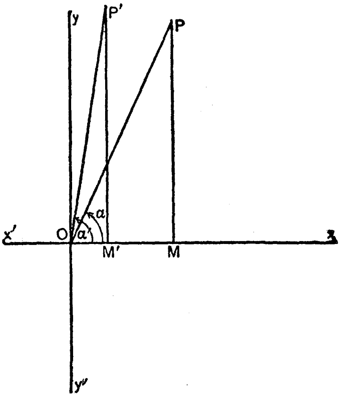

FIG. 15.

becomes greater than any value that may be assigned, however great, the cosine approaches the limit unity, etc.

25. Functions of 90°. Let the angle XOP, Fig. 15, be represented by α, and let OM, MP, and OP be respectively the abscissa, ordinate, and distance of P. Also, keeping the distance OP constant, let the line OP approach OY as its limiting position. Then,

$$\begin{array}{r}\alpha \text{ approaches the limit } 90^\circ,\\ OM \text{ approaches the limit zero,}\\ MP \text{ approaches the limit } OP.\end{array}$$

Therefore,

$$\lim_{\alpha \doteq 90^\circ} \sin\alpha = \text{limit}\ \frac{MP}{OP} = 1,$$

$$\lim_{\alpha \doteq 90^\circ} \cos\alpha = \text{limit}\ \frac{OM}{OP} = 0,$$

$$\lim_{\alpha \doteq 90^\circ} \tan\alpha = \text{limit}\ \frac{MP}{OM} = \infty,$$

$$\lim_{\alpha \doteq 90^\circ} \cot\alpha = \text{limit}\ \frac{OM}{MP} = 0,$$

$$\lim_{\alpha \doteq 90^\circ} \sec\alpha = \text{limit}\ \frac{OP}{OM} = \infty,$$

$$\lim_{\alpha \doteq 90^\circ} \csc\alpha = \text{limit}\ \frac{OP}{MP} = 1.$$

With the same understanding as in the preceding article these may be written

$$\begin{array}{lll}\sin 90^\circ = 1, & \tan 90^\circ = \infty, & \sec 90^\circ = \infty,\\ \cos 90^\circ = 0, & \cot 90^\circ = 0, & \csc 90^\circ = 1.\end{array} \tag{7}$$

26. The student should find, as in Arts. 24 and 25, the following:

$$\begin{array}{lll}\sin 180^\circ = 0, & \tan 180^\circ = 0, & \sec 180^\circ = -1,\\ \cos 180^\circ = -1, & \cot 180^\circ = \infty, & \csc 180^\circ = \infty.\end{array} \tag{8}$$

$$\begin{array}{lll}\sin 270^\circ = -1, & \tan 270^\circ = \infty, & \sec 270^\circ = \infty,\\ \cos 270^\circ = 0, & \cot 270^\circ = 0, & \csc 270^\circ = -1.\end{array} \tag{9}$$

27. Limiting Values of the Functions. We have seen (Art. 20) that all possible numerical values of the trigonometric functions are given by angles lying between 0° and 90°. Let us now see between what limits the values of the functions lie. From the discussion and figures of articles 24 to 26 we see that

the *sine* and *cosine* of an angle *lie between* -1 and $+1$,
the *tangent* and *cotangent* lie *between* $-\infty$ and $+\infty$,
the *secant* and *cosecant* lie *between* 1 and ∞ or *between* -1 and $-\infty$.

It is well to note also, for angles in the first quadrant, that as the angle increases the direct functions increase, the co-functions decrease.

A very convenient and simple way to remember the range of values and the signs of the trigonometric functions is by means of the unit circle, a circle with unit radius, which need not be actually drawn but merely visualized. Draw such a circle, Fig. 16, with its center at the origin of coordinates, and let XOP be any angle. Drop the perpendicular PM upon the axis of X, and draw LQ tangent to the circle at L and meeting OP produced in Q. Then, by definition,

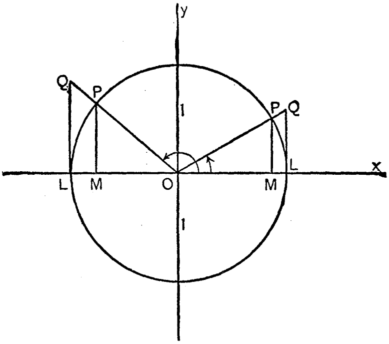

FIG. 16.

$$\sin XOP = \frac{MP}{OP} = \frac{MP}{1} = MP,$$

$$\cos XOP = \frac{OM}{OP} = \frac{OM}{1} = OM,$$

$$\tan XOP = \frac{LQ}{OL} = \frac{LQ}{\pm 1} = \pm LQ, \text{ etc.}$$

If now the line OP be pictured as revolving from the position OL, the sine of the angle XOP, namely MP, will be seen to increase from zero and approach unity as the angle approaches 90°. The cosine, namely OM, decreases from unity to zero, and the tangent (LQ) increases without limit. Also, as the angle increases beyond 90°, the directions of the lines MP and OM indicate the signs of the sine and cosine. The other functions follow directly from these two by virtue of the relations of Art. 12.

CHAPTER III

THE SOLUTION OF RIGHT TRIANGLES. LOGARITHMS AND COMPUTATION BY MEANS OF LOGARITHMS

28. Solution of Right Triangles. With the definitions of the trigonometric functions and tables giving their numerical values we are now prepared to solve right triangles; that is, to find the values of the unknown parts from those that are known. Two parts in addition to the right angle must be known, and one at least of these parts must be a side. We have then the general rule of procedure: *Select that trigonometric function which involves the two known parts and one unknown part.* The value of the unknown part can then be computed by elementary algebraic processes.

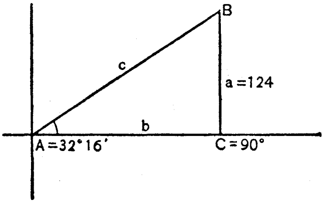

FIG. 17.

Example 1. Given $A = 32° \; 16'$, $a = 124$, $C = 90°$, find B, b, and c. See Fig. 17.

Obviously $B = 90° - A = 90° - 32° \; 16' = 57° \; 44'$. Then

$$\cot A = \frac{b}{a}, \qquad \sin A = \frac{a}{c},$$

or

$$b = a \cot A. \qquad c = \frac{a}{\sin A}.$$

From the tables we find

$$\cot A = 1.5839. \qquad \sin A = .5338.$$

Therefore,

$$b = 124 \times 1.5839 \qquad c = \frac{124}{.5338}$$

$$= 196.4. \qquad = 232.3.$$

Example 2. Given $a = 50$, $b = 60$, $C = 90°$, find A, B, and c.

In this case,

$$\tan A = \frac{a}{b} = \frac{50}{60} = .8333.$$

$$A = 39° \, 48'.$$

$$B = 90° - A = 50° \, 12'.$$

To find c we may use either

$$\sin A = \frac{a}{c} \qquad \text{or} \qquad c^2 = a^2 + b^2$$

$$c = \frac{a}{\sin A} \qquad c = \sqrt{a^2 + b^2}$$

$$= \frac{50}{.6402} \qquad = \sqrt{2500 + 3600}$$

$$= 78.1. \qquad = 78.1.$$

EXAMPLES

Solve the following right triangles:

1. $a = 250$, $A = 36° \, 22'$.
2. $a = 37.5$, $b = 40.1$.
3. $a = .55$, $c = .70$.
4. $B = 72° \, 6'$, $c = 502$.
5. $A = 59° \, 58'$, $b = 412$.
6. $B = 24° \, 33'$, $a = 211$.

PROBLEMS

7. What is the height of a flagpole if at a horizontal distance of 200 feet from the foot of the pole the angle of elevation of its top is 19° 28′?

8. A rope is stretched taut from the top of a building to the ground, and is found to make an angle of 58° 56′ with the horizontal. If the building is 61 feet high how long is the rope?

9. If a tree 74.3 feet high casts a shadow 42.6 feet long, how many degrees above the horizon is the sun?

10. A man walking on level ground finds, at a certain point, that the angle of elevation of the top of a tower is 30°. He walks directly toward the tower for a distance of 300 feet and then finds the angle of elevation of the top to be 60°. What is the height of the tower?

11. At a point, A, south of a tower the angle of elevation of the top of the tower is 60°. At another point 300 feet east of A the angle of elevation is 30°. What is the height of the tower?

12. The angles of a right triangle are 42° and 48°; the hypotenuse is 200 feet. What is the length of the perpendicular from the right angle to the hypotenuse?

13. The height of a gable roof is 20 feet, its width 42 feet. What is the pitch of the roof; that is, the angle it makes with the horizontal?

14. From where I stand a tree 50 feet away has an angle of elevation of 43° 31′. From the same point another tree, 75 feet distant, has an angle of elevation of 32° 20′. Which tree is the taller and by how much?

29. Logarithms. The solution of right triangles as thus explained is simple in theory but may become laborious in practice because of the arithmetic computation involved. Fortunately we have in logarithms a device for simplifying such computation. The base of a system of logarithms is, in general, any arbitrarily chosen number.* In practice two systems are used: the Briggsian or common system of which the base is 10; and the Napierian system of which the base is $e = 2.718 \cdots$. The logarithm of a number to a given base (a) is the exponent of the power to which the base (a) must be raised to produce the number. Thus, if $a^x = m$, then x is the logarithm of m to the base a; written $x = \log_a m$.

The word power is used here in its broader sense to include fractional and negative exponents. Defining fractional and negative exponents in such a way that the laws of exponents — $a^m a^n = a^{m+n}$; $(a^m)^n = a^{mn}$ — hold for negative numbers and fractions as well as for positive integers, values of x may be found to satisfy, approximately at least, such an equation as $a^x = b$, no matter what values a and b may have. Thus, given any number, a, by raising it to a suitable power, p, and extracting a suitable root, q, of the result, we can obtain any other number, b; that is, $\sqrt[q]{a^p} = b$. But this may be written $a^{\frac{p}{q}} = b$ or $a^x = b$, where $x = \frac{p}{q}$, the division of p by q being carried out to any desired number of decimal places. We then call x the logarithm of b to the base a.

---

* Some numbers, unity, for example, cannot be so used.

30. The Common System. For purposes of computation the common system, base 10, is used. Let us form a table of powers of 10 and express the relations in terms of logarithms.

$$\begin{array}{lll}
10^{-3} = .001, & \text{or} & \log_{10} .001 = -3. \\
10^{-2} = .01, & & \log_{10} .01 = -2. \\
10^{-1} = .1, & & \log_{10} .1 = -1. \\
10^{0} = 1, & & \log_{10} 1 = 0. \\
10^{1} = 10, & & \log_{10} 10 = 1. \\
10^{2} = 100, & & \log_{10} 100 = 2. \\
10^{3} = 1000, & & \log_{10} 1000 = 3,^{*} \\
\quad \text{etc.} & & \quad \text{etc.}
\end{array}$$

This table could be extended indefinitely in either direction. If we examine the table we notice that to produce a number between 1 and 10 we must raise the base 10 to a positive power between 0 and 1; to produce a number between 10 and 100, the exponent of the base must lie between 1 and 2; for a number between 100 and 1000, the exponent must lie between 2 and 3, and so on. In other words, the logarithm of a number between 1 and 10 lies between 0 and 1, and is, therefore, a fraction, always expressed as a decimal. The logarithm of a number between 10 and 100 lies between 1 and 2, or is 1 plus a decimal. The logarithm of a number between 100 and 1000 is 2 plus a decimal. The logarithm of a number is thus seen to consist, in general, of two parts, an integral part and a decimal part. The *integral part* is called the *characteristic* of the logarithm; the *decimal part* is called the *mantissa.* The results of our observations may be summarized thus:

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* Hereafter in this work we shall not write the base 10. Thus log 7 means $\log_{10} 7$. In general, however, except in works on trigonometry, if no base is written, $e = 2.718 \cdots$ is understood.

Number Between	Contains	Characteristic of Logarithm
1 and 10	1 integral digit	0
10 and 100	2 integral digits	1
100 and 1000	3 integral digits	2
. . . . . .	. . . .	. . .

Whence we formulate the law: *The characteristic of the logarithm of a number is one less than the number of digits in the integral part of the number.*

On the other hand, we observe from the table of this article that if a number contain no integral digits, that is, if it be purely decimal, its logarithm is negative. The characteristic in this case can be got by counting the number of zeros before the first significant figure, prefixing the minus sign. It is usual, and better, however, except for special purposes, not to write the characteristic of the logarithm of a decimal number in the form just stated, for reasons which will now be pointed out.

31. The Mantissa. In the common system the *mantissa* of the logarithm of a number can be made to depend only upon the sequence of digits in the number, and be independent of the position of the decimal point. Let us assume that we know the logarithm of 1.285 to be 0.1089. It follows, multiplying successively by ten, that

$$\begin{array}{llll}
10^{0.1089} = 1.285, & \text{or} & \log 1.285 & = 0.1089. \\
10^{1.1089} = 12.85, & & \log 12.85 & = 1.1089. \\
10^{2.1089} = 128.5, & & \log 128.5 & = 2.1089. \\
10^{3.1089} = 1285, & & \log 1285 & = 3.1089. \\
10^{4.1089} = 12850, & & \log 12850 & = 4.1089. \\
\cdots\cdots & & \cdots\cdots &
\end{array}$$

which verifies the law we have stated. If, however, we divide $10^{0.1089}$ successively by 10 we find

$$\begin{array}{lll} 10^{0.1089-1} = 10^{-0.8911} = .1285, & \text{or} & \log .1285 = -0.8911. \\ 10^{-1.8911} = .01285, & & \log .01285 = -1.8911. \\ 10^{-2.8911} = .001285, & & \log .001285 = -2.8911. \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot & & \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \end{array}$$

This is the true form of the logarithm of a purely decimal number, and for certain purposes this is the form which must be used.*

It is obvious from the preceding discussion that the mantissa corresponding to a given sequence of digits remains the same as long as the sequence contains one or more integral digits, but that as soon as the sequence is a purely decimal number the mantissa changes. To obviate this difficulty and to keep the mantissa the same for a given sequence of digits regardless of the position of the decimal point, we note that the number -0.8911 may be written, without change of value, in the form $9.1089-10$. We have added 10 and subtracted 10, and have therefore left the value unchanged. We may then say

$$\log .1285 = -0.8911 = 9.1089 - 10,$$

and if we agree to use the latter form † we see that the mantissa of the logarithm of .1285 (that is, 1089) is the same as the mantissa of the logarithm of the sequence 1285 when it contains integral digits. We may now write

$$\begin{array}{ll} \log 1.285 = 0.1089 & \log .1285 = 9.1089 - 10 \\ \log 12.85 = 1.1089 & \log .01285 = 8.1089 - 10 \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \end{array}$$

and make the statement: *In the common system the mantissa of a logarithm is unique for a given sequence of digits. The*

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* For example, in dividing one logarithm by another.

† This form, $9.1089-10$, is perfectly convenient as long as the operations to be performed are addition and subtraction, which are the usual operations in dealing with logarithms.

characteristic is one less than the number of integral digits. If a number be purely decimal, count the decimal point and the zeros before the first significant figure. The result subtracted from 10 minus 10 will be the characteristic.

32. Four Computation Theorems. The use of logarithms in computation depends upon the four following theorems:

I. *In any system the logarithm of a product is equal to the sum of the logarithms of its factors.*

To prove, $\log_a mn \cdots s = \log_a m + \log_a n + \cdots + \log_a s.$

Let $\log_a m = x$ then $a^x = m$

$\log_a n = y \qquad a^y = n$

$\cdot \quad \cdot \quad \cdot \quad \cdot \qquad \cdot \quad \cdot \quad \cdot$

$\log_a s = z \qquad a^z = s.$

Whence $a^x \cdot a^y \cdots a^z = a^{x+y+\cdots+z} = mn \cdots s,$

or, by the definition of a logarithm,

$$\log_a mn \cdots s = x + y + \cdots + z.$$

That is, $\log_a mn \cdots s = \log_a m + \log_a n + \cdots + \log_a s.$

This theorem replaces the operation of multiplication by the simpler operation of addition.

II. *In any system the logarithm of a quotient is equal to the logarithm of the dividend minus the logarithm of the divisor.*

To prove, $\log_a \frac{m}{n} = \log_a m - \log_a n.$

Let $\log_a m = x$ then $a^x = m$

$\log_a n = y \qquad a^y = n.$

Whence

$$\frac{a^x}{a^y} = a^{x-y} = \frac{m}{n},$$

or, by the definition of a logarithm,

$$\log_a \frac{m}{n} = x - y.$$

That is,

$$\log_a \frac{m}{n} = \log_a m - \log_a n.$$

This theorem replaces the operation of division by the simpler operation of subtraction.

III. *In any system the logarithm of a power of a number is equal to the exponent of the power times the logarithm of the number.*

To prove, $\log_a m^n = n \log_a m.$

Let $\log_a m = x$ or $a^x = m.$

Whence $(a^x)^n = a^{nx} = m^n,$

or, by the definition of a logarithm,

$$\log_a m^n = nx.$$

That is $\log_a m^n = n \log_a m.$

This theorem replaces the operation of involution, or successive multiplications, by the simpler operation of a single multiplication.

IV. *In any system the logarithm of a root of a number is equal to the quotient of the logarithm of the number by the index of the root.*

To prove $\log_a \sqrt[n]{m} = \frac{\log_a m}{n}.$

Let $\log_a m = x$ or $a^x = m.$

Whence $\sqrt[n]{a^x} = a^{\frac{x}{n}} = \sqrt[n]{m},$

or, by the definition of a logarithm,

$$\log_a \sqrt[n]{m} = \frac{x}{n}.$$

That is $$\log_a \sqrt[n]{m} = \frac{\log_a m}{n}.$$

This theorem replaces the operation of evolution, or extraction of roots, by the simpler operation of division.

Another theorem, important in the theory of logarithms, but of which no application is made in the study of trigonometry is the following:

$$\log_a m = \frac{\log_b m}{\log_b a}.$$

Proof: Let $\log_b m = x$ then $b^x = m$

$\log_b a = y$ $b^y = a.$

Whence $$m = b^x = (b^y)^{\frac{x}{y}} = a^{\frac{x}{y}}$$

or, $$\log_a m = \frac{x}{y} = \frac{\log_b m}{\log_b a}.$$

By means of this theorem the logarithm of a number to any base can be found if the logarithms of numbers to some one base are known. Thus, assuming that logarithms to the base 10 are known,

$$\log_e 71.24 = \frac{\log_{10} 71{\cdot}24}{\log_{10} e} = \frac{\log_{10} 71.24}{\log_{10} 2.718} = \frac{1.8527}{0.4343} = 4.2659.$$

As a corollary of the above theorem we have, putting $b = m$,

$$\log_a m = \frac{1}{\log_m a}.$$

33. Special Properties of Logarithms. In addition to the preceding theorems we may note the following properties of logarithms:

1. In any system the logarithm of 1 is 0. For, by the definition of zero exponent, $a^0 = 1$. Therefore, $\log_a 1 = 0$.

2. In any system the logarithm of the base is 1.

For $a^1 = a$. Therefore, $\log_a a = 1$.

3. In any system whose base is greater than 1 the logarithm of 0 is $-\infty$. For, $a > 1$, $a^{-\infty} = \frac{1}{a^{\infty}} = \frac{1}{\infty} = 0$. Therefore, $\log_a 0 = -\infty$. That is, the base of the system being greater than unity, the logarithm of a number which becomes smaller than any assigned number however small, is negative and numerically greater than any assigned number however great.

4. The cologarithm of a number is the logarithm of the reciprocal of the number.

Thus, the base being 10,

$$\text{colog}\ n = \log\frac{1}{n} = \log 1 - \log n,$$

or,

$$\text{colog}\ n = 0 - \log n,$$

which may be written,

$$\text{colog}\ n = (10 - 10) - \log n.$$

Therefore, to find the *cologarithm* of a number to the base 10 *subtract the logarithm* of the number *from 10 − 10.*

It may be noted that

$$\log\frac{m}{n} = \log m \cdot \frac{1}{n} = \log m + \log\frac{1}{n} = \log m + \text{colog}\ n.$$

Therefore we may, instead of subtracting the logarithm of a number, add its cologarithm. It is found convenient to do so in most cases.

34. The following example will illustrate the use of logarithms in making numerical computations.

Example.

Find the value of

$$\sqrt[3]{\frac{.0005616 \times \sqrt[7]{-424.65}}{(6.73)^4 \times (.03194)^{\frac{5}{6}}}}$$

We note first that, with the definition of logarithms we have adopted, negative numbers have no logarithms. But the numerical result of operations of multiplication and division is the same no matter what the combination of algebraic signs. We therefore find the numerical value of any expression, treating all numbers as positive, and determine the algebraic sign of the result by considering the operations indicated. Thus, in the above example the factors are all positive except $\sqrt[7]{-424.65}$. Therefore, the number of which we are to extract the cube root is negative and the final result will be negative.

$$\begin{aligned} \log .0005616 &= 6.7494 - 10 \\ \tfrac{1}{7}\log 424.65 &= 0.3754 \\ 4 \operatorname{colog} 6.73 &= 6.6880 - 10 \\ \tfrac{5}{6}\operatorname{colog} .03194 &= 1.2464 \\ &\, 3\overline{)5.0592 - 10} \\ \log N &= 8.3531 - 10 \\ N &= .02255. \end{aligned}$$

Therefore

$$\sqrt[3]{\frac{.0005616 \times \sqrt[7]{-424.65}}{(6.73)^4 \times (.03194)^{\frac{5}{6}}}} = -.02255.$$

NOTE. The colog 6.73 = 9.1720 – 10, which being multiplied by four gives 36.6880 – 40; subtracting and adding 30 this becomes 6.6880 – 10, the desired form of "a number minus 10." Similarly to divide 5.0592 – 10 by three we first add and subtract 20. Also, in finding five sixths of the cologarithm of .03194, we first multiply by 5 and then divide by 6, in order that any error arising from inexact division by 6 may not be increased 5-fold.

35. We may now return to the problems of Art. 28 and solve them by the use of logarithms.

Example 1. Given $A = 32° 16'$, $a = 124$, $C = 90°$, find B, b, and c.

As before,

$$b = a \cot A, \qquad c = \frac{a}{\sin A}.$$

Therefore

$$\log b = \log a + \log \cot A, \qquad \log c = \log a - \log \sin A.$$

$$\begin{array}{rlcrl}
\log a & = 2.0934 & \qquad & \log a & = 12.0934 - 10 \\
\log \cot A & = 0.1997 & & \log \sin A & = 9.7274 - 10 \\
\hline
\log b & = 2.2931 & & \log c & = 2.3660 \\
b & = 196.4, & & c & = 232.3.
\end{array}$$

Example 2. Given $a = 50$, $b = 60$, $C = 90°$.
As before,

$$\tan A = \frac{a}{b}, \text{ therefore } \log \tan A = \log a - \log b.$$

$$\begin{array}{rl}
\log a & = 11.6990 - 10 \\
\log b & = 1.7782 \\
\hline
\log \tan A & = 9.9208 - 10 \\
A & = 39° \, 48'.
\end{array}$$

Also

$$c = \frac{a}{\sin A}, \text{ or } \log c = \log a - \log \sin A.$$

$$\begin{array}{rl}
\log a & = 11.6990 - 10 \\
\log \sin A & = 9.8063 - 10 \\
\hline
\log c & = 1.8927 \\
c & = 78.1.
\end{array}$$

It must be emphasized that results obtained by logarithmic computation are approximate. The value of the logarithm of a number cannot, in general, be found exactly, but only approximately to four, five, or any desired number of decimal places. *The results of numerical computation by means of logarithms are not,* in any case, *correct beyond the number of decimal places in the logarithms* used to make the computation. In the same way, the values of the trigonometric functions being, in general, not exact but approximate to four, five, or more decimal places, the solutions of triangles got by their use, with or without logarithms, are *approximate solutions only, to the degree of accuracy of the tables used.*

Indeed, in all but the simplest problems in applied mathematics the results are necessarily approximate, the data of a problem being themselves approximate. It is useless to try to make results "more accurate" by using tables of logarithms or other functions carried to seven places when the data are correct only to, say, three figures. In general if data are given to three figures, three-place tables should be used; if to seven figures, seven-place tables, etc. On the other hand, no matter to how many figures the data may be given, if we are using, say, four-place tables, the data should be used and results found to four figures only. To illustrate these points the following simple example will be worked in four ways: 1. by actual multiplication; 2. by using four-place tables; 3. by using five-place tables; 4. by using seven-place tables.

Example. Find the value of

$$123045 \times 200368.$$

1. By actual multiplication the result is 24,654,280,560.

2.

$$\begin{array}{rl}
\log 123045 & \\
123|0 = & 5.0899 \\
\log 200368 & \\
200|4 = & 5.3019 \\
\hline
\log \text{product} = & 10.3918 \\
\text{product} = & 24{,}650{,}000{,}000
\end{array}$$

which agrees with the first result to four figures.

3.

$$\begin{array}{rl}
\log 1230|45 = & 5.09007 \\
\log 2003|68 = & 5.30183 \\
\hline
\log \text{product} = & 10.39190 \\
\text{product} = & 24{,}655{,}000{,}000
\end{array}$$

which does not agree with the first result to the fifth figure. It will be noted that there was an accumulation of errors all in one direction. Result 3 is nearer to result 1, however, than is result 2.

4.
$$\begin{array}{rl} \log 12304\,|\,5 = & 5.0900640 \\ \underline{\log 20036\,|\,8 =} & \underline{5.3018284} \\ \log \text{product} = & 10.3918924 \\ \text{product} = & 24{,}654{,}280{,}000 \end{array}$$
which agrees with the first result to seven figures.

EXAMPLES

What is the value of

1. $10^{\log 7.218}$. 2. $\log 10^{2.6994}$.

3. Given $\log 2 = 0.3010$, $\log 3 = 0.4771$, find $\log 12$.

4. Prove $10^{\log a + 1} = 10\,a$.

5. Is $\log 14 = \log 2 \cdot \log 7$? Why?

6. Is $\dfrac{\log 10}{\log 3} = \dfrac{10}{3}$? Why?

7. Is $\dfrac{\log 10}{\log 3} = \log \left(\dfrac{10}{3}\right)$? Why?

Find the value of

8. $\dfrac{\log .00365}{\log .05312}$. 9. $\dfrac{\log 77.95}{\log .00684}$.

Find the value of x in the following equations:

10. $\log_{10} x = 3$. 12. $\log_e x = 2$.

11. $\log_{10} x = \frac{3}{2}$. 13. $\log_e x = \frac{5}{4}$.

14. $\log_{10} x \cdot \log_e x = \log_{10} x^2$.

15. $a \log_{10} x - b \log_{10} x = a^2 - b^2$.

16. $\log_e x^3 - \log_e x^2 = 5$.

17. $\frac{1}{2} \log_{10} x^{10} - \log_{10} x^3 = 4$.

Express as the logarithm of a fraction:

18. $\log (x^2 - a^2)^2 - \log (x^2 - a^2) - \log (x + a)$.

19. $\log \sqrt{x^2 + a^2} - \log \sqrt[4]{x^2 + a^2} + \log (x^2 + a^2)^{\frac{3}{4}}$.

Solve the equations:

20. $e^x + e^{-x} = 2$. 21. $e^x - e^{-x} = 0$.

22. $e^{2(x-1)} - 2\,e^{x-1} + 1 = 0$. 23. $e^{2(x-1)} + 2\,e^{x-1} + 1 = 0$.

24. Given $10^x = 400$, prove that $x = 2 + \log_{10} 4$.

25. Solve the equation
$$a^2 e^{-ax} - b^2 e^{-bx} = 0. \qquad (\text{Assume } a > b)$$

Find the value of $a^2e^{-ax} - b^2e^{-bx}$:

26. When $x = \dfrac{\log a - \log b}{a - b}$. $(a > b)$

27. When $x = \dfrac{2(\log a - \log b)}{a - b}$. $(a > b)$

Compute the values of the following :

28. $\log_e 1$. 29. $\log_e 2$. 30. $\log_e 3$. 31. $\log_e 4$.

By means of logarithms compute the values of :

32. $\sqrt[3]{.01236}$. 33. $\sqrt{1.1193}$. 34. $\sqrt[5]{-.002807}$.

35. $\dfrac{(56.333)^{\frac{2}{3}}}{\sqrt{11.11}}$. 36. $\dfrac{99.02}{\sqrt[4]{.02983}}$. 37. $\dfrac{(-16.65)^4}{\sqrt[3]{-.00986}}$.

38. $\dfrac{\log 771.2}{40.04}$. 39. $[\log (.00915)]^{\frac{1}{2}}$. 40. $\dfrac{693.08}{\log .00598}$.

41. $\dfrac{62.85 \times \sqrt{3111.59}}{-999.9 \times .002008}$. 42. $\dfrac{(56.3)^3 \times \sqrt[3]{56.3}}{\sqrt{.08888} \times 40.19}$.

43. $\sqrt{(.001)^3}$. 44. $(.01)^{\frac{5}{7}}$. 45. $(.0001)^{-\frac{1}{2}}$.

46. $\sqrt[3]{\dfrac{485.7 \times 22.01 \times 11.79}{-55.5 \times -66.66}}$. 48. $\dfrac{(.0002635)^{-\frac{1}{3}}}{(5362)^{-\frac{1}{2}}}$.

47. $\dfrac{\sqrt{.00298 \times .00384}}{\sqrt[4]{632 \times .06302}}$. 49. $\dfrac{(88)^2 \times (999)^{\frac{1}{2}}}{\sqrt[3]{1000000}}$.

EXAMPLES: SOLUTION OF RIGHT TRIANGLES

Solve the following right triangles, and find their areas :

1. $B = 24° 23'$, $b = .02126$.
2. $B = 55° 45'$, $c = 4116$.
3. $B = 43° 30'$, $a = 26185$.
4. $a = 77.38$, $c = 91.08$.
5. $B = 76° 34'$, $b = 2423$.
6. $A = 67° 47'$, $c = .00954$.
7. $b = .2072$, $a = .4212$.
8. $A = 82° 6'$, $b = .08937$.
9. $a = .8478$, $c = 1.234$.
10. $B = 60° 14'$, $c = .007745$.
11. $A = 14° 53'$, $a = 1353$.
12. $B = 39° 22'$, $a = 121.2$.
13. $b = 156.6$, $c = 856.4$.
14. $B = 43° 46'$, $a = 66650$.
15. $B = 74° 17'$, $b = .00002039$.
16. $A = 29° 56'$, $c = .0007814$.
17. $b = 8.243$, $c = 9.275$.
18. $B = 58° 39'$, $c = 35.73$.
19. $A = 35° 8'$, $a = 17270$.
20. $b = 3814$, $a = 3651$.

PROBLEMS

21. A road rises 348.9 feet in a horizontal distance of one half mile. Another road rises the same height in a distance of 3019 feet along the road. Which road is the steeper and by how much?

22. From a ship sailing due east at the rate of 7.6 miles per hour a headland bears due north at 10.35 A.M. At 12.46 P.M. the headland bears 33° west of north. How far was the headland from the ship in each position?

23. At a distance of 502.3 feet, horizontally, from the center of a bridge the sidewalk rises at an angle of elevation of 5°. The roadway, beginning 203.5 feet farther away from the center, has an angle of elevation of 4° 25′. If a pedestrian and a team enter the bridge at the same moment, which will reach the center first, the man, walking 3.4 miles per hour, or the team, going 5.6 miles per hour?

24. A flagpole 20 feet long stands on the corner of a building 143.6 feet high. Find the angle subtended by the flagpole from a point 100 feet distant from the foot of the building in a horizontal line.

25. If the radius of a circle is 835.4 feet, what is the length of the chord which subtends an arc of 45° 37′?

26. In a circle whose radius is 35.37 inches is inscribed a regular polygon of fifteen sides. Find the length of a side.

27. A tree 214.8 feet high casts a shadow 167.4 feet long. How many degrees is the sun above the horizon? What is the time of day if the sun rose at six o'clock and will set at six o'clock?

[Assume that the sun passes through the zenith.]

28. A gable roof is 23.4 feet high and 90.6 feet broad. By how much must the height be reduced to reduce the pitch of the roof 40 per cent?

NOTE. The pitch of a roof is the angle between the slope of the roof and the horizontal line.

29. From the top of a cliff 378.6 feet above the sea, the angles of depression of a boat and a buoy, in line with the observer, are found to be 29° 20′ and 11° 50′ respectively. Is the boat or the buoy farther from the base of the cliff? How much farther?

30. The point *B* is 1249 feet due east of the point *A*, and the point *C* is 376 feet due east of *B*. The angle of elevation of *B* above *A* is 9° 13′; of *C* above *B*, 7° 23′. A railroad runs from *A* to *C* via *B*. What is the increase in altitude from *A* to *C*?

31. If, in Example 30, the railway could, by grading, be made to run in a straight line from A to C, what would be the angle of elevation of the new route?

32. How much shorter would the railway of Example 31 be than the railway of Example 30?

33. Taking the earth as a sphere of radius 3956 miles, what is the length of the radius of the Arctic Circle, latitude 66° 32′ N.?

34. Taking the earth as a sphere of radius 3956 miles, what is the latitude of a place which is 2113 miles from the earth's axis?

35. A vessel sailing due south at a uniform rate observes at 7.15 A.M. that a lighthouse bears 70° east of south. At 8.05 A.M. the lighthouse is 12.75 miles due east from the ship. How far from the ship, and in what direction, will the lighthouse be at 9.30 A.M.?

36. A ship sailing due south at a uniform rate observes, at 6 A.M., a lighthouse 11.25 miles away, due east. At 6.30 A.M. the lighthouse bears 17° 57′ north of east. What will be the bearings of the lighthouse from the ship at 9 A.M.? How fast does the vessel sail?

37. Taking the Earth as a sphere with diameter 7912 miles, what is the distance of the farthest point on the Earth's surface visible from the top of a mountain 8200 feet in height?

38. The towns B and C lie due east from the town A, B being halfway from A to C, which are 5 miles apart. The towns B, C, and D are equally distant from each other. How far is D from A and in what direction?

39. A ray of light from a source A, perpendicularly above the edge of a horizontal mirror, strikes the mirror, which is 102 mm. broad, at a point two thirds of the way from that edge. The ray is then reflected to E at a perpendicular distance 25.7 mm. above the other edge of the mirror. Find the length of the path of the ray from A to E.

40. From a window of a house, on a level with the bottom of a spire, the angle of elevation of the top of the spire was 41°. From another window, 20.5 feet directly above the former, the like angle was 37° 31′. What was the height of the spire?

41. Having at a certain (unknown) distance measured the angle of elevation of a cliff, a surveyor walked 60 yards on a level toward the cliff. The angle of elevation from this second station was the complement of the former angle. The surveyor then walked 20 yards nearer the cliff, in the same line, and found the angle of elevation from the third station to be double the first angle. How high was the cliff?

CHAPTER IV

FUNDAMENTAL IDENTITIES

36. In this chapter we shall discuss some of the important relations of analytical trigonometry. The number of such relations is, of course, unlimited, but there are a few, of frequent occurrence and of fundamental importance, upon which the others depend; it is this fundamental group with which we shall now deal. Let us first observe how the need for some of the relations may arise. We have seen (Art. 27) that as the angle increases from 0° the sine of the angle also increases. But does the sine increase at the same rate as the angle, so that, for instance, if the angle be made twice as large the sine also becomes twice as large? This is obviously not so, for, as we have seen, the sine of 60° is not twice the sine of 30°. What then are the relations, if there be any such, by which we may find the functions of twice an angle when the functions of the angle are given? or again, is there any relation connecting the functions of the sum of two angles with the functions of the angles separately? Such questions as these we shall now proceed to answer.

37. The Addition Formulæ. Let x and y be two acute angles, whose sum may be an angle either in the first quadrant or in the second. Construct, Fig. 18, the angle XOP equal to x and add to it the angle POQ equal to y. Then the angle XOQ is equal to $x+y$. From any point, A, in the terminal side of the combined angle $x+y$ draw AB perpendicular to the axis of x which is the initial side of the angle x. Then OB, BA, and OA are respectively the abscissa, ordinate, and distance of the point A and we may write any function of the angle $x+y$. But as we wish to express the functions of $x+y$ in terms of the functions of x and y, we proceed to draw lines which will give us those functions. Thus, from A draw AC perpendicular to the terminal side of the angle x, and from C draw CD perpendicular to the axis of x and CE perpendicular to AB.

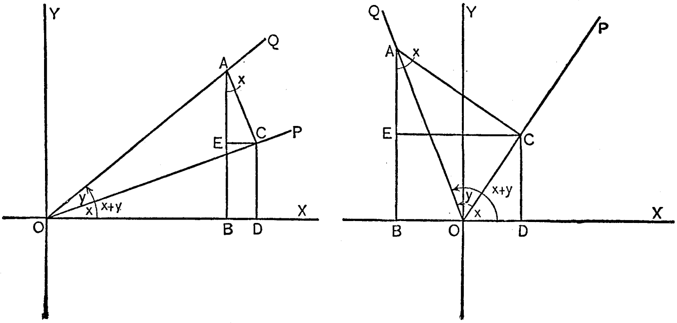

FIG. 18.

Then since AE is perpendicular to OX and AC to OP, the angle EAC is equal to the angle x, each of the angles being acute. We now have

$$\sin(x+y)=\frac{BA}{OA}=\frac{BE+EA}{OA}=\frac{DC}{OA}+\frac{EA}{OA}.$$

But these last two ratios are not functions of any of the angles in the figure. To obtain a function of x or y we must use with DC either OD or OC, and with OA either OC or CA. Therefore we shall multiply and divide $\frac{DC}{OA}$ by the common line OC. Similarly with EA and OA we use CA. Thus we may write

$$\sin(x+y)=\frac{DC}{OC}\cdot\frac{OC}{OA}+\frac{EA}{CA}\cdot\frac{CA}{OA}$$

or

$$\sin(x+y)=\sin x\cos y+\cos x\sin y.$$

In the same way

$$\cos(x+y)=\frac{OB}{OA}=\frac{OD-BD}{OA}=\frac{OD}{OA}-\frac{EC}{OA}$$

$$=\frac{OD}{OC}\cdot\frac{OC}{OA}-\frac{EC}{AC}\cdot\frac{AC}{OA}{}^{*},$$

or

$$\cos(x+y)=\cos x\cos y-\sin x\sin y.$$

38. The Addition Formulæ (*continued*). Again let x and y be two acute angles where x may be either greater or less than y. Construct, Fig. 19, the angle XOP equal to x and from it subtract the angle QOP equal to y. Then the angle XOQ is equal to $x-y$. From A, any point in the terminal side of the combined angle, draw AB perpendicular to the axis of x which is the initial side of the angle x. Then OB, BA, and OA are, respectively, the abscissa, ordinate and distance of A. From A draw AC perpendicular to the terminal side of the angle x, and from C draw CD perpendicular to the axis of x and CE perpendicular to BA pro-

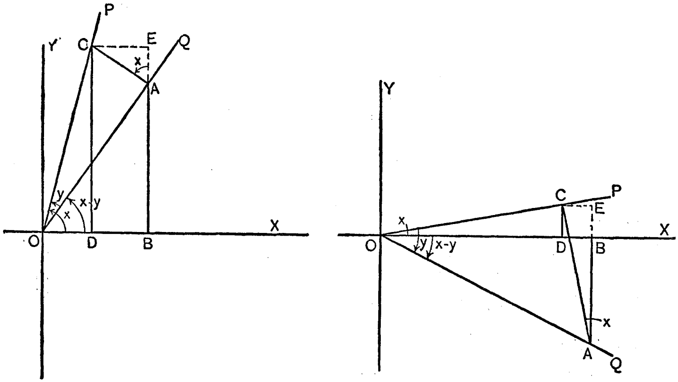

FIG. 19.

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[*] Note that in $\frac{EC}{AC}$ we use AC as the positive direction of the line, therefore AC must be positive in the ratio $\frac{AC}{OA}$ also.

duced. Then since AE is perpendicular to OX and AC to OP, the angle EAC is equal to the angle x, each being acute. We now have as in Art. 37,

$$\sin(x-y)=\frac{BA}{OA}=\frac{BE-AE}{OA}=\frac{DC}{OA}-\frac{AE}{OA}$$

$$=\frac{DC}{OC}\cdot\frac{OC}{OA}-\frac{AE}{AC}\cdot\frac{AC}{OA},$$

or,

$$\sin(x-y)=\sin x\cos y-\cos x\sin y.$$

Also

$$\cos(x-y)=\frac{OB}{OA}=\frac{OD+DB}{OA}=\frac{OD}{OA}+\frac{CE}{OA}$$

$$=\frac{OD}{OC}\cdot\frac{OC}{OA}+\frac{CE}{CA}\cdot\frac{CA}{OA},$$

or,

$$\cos(x-y)=\cos x\cos y+\sin x\sin y.$$

39. We have thus proved the formulæ

$$\begin{aligned}\sin(x\pm y)&=\sin x\cos y\pm\cos x\sin y,\\ \cos(x\pm y)&=\cos x\cos y\mp\sin x\sin y,\end{aligned}\qquad(10)$$

for values of x and y less than 90°. It now remains to be proved that these relations are true for all values of x and y. This may be done by a geometric construction as in the cases given, but the following method is preferable.

40. Let x be an angle in the second quadrant and y an angle in the third quadrant. Then we may put $x=90°+a$ and $y=180°+b$, where a and b are acute. We may now write

$$\begin{aligned}\cos(x+y)&=\cos(\overline{90°+a}+\overline{180°+b})\\ &=\cos(270°+\overline{a+b})\\ &=\sin(a+b). &&\text{(Art. 19)}\\ &=\sin a\cos b+\cos a\sin b. &&\text{(Art. 39)}\end{aligned}$$

But $a=-90°+x$ and $b=-180°+y$.

Therefore

$$\cos(x+y) = \sin(-90^\circ + x)\cos(-180^\circ + y) + \cos(-90^\circ + x)\sin(-180^\circ + y)$$

$$= (-\cos x)(-\cos y) + (\sin x)(-\sin y) \qquad \text{(Art. 19)}$$

or

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

which is the same as the relation of Art. 39.

Again, let x be an angle in the first quadrant and y an angle in the third. We may put $y = 180^\circ + b$, where b is acute, and write

$$\begin{aligned}\sin(x-y) &= \sin(x - \overline{180^\circ + b})\\ &= \sin(-180^\circ + \overline{x - b})\\ &= -\sin(x-b) && \text{(Art. 19)}\\ &= -\sin x \cos b + \cos x \sin b. && \text{(Art. 39)}\end{aligned}$$

But $b = -180^\circ + y$, and therefore,

$$\begin{aligned}\sin(x-y) &= -\sin x \cos(-180^\circ + y) + \cos x \sin(-180^\circ + y)\\ &= -\sin x(-\cos y) + \cos x(-\sin y) && \text{(Art. 19)}\\ &= \sin x \cos y - \cos x \sin y.\end{aligned}$$

Thus it may be proved that the equations of Art. 39 are true for all values of x and y.

The importance of these four relations, (10) of Art. 39, can hardly be over-emphasized. From them, together with those given in Art. 12, may be derived all other trigonometric identities. The method of so doing is shown in the following articles, and is illustrated by the following examples:

Example 1. Prove the relation

$$\sin(45^\circ + a)\cos(45^\circ - b) + \cos(45^\circ + a)\sin(45^\circ - b) = \cos(a - b).$$

This is simply a case of the first formula of (10) where

$$x = 45° + a, \ y = 45° - b.$$

We may write

$$\sin(45° + a)\cos(45° - b) + \cos(45° + a)\sin(45° - b)$$

$$= \sin(\overline{45° + a} + \overline{45° - b}) = \sin(90° + a - b) = \cos(a - b).$$

EXAMPLES

Prove that

1. $\sin 105° + \cos 105° = \cos 45°$.
2. $\cos(45° - x)\cos(45° + x) - \sin(45° - x)\sin(45° + x) = 0$.
3. $\sin x \cos(90° - x) - \cos x \sin(90° - x) = -\cos 2x$.
4. $\cos(30° - 45°) - \cos(30° + 45°) = \sin 45°$.
5. Given $\sin x = \frac{3}{5}$, $\cos y = \frac{3}{4}$, find $\sin(x + y)$.
6. Given $\cos x = \frac{1}{3}$, $\cos y = \frac{1}{4}$, find $\cos(x - y)$.

Given $\tan x = 2$, $\tan y = 3$, find

7. $\sin(x + y)$. 8. $\cos(x + y)$. 9. $\sin(x - y)$. 10. $\cos(x - y)$.

41. Tangent of a Sum. To derive an expression for the tangent of the sum or difference of two angles we proceed as follows:

$$\tan(x \pm y) = \frac{\sin(x \pm y)}{\cos(x \pm y)}$$

$$= \frac{\sin x \cos y \pm \cos x \sin y}{\cos x \cos y \mp \sin x \sin y}$$

$$= \frac{\dfrac{\sin x \cos y}{\cos x \cos y} \pm \dfrac{\cos x \sin y}{\cos x \cos y}}{\dfrac{\cos x \cos y}{\cos x \cos y} \mp \dfrac{\sin x \sin y}{\cos x \cos y}}.$$

or

$$\boldsymbol{\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}}. \qquad (11)$$

In a similar manner may be proved

$$\cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}. \qquad (12)$$

EXAMPLES

Prove that

1. $\tan(45^\circ + x) = \dfrac{1 + \tan x}{1 - \tan x}$.

2. $\tan(45^\circ + x)\tan(135^\circ + x) + 1 = 0$.

3. $\tan(45^\circ + x)\tan(45^\circ - x) = 1$.

4. Given $\tan a = 2$, $\tan b = 4$, find $\tan(a + b)$.

5. Given $\sin a = \frac{1}{2}$, $\cos b = \frac{1}{3}$, find $\tan(a - b)$.

6. Given $\sec a = 3$, $\csc b = 4$, find $\tan(a + b)$.

7. Given $\tan a = \frac{5}{6}$, $\tan b = \frac{1}{11}$, find $a + b$.

8. Given $\sin a = \frac{3}{5}$, $\cos b = \frac{4}{5}$, find $\tan(a + b)$.

9. Given $\sin a = \frac{3}{5}$, $\sin b = \frac{4}{5}$, find $a + b$.

Prove the following identities.

10. $\dfrac{\sin(x + y)}{\cos(x - y)} = \dfrac{\cot x + \cot y}{1 + \cot x \cot y}$.

11. $\dfrac{\tan x - \tan(x - y)}{1 + \tan x \tan(x - y)} = \tan y$.

42. Functions of the Double Angle. The equations of Arts. 39 and 41 being true for all values of x and y, let us assume that $y = x$. Substituting x for y in the functions of the sum of two angles we obtain

$$\sin 2x = 2 \sin x \cos x \tag{13}$$

$$\cos 2x = \cos^2 x - \sin^2 x. \tag{14}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}. \tag{15}$$

$$\cot 2x = \frac{\cot^2 x - 1}{2 \cot x}. \tag{16}$$

43. The student should clearly understand that the equations of Art. 42 give the values of functions of twice an angle in terms of functions of the angle, no matter what the value or form of the angle may be. For example, the following relations are all true, being merely the equations of Art. 42 changed slightly in form.

$$2 \sin \frac{a}{2} \cos \frac{a}{2} = \sin a.$$

$$\cos \frac{3x}{2} = \cos^2 \frac{3x}{4} - \sin^2 \frac{3x}{4}.$$

$$\tan (2\alpha + \beta) = \frac{2 \tan \left(\alpha + \frac{\beta}{2}\right)}{1 - \tan^2 \left(\alpha + \frac{\beta}{2}\right)}.$$

44. Functions of the Half-angle. We may write the two proved relations

$$\sin^2 x + \cos^2 x = 1 \quad \text{and} \quad \cos 2x = \cos^2 x - \sin^2 x$$

in the form

$$\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} = 1.$$

$$\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} = \cos x.$$

Subtracting and adding these, we have

$$\begin{aligned} 2 \sin^2 \frac{x}{2} &= 1 - \cos x. \\ 2 \cos^2 \frac{x}{2} &= 1 + \cos x. \end{aligned} \tag{17}$$

Dividing the last two equations one by the other we obtain

$$\begin{aligned} \tan^2 \frac{x}{2} &= \frac{1 - \cos x}{1 + \cos x}, \\ \cot^2 \frac{x}{2} &= \frac{1 + \cos x}{1 - \cos x}. \end{aligned} \tag{18}$$

Thus we have equations which give the sine, cosine, tangent, and cotangent of one half an angle in terms of the cosine of that angle.

EXAMPLES

Given $\sin\theta = \frac{1}{3}$, find

1. $\sin 2\theta$.
2. $\cos 2\theta$.
3. $\tan 2\theta$.
4. $\sin\frac{\theta}{2}$.
5. $\cos\frac{\theta}{2}$.
6. $\tan\frac{\theta}{2}$.

Prove the following identities.

7. $\cos^4 x - \sin^4 x = \cos 2x$.
8. $(\sin x + \cos x)^2 = 1 + \sin 2x$.
9. $\tan x = \dfrac{\sin 2x}{1 + \cos 2x}$.
10. $\sin^2\dfrac{x}{2} = \dfrac{\sec x - 1}{2\sec x}$.
11. $\tan\dfrac{x}{2} = \dfrac{1 - \cos x}{\sin x}$.
12. $\tan\dfrac{x}{2} = \dfrac{\sin x}{1 + \cos x}$.
13. $\dfrac{\cos 2x}{1 + \sin 2x} = \tan(45^\circ - x)$.
14. $2\cos^2\dfrac{x}{2} = \dfrac{1 + \sec x}{\sec x}$.
15. $\tan(45^\circ + x) + \tan(45^\circ - x) = 2\sec 2x$.

45. Sum of Sines or Cosines. By addition and subtraction of the two equations

$$\sin(x + y) = \sin x\cos y + \cos x\sin y,$$
$$\sin(x - y) = \sin x\cos y - \cos x\sin y,$$

we obtain

$$\sin(x + y) + \sin(x - y) = 2\sin x\cos y,$$
$$\sin(x + y) - \sin(x - y) = 2\cos x\sin y.$$

If now we let $x + y = \alpha$, $x - y = \beta$, so that $x = \frac{1}{2}(\alpha + \beta)$ and $y = \frac{1}{2}(\alpha - \beta)$, we obtain from the last two identities

$$\begin{aligned}\sin\alpha + \sin\beta &= 2\sin\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta),\\ \sin\alpha - \sin\beta &= 2\cos\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta).\end{aligned} \tag{19}$$

Proceeding in the same way with the equations

$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y,$$

we obtain two more equations of importance

$$\begin{aligned}\cos\alpha + \cos\beta &= 2\cos\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta),\\ \cos\alpha - \cos\beta &= -2\sin\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta).\end{aligned} \tag{20}$$

EXAMPLES

Express each of the following as the algebraic sum of sines or cosines.

1. $\sin 6x \cos 2x$.
2. $\cos 4x \sin 2x$
3. $\cos \frac{x}{2} \sin \frac{3x}{2}$.
4. $\sin(x + 2y)\cos(x - y)$.
5. $\sin(30^\circ + x)\sin(30^\circ - x)$.
6. $\cos 3x \cos(x - y)$.

Prove the following identities.

7. $\cos(30^\circ - x)\cos(60^\circ - x) = \frac{1}{4}(2\sin 2x + \sqrt{3})$.
8. $\cos 3x \sin 2x - \cos 4x \sin x = \cos 2x \sin x$.
9. $\sin x \cos(x + y) - \cos x \sin(x - y) = \cos 2x \sin y$.

46. Identities and Equations. It should be borne in mind that all of the equations of this chapter are *identities*, that is, they are true no matter what values the angles may have. We shall deal later on, in Chapter VII, with trigonometric equations of condition, where we shall find that not every value but only particular values of the angles involved will satisfy the equations. Also, in connection with this chapter attention should be again called to the group of fundamental identities in Art. 12.

ILLUSTRATIVE EXAMPLES

Example 1. Prove that $\sec 2x = 1 + \tan x \tan 2x$.

$$\sec 2x = \frac{1}{\cos 2x} = \frac{1}{\cos^2 x - \sin^2 x} = \frac{\frac{1}{\cos^2 x}}{1 - \tan^2 x} = \frac{\sec^2 x}{1 - \tan^2 x}$$

$$= \frac{1 + \tan^2 x}{1 - \tan^2 x} = 1 + \frac{2\tan^2 x}{1 - \tan^2 x}$$

$$= 1 + \tan x \cdot \frac{2\tan x}{1 - \tan^2 x} = 1 + \tan x \tan 2x.$$

By the above method we begin with sec 2 x and deduce or derive the required result. Another method of procedure is as follows:

Assume that

$$\sec 2x = 1 + \tan x \tan 2x.$$

Then
$$\begin{aligned}\sec 2x &= 1 + \frac{2\tan^2 x}{1-\tan^2 x}\\ &= \frac{1+\tan^2 x}{1-\tan^2 x}\\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x - \sin^2 x}\\ &= \frac{1}{\cos 2x}\\ &= \sec 2x.\end{aligned}$$

Therefore, the original assumption is correct.

Example 2. Prove that $\csc 2x = \frac{1}{2}\sec x \csc x$.
First method.

$$\csc 2x = \frac{1}{\sin 2x} = \frac{1}{2\sin x\cos x} = \frac{\sec x \csc x}{2}.$$

Second method. Take the reciprocals of both members.

$$\sin 2x = 2\sin x \cos x.$$

EXAMPLES

Without the use of tables find the following:

1. Sine and cosine of 15°.
2. Sine and cosine of 22° 30′.
3. Tangent of 15°.
4. Tangent of 22° 30′.
5. Find the value of $\sin 3x$ in terms of $\sin x$.
6. Find the value of $\cos 3x$ in terms of $\cos x$.
7. Find the value of $\tan 3x$ in terms of $\tan x$.
8. Find the value of $\tan 4x$ in terms of $\tan x$.
9. Find the value of $\sin 4x$ in terms of functions of x.
10. Find the value of $\cos 4x$ in terms of functions of x.
11. Given $\sin 4x = a$, $\cos 4x = b$, find $\sin 8x$ and $\cos 8x$.
12. Given $\tan 3x = a$, find $\tan 6x$.

Prove the following identities.

13. $\sin(90° + x + y) = \cos x \cos y - \sin x \sin y.$

14. $\sin\frac{a}{2}\cos\frac{b}{2} + \cos\frac{a}{2}\sin\frac{b}{2} = \sin\frac{1}{2}(a+b).$

15. $\cos 6a = 1 - 2\sin^2 3a.$

16. $\dfrac{\sin 4a + \sin 2a}{\sin 4a - \sin 2a} = \dfrac{3 - \tan^2 a}{1 - 3\tan^2 a}.$

17. $\dfrac{1}{1+\cos^2\theta} = \dfrac{\sec^2\theta}{\tan^2\theta + 2}.$

18. $\sin^2 x \cos^2 x = \dfrac{1 - \cos 4x}{8}.$

19. $\cos 5x \cos 2x = \frac{1}{2}(\cos 7x + \cos 3x).$

20. $\sec^2 x \csc^2 x = \sec^2 x + \csc^2 x.$

21. $\dfrac{\sin 3x + \sin 5x}{\sin 4x} = 2\cos x.$

22. $\cos 4x \sin x = \frac{1}{2}(\sin 5x - \sin 3x).$

23. $\dfrac{\sin 4x + 2\sin 3x + \sin 2x}{\sin 3x} = 4\cos^2\frac{x}{2}.$

24. $\sin x = \dfrac{2\tan\frac{1}{2}x}{1 + \tan^2\frac{1}{2}x}.$

25. $\sin x = \dfrac{2}{\cot\frac{1}{2}x + \tan\frac{1}{2}x}.$

26. $\tan 2x = \dfrac{2}{\cot x - \tan x}.$

27. $\dfrac{\sin(x+2y) - \sin(x-2y)}{\sin y} = 4\cos x\cos y.$

28. $\dfrac{\cos(2x-y) - \cos(2x+y)}{\cos x} = 4\sin x \sin y.$

29. $\tan^2\frac{x}{2} + 2\cot x \tan\frac{x}{2} = 1.$

30. $\tan^2\frac{x}{2} - 2\csc x \tan\frac{x}{2} + 1 = 0.$

31. $\cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}.$

32. $\tan\frac{1}{2}x = \csc x - \cot x.$

33. $\cot\frac{x}{2} = \csc x + \cot x.$

34. $\tan\frac{1}{2}x = \frac{1+\sin x-\cos x}{1+\sin x+\cos x}$.

35. $\frac{\sec^2(x+45^\circ)}{2\tan(x+45^\circ)} = \sec 2x$.

36. $\frac{\cos x-\cos 3x}{\sin 3x-\sin x} = \tan 2x$.

37. $\sin x(1+\tan x)+\cos x(1+\cot x) = \sec x+\csc x$.

38. $\cot x-\cot 2x = \sec 2x$.

39. $\frac{1+\sin 2x}{\cos 2x} = \frac{\cos x+\sin x}{\cos x-\sin x}$.

40. $\sin x\cos^3 x-\cos x\sin^3 x = \frac{\sin 4x}{4}$.

41. Given $\sin x = \frac{e^{x\sqrt{-1}}-e^{-x\sqrt{-1}}}{2\sqrt{-1}}$, find the value of $\cos x$.

42. Given $r^2 = a^2\sin 2\theta$,

Prove
$$\frac{r^2\cos\theta+a^2\cos 2\theta\sin\theta}{a^2\cos 2\theta\cos\theta-r^2\sin\theta} = \tan 3\theta.$$

43. Given $r = a\sec^2\frac{\theta}{2}$,

Prove
$$\frac{r\cos\theta+a\sec^2\frac{\theta}{2}\tan\frac{\theta}{2}\sin\theta}{a\sec^2\frac{\theta}{2}\tan\frac{\theta}{2}\cos\theta-r\sin\theta} = -\cot\frac{\theta}{2}.$$

44. Given $r = a(1-\cos\theta)$,

Prove
$$\frac{r\cos\theta+a\sin^2\theta}{a\sin\theta\cos\theta-r\sin\theta} = \tan\frac{3\theta}{2}.$$

Prove the following identities.

45. $\frac{\cos 3x}{\sin x}+\frac{\sin 3x}{\cos x} = 2\cot 2x$.

46. $1+\cos 2x\cos 2y = 2(\sin^2 x\sin^2 y+\cos^2 x\cos^2 y)$.

47. $\frac{\sin^2 x\cos^2 y-\cos^2 x\sin^2 y}{\cos^2 x\cos^2 y-\sin^2 x\sin^2 y} = \tan(x+y)\tan(x-y)$.

48. $\frac{\sin^3 x-\cos^3 x}{\sin x-\cos x} = 1+\frac{1}{2}\sin 2x$.

49. $\frac{\cos 2\theta+\cos\theta+1}{\sin 2\theta+\sin\theta} = \cot\theta$.

50. $(\sec 2\theta + 1)\sqrt{\sec^2\theta - 1} = \tan 2\theta.$

51. $\dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} - \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = 2\tan 2\theta.$

52. $\tan 2\theta - \sec\theta \sin\theta = \tan\theta \sec 2\theta.$

53. $\dfrac{1 - \cos x + \cos y - \cos(x + y)}{1 + \cos x - \cos y - \cos(x + y)} = \dfrac{\tan\frac{1}{2}x}{\tan\frac{1}{2}y}.$

54. Given $\left.\begin{array}{l} x = -3\cos^2\theta\sin\theta \\ y = 3\sin^2\theta\cos\theta \end{array}\right\}$ Prove that $2\sqrt{x^2 + y^2} = 3\sin 2\theta.$

55. Given $x = a\cos\theta - r\sin\theta,$ $a = -\dfrac{2\sin 2\theta}{\sqrt{2\cos 2\theta}},$
$y = a\sin\theta + r\cos\theta,$
$r^2 = 2\cos 2\theta,$ Prove, $x^2 + y^2 = 2\sec 2\theta.$

CHAPTER V

THE CIRCULAR OR RADIAN MEASURE OF AN ANGLE. INVERSE TRIGONOMETRIC FUNCTIONS

47. Circular or Radian Measure of an Angle. Any convenient unit may be chosen for the measurement of angles. We have hitherto used the degree, subdivided into minutes and seconds, as the unit,* but we shall now introduce another unit called the *radian*, the unit angle in the circular measure of angles.

The *radian* is an *angle at the centre of a circle whose subtending arc is equal to the radius* of the circle.

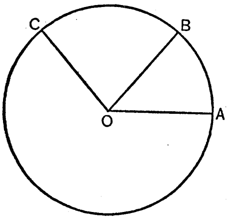

Fig. 20.

It is obvious that the radian is a constant angle, is the same in all circles, since the ratio of the circumference of a circle to its radius is constant.

In Fig. 20 let the angle AOB be a radian, that is, let the arc AB be equal to OA, the radius of the circle. Also let the angle AOC be an angle to be measured in radians. To measure a quantity is to find its ratio to another quantity of the same kind chosen as the unit. Therefore,

---

* It may be noted that the right angle is used as a unit in the study of geometry, partly because it is an angle easily constructed.

$$\text{Circular measure } AOC = \frac{AOC}{AOB} \text{ radians.}$$

But $$\frac{AOC}{AOB} = \frac{\text{arc } AC}{\text{arc } AB} = \frac{\text{arc } AC}{\text{radius } OA}.$$

Therefore, *to measure an angle in circular measure,* or in other words to express the angle in radians, *find the ratio of the arc subtending the angle* in any circle *to the radius* of the circle.

If we represent the angle, measured in radians, by x, the length of the arc subtending the angle by s, and the radius of the circle by r, we have the relation $x = \frac{s}{r}$. This is an algebraic equation involving three quantities. If any two of the quantities are known the third can be found. Thus

$$\boldsymbol{x = \frac{s}{r}, \quad s = rx, \quad r = \frac{s}{x}.}$$

Example 1. What is the radius of a circle in which an arc of 12 inches subtends an angle of $1\frac{1}{2}$ radians?

$$r = \frac{s}{x} = \frac{12}{1\frac{1}{2}} = 8 \text{ inches.}$$

Example 2. If the radius of a circle is 15 feet what length of arc subtends an angle of two-thirds of a radian?

$$s = 15 \times \tfrac{2}{3} = 10 \text{ feet.}$$

We know that the ratio of a semicircumference to its radius is $\pi = 3.1416$. It follows, therefore, that the angle which is sometimes called a straight angle, and which is expressed as 180°, may also be expressed as π radians. Thus,

$$\pi \text{ radians} = 180°$$

$$1 \text{ radian} = \frac{180°}{\pi} = 57.296°, \text{ approximately.}$$

Also $$180° = \pi \text{ radians}$$

$$1° = \frac{\pi}{180} \text{ radians.}$$

By means of these two relations we can readily reduce a given angle from either system of measurement to the other.

Example 1. Express the angle 7 radians * in degrees.

$\pi = 180°$. Therefore,

$$1 = \frac{180°}{\pi}. \qquad 7 = 7 \times \frac{180°}{\pi} = \frac{1260°}{\pi} = 401.03°.$$

Example 2. Express the angle $\frac{2\pi}{3}$ in degrees.

Since $\pi = 180°$, $\frac{2\pi}{3} = \frac{2}{3} \times 180° = 120°.$

Example 3. Express the angle $110°\ 32'\ 30''$ in circular measure. $110°\ 32'\ 30'' = 110\frac{13}{24}° = \frac{2653}{24}°.$

Since $180° = \pi$, therefore,

$$1° = \frac{\pi}{180}, \qquad \frac{2653°}{24} = \frac{\pi}{180} \times \frac{2653}{24} = \frac{2653\,\pi}{4320}.$$

An angle in circular measure is usually expressed as a multiple of π.

EXAMPLES

Express the following angles in degrees.

1. 6.5 radians.
2. $\frac{2\pi}{5}$.
3. $\frac{\pi}{7}$.
4. 3.8 radians.
5. $\frac{7\pi}{6}$.
6. $\frac{5\pi}{8}$.

Express the following angles in circular measure.

7. $270°$.
8. $13°\ 24'$.
9. $25°\ 16'$.
10. $-450°$.
11. $208°\ 30'$.
12. $-98°$.
13. What is the ratio of a radian to a right angle ?

---

* The name radian is often omitted. An angle written $\frac{\pi}{3}$ and read "pi over three," means "pi over three" radians, or about 1.05 radians.

How many right angles are there in each of the following angles?

14. $\frac{\pi}{5}$. **15.** $\frac{3\pi}{7}$. **16.** $\frac{2\pi}{3}$. **17.** 5 radians.

18. Through how many radians do the minute and hour hands of a clock turn in 30 minutes?

19. Through how many radians does the minute hand of a clock turn in 35 minutes?

20. Through how many radians does the hour hand of a clock turn in 18 minutes?

21. The front wheel of a cart is 2 feet in diameter, the hind wheel 3 feet. Through how many radians will the hind wheel turn while the front wheel is turning through 600°?

22. Through how many radians does the earth revolve about its axis in a week? Is the result the same in 45° north latitude as at the equator?

23. A wheel turns 50 revolutions per minute. Express its angular velocity in radians per second.

24. A wheel has an angular velocity of 20 radians per second. How many revolutions does it make per minute?

25. Through how many miles will a point on the equator of the earth travel as the earth turns through $1\frac{1}{2}$ radians?

26. Through how many miles will a point at 45° north latitude travel as the earth turns through one radian?

27. The radius of a graduated quadrant is 2 feet, and the graduations are $5'$ apart. What is the distance between successive graduations?

28. What must be the radius of a graduated quadrant if the distance between graduations $5'$ apart is to be $\frac{1}{10}$ inch?

48. Inverse Trigonometric Functions. Let us suppose that y is the sine of the angle x. We express this briefly in mathematical symbols as $y = \sin x$. Suppose now that we wish to make the inverse statement that x is the angle whose sine is y. To express this in mathematical symbols we write $x = \sin^{-1} y$, where, it must be noted, the *minus unity is not an exponent.* Having expressed our idea in symbols we next note that x depends upon y for its value, is a function of y, and we name the function the anti-sine or inverse sine. Similarly $a = \tan^{-1} b$ means that a is the angle whose tangent is b, and we say that a is the anti-tangent of b. In this way we have a group of six inverse trigonometric functions,

$$\sin^{-1} x, \qquad \tan^{-1} x, \qquad \sec^{-1} x,$$
$$\cos^{-1} x, \qquad \cot^{-1} x, \qquad \csc^{-1} x.$$

These six quantities, it must be remembered, are angles.

49. General Value of an Angle. Identities connecting the various inverse trigonometric functions exist and may be derived or proved by methods analogous to those of Chapter IV. Before taking them up, however, one important difference between the direct and the inverse trigonometric functions must be noted.

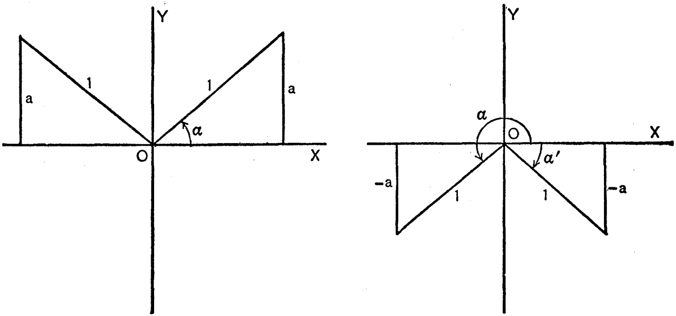

Fig. 21.

If $y = \sin x$, and if we give x a particular value, say 30°, then y will have one and only one value, one-half. On the other hand, if $x = \sin^{-1} y$ and if we give y a particular value, say $\frac{1}{2}$, then x does not have one value only but an infinite number of values, 30°, 150°, 390°, $-210°$ etc. This being so it is well to get an expression that will represent all the angles which have a given value of the sine, cosine, etc.

Let $\sin x = \pm a$, a being a positive number, or $x = \sin^{-1}(\pm a)$ and let α be the smallest angle* which has for its sine the value $\pm a$. By Fig. 21 we see that the possible values of x are

---

* That is, α if we use positive angles only; α' if negative angles also are used. Either method may be adopted.

$$\alpha,\quad \pi-\alpha,\quad 2\pi+\alpha,\quad 3\pi-\alpha,\quad 4\pi+\alpha,\cdots,$$
$$-\pi-\alpha,\quad -2\pi+\alpha,\quad -3\pi-\alpha,\quad -4\pi+\alpha,\cdots,$$

which may be written in the general form

$$x=\sin^{-1}a=n\pi+(-1)^n\alpha \tag{21}$$

where n is any positive or negative integer, including zero, and α is the least angle whose sine is a. This is called the *general value* of the angle and α is called the *principal value.*

Since the cosecant is the reciprocal of the sine we may write

$$x=\csc^{-1}a=n\pi+(-1)^n\alpha. \tag{22}$$

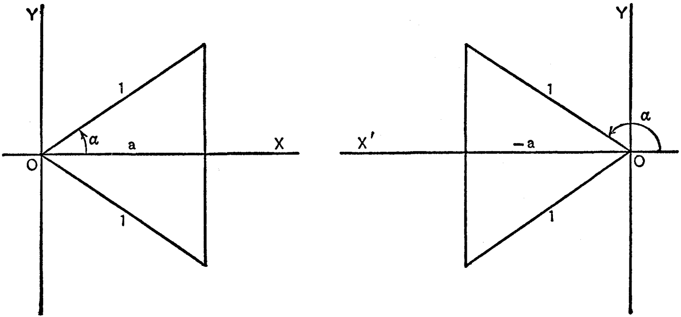

FIG. 22.

Let $\cos x=\pm a$ or $x=\cos^{-1}(\pm a)$ and let α be the least angle whose cosine is $\pm a$. By Fig. 22 we see that possible values of x are

$$\alpha,\ 2\pi\pm\alpha,\ 4\pi\pm\alpha,\cdots,$$

or, in the general form

$$x=\cos^{-1}a=2n\pi\pm\alpha \tag{23}$$

where n is any positive or negative integer, including zero, and α is the least angle whose cosine is a.

Since $\sec x=\dfrac{1}{\cos x}$ we may write for the general value, α

being the principal value,

$$x = \sec^{-1} a = 2\,n\pi \pm \alpha. \tag{24}$$

Let $\tan x = \pm a$ or $x = \tan^{-1}(\pm a)$, and let α be the least angle whose tangent is $\pm a$. By Fig. 23 we see that possible values of x are

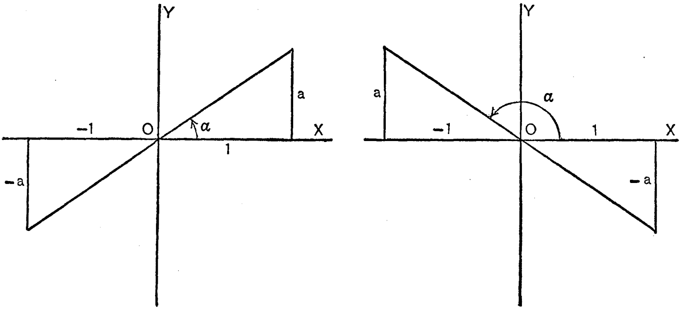

FIG. 23.

$$\alpha, \quad \pi + \alpha, \quad 2\,\pi + \alpha, \quad 3\,\pi + \alpha, \cdots,$$
$$-\pi + \alpha, \quad -2\,\pi + \alpha, \quad -3\,\pi + \alpha, \cdots,$$

or, for the general value,

$$x = \tan^{-1} a = n\pi + \alpha \tag{25}$$

where n is any positive or negative integer, including zero, and α is the principal value.

Giving n and α the same meaning we may write, since $\cot x = \dfrac{1}{\tan x}$,

$$x = \text{cct}^{-1} a = n\pi + \alpha. \tag{26}$$

One need not make use of the formulæ, 21–26, but may proceed as follows: Find the *two smallest angles*, positive or negative, which correspond to the given value of the function. If we call these angles α and β then the complete series of angles will be given by

$$2\,n\pi + \alpha \text{ and } 2\,n\pi + \beta.$$

Example 1. Write the general value of $\cos^{-1} .9205$.

From the tables of trigonometric functions we find $\alpha = 23°$. Therefore

$$x = \cos^{-1} .9205 = 2\,n\pi \pm 23° = 2\,n\pi \pm \frac{23\,\pi}{180}.$$

Example 2. Write the general value of $\sin^{-1} 1$. We know $\alpha = 90° = \frac{\pi}{2}$. Therefore

$$x = \sin^{-1} 1 = n\pi + (-1)^n \frac{\pi}{2}.$$

Example 3. Prove the identity

$$2 \sin^{-1} a = \sin^{-1} (2\,a\sqrt{1 - a^2}).$$

Let $\sin^{-1} a = x$, then $\sin x = a$. Substituting these values in the formula to be proved we have

$$2\,x = \sin^{-1} (2 \sin x\sqrt{1 - \sin^2 x}),$$

or

$$\sin 2\,x = 2 \sin x\sqrt{1 - \sin^2 x} = 2 \sin x \cos x.$$

Q. E. D.

Or, we may proceed as follows:

We know $\sin 2\,x = 2 \sin x \cos x$, which may be written

$$2\,x = \sin^{-1}(2 \sin x \cos x).$$

Let $\sin^{-1} a = x$, $\sin x = a$ and substitute:

$$2 \sin^{-1} a = \sin^{-1} (2\,a\sqrt{1 - a^2}).$$ Q. E. D.

Example 4. Find the principal value of $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3}$. We note that this is the sum of two angles each given by the value of its tangent. We therefore write, formula (11),

$$\tan (\tan^{-1} \tfrac{1}{2} + \tan^{-1} \tfrac{1}{3}) = \frac{\tan (\tan^{-1} \frac{1}{2}) + \tan(\tan^{-1} \frac{1}{3})}{1 - \tan(\tan^{-1} \frac{1}{2}) \cdot \tan (\tan^{-1} \frac{1}{3})}$$

$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}} = 1.$$

Therefore, $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \tan^{-1} 1 = \frac{\pi}{4}$.

EXAMPLES

Write the general values of the following angles:

1. $\sin^{-1}\frac{1}{2}$.
3. $\tan^{-1}(-1)$.
5. $\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$.
7. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.
2. $\cos^{-1}0$.
4. $\cot^{-1}\frac{1}{\sqrt{3}}$.
6. $\csc^{-1}\sqrt{2}$.
8. $\cot^{-1}(-\sqrt{3})$.

Find the value of

9. $\sin(\sin^{-1}a)$.
11. $\tan(\tan^{-1}y)$.
13. $2\cos(\cos^{-1}.523)$.
10. $\cos^{-1}(\cos x)$.
12. $\sec^{-1}(\sec 30°)$.
14. $\cot(\cot^{-1}2.718)$.
15. $\sin^{-1}(\cos 35°)$.
18. $\cos\left(\tan^{-1}1+\sec^{-1}\frac{2}{\sqrt{3}}\right)$.
16. $\tan^{-1}(\cot 40°)$.
19. $\tan(\sin^{-1}1+\cos^{-1}\frac{1}{2})$.
17. $\sin\left(\sin^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{\sqrt{2}}\right)$.
20. $\cot\left(\tan^{-1}\sqrt{3}+\tan^{-1}\frac{1}{\sqrt{3}}\right)$.
21. Prove that $x=\sec^{-1}\sqrt{1+\tan^2 x}$.
22. Prove that $\tan^{-1}y=\sec^{-1}\sqrt{1+y^2}$.

Prove the following:

23. $\tan^{-1}(\sqrt{2}+1)-\tan^{-1}(-\sqrt{2}-1)=135°$.
24. $\tan^{-1}\sqrt{3}-\tan^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\tan^{-1}(-3\sqrt{3})$.
25. $\tan^{-1}\frac{1}{3}-\tan^{-1}(-1)=\tan^{-1}2$.
26. $\tan^{-1}\frac{a+b}{a-b}-\tan^{-1}\frac{b+a}{b-a}=\tan^{-1}\frac{b^2-a^2}{2ab}$.
27. $\tan^{-1}\frac{b}{\sqrt{a^2-b^2}}-\tan^{-1}\frac{-b}{\sqrt{a^2-b^2}}=\tan^{-1}\frac{2b\sqrt{a^2-b^2}}{a^2-2b^2}$.
28. $\tan^{-1}\left(-\frac{a^2}{b^2}\right)-\tan^{-1}\left(\frac{b^2}{a^2}\right)=\frac{\pi}{2}$.
29. $\tan^{-1}\frac{2b\sqrt{a^2-b^2}}{a^2-2b^2}=2\sin^{-1}\frac{b}{a}$.
30. $\sin^{-1}x+\cos^{-1}y=\tan^{-1}\frac{xy+\sqrt{1-x^2}\sqrt{1-y^2}}{y\sqrt{1-x^2}-x\sqrt{1-y^2}}$.
31. $\sec^{-1}x-\csc^{-1}y=\cos^{-1}\frac{\sqrt{x^2-1}+\sqrt{y^2-1}}{xy}$,

CHAPTER VI

THE SOLUTION OF GENERAL TRIANGLES

50. Four Cases. — As in the case of right triangles the solution of any triangle means the finding of the values of unknown parts from the parts that are known. Of the six parts (three angles and three sides) there must be given three, one of which at least is a side, in order that the triangle may be solved. Consider any triangle, Fig. 24.

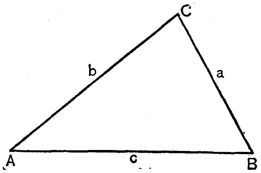

FIG. 24.

The following cases may be enumerated:

I. *Given a side and two angles; say,* a, A, B.

II. *Given two sides and the angle opposite one of them; say,* a, b, A.

III. *Given two sides and the included angle; say,* a, b, C.

IV. *Given the three sides;* a, b, c.

51. The Law of Sines. — Cases I and II may be solved by means of the following theorem.

In any triangle the sides are proportional to the sines of the opposite angles. That is, Fig. 25,

$$a : b : c = \sin A : \sin B : \sin C. \tag{27}$$

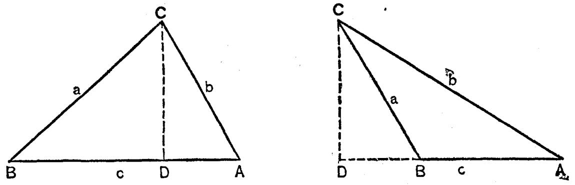

FIG. 25.

Proof: In the triangle BAC draw CD perpendicular to BA. Then

$$\sin A = \frac{DC}{AC} = \frac{DC}{b} \text{ and } \sin B = \frac{DC}{BC} = \frac{DC}{a}.$$

Therefore

$$DC = b \sin A = a \sin B.$$

Whence

$$a : b = \sin A : \sin B.$$

Similarly the theorem may be proved for the other pairs of sides and angles.

52. Case I. *Given a side and two angles ;* $a, A, C.$

To find the third angle we have

$$B = 180° - (A + C).$$

To find B and C we have

$$\frac{b}{a} = \frac{\sin B}{\sin A}, \quad \frac{c}{a} = \frac{\sin C}{\sin A},$$

selecting in each case that proportion, from (27), which involves an unknown side, b or c, and three known parts.

From these two proportions we have

$$\log b = \log a + \log \sin B + \text{colog} \sin A$$

$$\log c = \log a + \log \sin C + \text{colog} \sin A$$

Example. Given $a = 412.7$, $A = 50° 38'$, $C = 69° 13'$, find B, b, c.

$$B = 180° - 119° 51' = 60° 9'.$$

$$\begin{array}{rlcrl}
\log a &= 2.6157 & \qquad & \log a &= 2.6157 \\
\log \sin B &= 9.9382 & & \log \sin C &= 9.9708 \\
\text{colog} \sin A &= 0.1118 & & \text{colog} \sin A &= 0.1118 \\
\hline
\log b &= 2.6657 & & \log c &= 2.6983 \\
b &= 463.1 & & c &= 499.2
\end{array}$$

53. Case II. *Given two sides and the angle opposite one of them; a, b, A.*

We have, to find B,

$$\frac{\sin B}{\sin A} = \frac{b}{a}.$$

Whence, $\log \sin B = \log \sin A + \log b + \text{colog } a$.

Also, $C = 180° - (A + B)$.

Then $\frac{c}{a} = \frac{\sin C}{\sin A}$.

Whence $\log c = \log a + \log \sin C + \text{colog} \sin A$.

Example. Given $a = 31.24$, $b = 49$, $A = 32° 18'$, find B, C, c.

$$\begin{array}{rl}
\log \sin A &= 9.7278 \\
\log b &= 1.6902 \\
\text{colog } a &= 8.5053 \\
\hline
\log \sin B &= 9.9233 \\
B &= 56° 56'
\end{array}$$

But since B is found from the log sine it may have two values; namely, $56° 56'$ and $180° - 56° 56' = 123° 4'$. To determine which value is correct or whether both are possible we recall the theorem of geometry which states that if the given angle is acute and the side opposite is less than the other given side, then it may be possible to construct two triangles from the given parts, two sides and an opposite angle. In the above example the given angle A is acute and its opposite side a is less than b; there are two solutions and both values of B must be used. Figure 26 explains the case graphically.

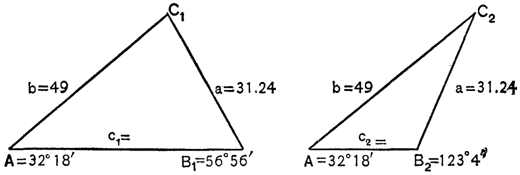

FIG. 26.

Consequently there are two values of C, namely,

$$\begin{aligned} C_1 &= 180° - (A + B_1) \\ &= 90°\ 46' \end{aligned} \qquad \begin{aligned} C_2 &= 180° - (A + B_2) \\ &= 24°\ 38' \end{aligned}$$

and two values of c, got as follows:

$$\begin{array}{r@{\;=\;}l} \log a & 1.4947 \\ \log \sin C_1 & 0.0000 \\ \text{colog} \sin A & 0.2722 \\ \hline \log c_1 & 1.7669 \\ c_1 & 58.46 \end{array} \qquad \begin{array}{r@{\;=\;}l} \log a & 1.4947 \\ \log \sin C_2 & 9.6199 \\ \text{colog} \sin A & 0.2722 \\ \hline \log c_2 & 1.3868 \\ c_2 & 24.37 \end{array}$$

If the given angle be obtuse there will be only one solution. If the given angle, A, be acute and the side a be greater than the side b, there will be one solution only. If A be acute and a be equal to the perpendicular from C to AB, there will be only one solution, a right triangle. In this case $B = 90°$ and $\log \sin B = 0.0000$. If, A being acute, a be less than the perpendicular from C to AB, there is no solution. In this case $\log \sin B$ will be greater than zero, which is impossible since $\sin B$ cannot be greater than unity.

54. Case III may be solved by means of the theorem following:

In any triangle the sum of two sides is to their difference as the tangent of half the sum of the angles opposite the two sides is to the tangent of half their difference.

Proof: By Art. 51

$$a : b = \sin A : \sin B.$$

Whence, by composition and division,

$$\frac{a+b}{a-b} = \frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\sin \frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)}$$

or,

$$\frac{a+b}{a-b} = \frac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}. \tag{28}$$

55. Case III.* *Given two sides and the included angle; b, c, A.*

By Art. 54 we have

$$\frac{\tan\frac{1}{2}(B-C)}{\tan\frac{1}{2}(B+C)} = \frac{b-c}{b+c}.$$

The sides b and c are known, and also

$$\tfrac{1}{2}(B+C) = \tfrac{1}{2}(180° - A) = 90° - \tfrac{1}{2}A,$$

since $\quad A + B + C = 180°.$

Therefore we may write

$$\tan\tfrac{1}{2}(B-C) = \frac{b-c}{b+c}\cdot\tan\tfrac{1}{2}(B+C),$$

or,

$$\log\tan\tfrac{1}{2}(B-C) = \log(b-c) + \text{colog}\,(b+c) + \log\tan\tfrac{1}{2}(B+C).$$

Thus $\frac{1}{2}(B-C)$ is found, and by finding the sum and difference of $\frac{1}{2}(B+C)$ and $\frac{1}{2}(B-C)$ the values of B and C are known. Finally, to determine a we have, as in Case I:

$$a : b = \sin A : \sin B.$$

Example. Given $b = .06239$, $c = .02348$, $A = 110° 32'$; find B, C, a.

$$\tfrac{1}{2}(B+C) = 90° - \tfrac{1}{2}A = 90° - 55° 16' = 34° 44'.$$

$$b + c = .08587 \qquad b - c = .03891.$$

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* See also Art. 61 following.

Whence we have

$$\begin{aligned} \log(b-c) &= 8.5900 \\ \operatorname{colog}(b+c) &= 1.0661 \\ \underline{\log\tan\tfrac{1}{2}(B+C)} &= \underline{9.8409} \\ \log\tan\tfrac{1}{2}(B-C) &= 9.4970 \\ \tfrac{1}{2}(B-C) &= 17^\circ\ 26'. \end{aligned}$$

And as $\frac{1}{2}(B+C) = 34^\circ\ 44'$

We have $B = 52^\circ\ 10'$ $C = 17^\circ\ 18'$.

Then

$$\begin{aligned} \log b &= 8.7951 \\ \log\sin A &= 9.9715 \\ \underline{\operatorname{colog}\sin B} &= \underline{0.1025} \\ \log a &= 8.8691 \\ a &= .07398 \end{aligned}$$

56. The Law of Cosines. Case IV may be solved by means of the following theorem:

In a triangle the square of any side is equal to the sum of the squares of the other two sides minus twice the product of those sides by the cosine of their included angle.

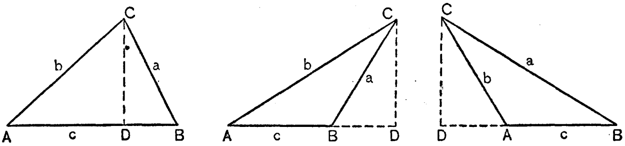


FIG. 27.

That is, Fig. 27, where CD is perpendicular to AB,

$$a^2 = b^2 + c^2 - 2\,bc\cos A.$$

We have proved the geometrical theorem

$$a^2 = b^2 + c^2 - 2\,c \cdot AD.^*$$

But $\cos A = \dfrac{AD}{b}$, or $AD = b\cos A$.

---

* Note that in the first two triangles of Fig. 27, AD, the projection of b, is read left to right and is positive; in the third triangle from right to left and is negative.

Therefore, $a^2 = b^2 + c^2 - 2\,bc \cos A.$ Q.E.D.

Obviously
$$\cos A = \frac{b^2 + c^2 - a^2}{2\,bc} \tag{29}$$

and in the same way

$$\cos B = \frac{c^2 + a^2 - b^2}{2\,ca} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2\,ab}$$

so that the three angles may be found.

57. The objection to the formulæ of Art. 56 is that they are not adapted to logarithmic computation. To remove this objection we proceed as follows: From (29) we have

$$1 - \cos A = 1 - \frac{b^2 + c^2 - a^2}{2\,bc} = \frac{a^2 - b^2 + 2\,bc - c^2}{2\,bc} = \frac{a^2 - (b-c)^2}{2\,bc}$$
$$= \frac{(a - b + c)(a + b - c)}{2\,bc},$$

or, by (17), Art. 44,

$$2 \sin^2 \tfrac{1}{2} A = \frac{(a - b + c)(a + b - c)}{2\,bc}.$$

Let $a + b + c = 2\,s.$

Then $a - b + c = 2(s - b),\ \ a + b - c = 2(s - c),$

Whence $2 \sin^2 \tfrac{1}{2} A = \dfrac{2(s - b) \cdot 2(s - c)}{2\,bc},$

or
$$\sin \tfrac{1}{2} A = \sqrt{\frac{(s - b)(s - c)}{bc}}. \tag{30}$$

Similarly
$$\sin \tfrac{1}{2} B = \sqrt{\frac{(s - c)(s - a)}{ca}}, \qquad \sin \tfrac{1}{2} C = \sqrt{\frac{(s - a)(s - b)}{ab}}.$$

These formulæ may be used for logarithmic computation.

58. Again, from (29), Art. 56, we have

$$1 + \cos A = 1 + \frac{b^2 + c^2 - a^2}{2\,bc} = \frac{(b + c)^2 - a^2}{2\,bc}$$

$$= \frac{(a + b + c)(b + c - a)}{2\,bc},$$

or, by (17), Art. 44,

$$2 \cos^2 \tfrac{1}{2} A = \frac{(a + b + c)(b + c - a)}{2\,bc}.$$

As before, letting $a + b + c = 2\,s$, this becomes

$$2 \cos^2 \tfrac{1}{2} A = \frac{2\,s \cdot 2(s - a)}{2\,bc},$$

or

$$\cos \tfrac{1}{2} A = \sqrt{\frac{s(s - a)}{bc}}. \qquad (31)$$

Similarly

$$\cos \tfrac{1}{2} B = \sqrt{\frac{s(s - b)}{ca}}, \qquad \cos \tfrac{1}{2} C = \sqrt{\frac{s(s - c)}{ab}}.$$

59. These formulæ also may be used for logarithmic computation, but a more convenient set is obtained by dividing the formulæ of Art. 57 by those of Art. 58. We thus obtain

$$\tan \tfrac{1}{2} A = \sqrt{\frac{(s - b)(s - c)}{s(s - a)}} \qquad (32)$$

$$\tan \tfrac{1}{2} B = \sqrt{\frac{(s - c)(s - a)}{s(s - b)}}, \quad \tan \tfrac{1}{2} C = \sqrt{\frac{(s - a)(s - b)}{s(s - c)}}.$$

A comparison of these three sets of formulæ (30), (31), (32), will show that for the complete solution of a triangle when the three sides are given, the first set (30), requires six different logarithms, the second set seven, the third set four. In addition to this slight advantage the tangent set, (32), gives more accurate results than the other two when the angles involved happen to be very small or very near ninety degrees.

60. Case IV. We will now solve a triangle when the *three sides* are known.

Example. Given $a = 10$, $b = 12$, $c = 14$; find A, B, C.

Here $2s = a + b + c = 36$, so that we have

$$\begin{array}{rll} s = 18, & \log s = 1.2553, & \operatorname{colog} s = 8.7447 - 10. \\ s - a = 8, & \log(s-a) = 0.9031, & \operatorname{colog}(s-a) = 9.0969 - 10. \\ s - b = 6, & \log(s-b) = 0.7782, & \operatorname{colog}(s-b) = 9.2218 - 10. \\ s - c = 4, & \log(s-c) = 0.6021, & \operatorname{colog}(s-c) = 9.3979 - 10. \end{array}$$

$$\begin{array}{rr} \log(s-b) = & 0.7782 \\ \log(s-c) = & 0.6021 \\ \operatorname{colog} s = & 8.7447 \\ \underline{\operatorname{colog}(s-a) =} & \underline{9.0969} \\ & 2\overline{)19.2219} \end{array} \qquad \begin{array}{rr} \log(s-c) = & 0.6021 \\ \log(s-a) = & 0.9031 \\ \operatorname{colog} s = & 8.7447 \\ \underline{\operatorname{colog}(s-b) =} & \underline{9.2218} \\ & 2\overline{)19.4717} \end{array}$$

$$\begin{array}{rl} \log \tan \frac{1}{2} A = & 9.6110, \\ \frac{1}{2} A = & 22^\circ\ 13', \\ A = & 44^\circ\ 26', \end{array} \qquad \begin{array}{rl} \log \tan \frac{1}{2} B = & 9.7359, \\ \frac{1}{2} B = & 28^\circ\ 34', \\ B = & 57^\circ\ 8', \end{array}$$

$$\begin{array}{rr} \log(s-a) = & 0.9031 \\ \log(s-b) = & 0.7782 \\ \operatorname{colog} s = & 8.7447 \\ \underline{\operatorname{colog}(s-c) =} & \underline{9.3979} \\ & 2\overline{)19.8239} \end{array}$$

$$\begin{array}{rl} \log \tan \frac{1}{2} C = & 9.9120, \\ \frac{1}{2} C = & 39^\circ\ 14', \\ C = & 78^\circ\ 28'. \end{array}$$

Check: $A + B + C = 180^\circ\ 2'$.

A common method of solving Case IV is by means of an auxiliary quantity,

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

We may write

$$\tan \tfrac{1}{2} A = \frac{r}{s-a}, \qquad \tan \tfrac{1}{2} B = \frac{r}{s-b}, \qquad \tan \tfrac{1}{2} C = \frac{r}{s-c}.$$

In using this method $\log r$ is first found, whence the log-tangents of the three half-angles are readily obtained.

61. Case III; Other Methods of Solution. The formulæ of Art. 56 may sometimes be used to advantage in solving Case III.

Example (see Art. 55). Given $b = .06239$, $c = .02348$, $A = 110° 32'$, to solve the triangle.

We have $$a^2 = b^2 + c^2 - 2\,bc \cos A.$$

Then
$$a^2 = (.06239)^2 + (.02348)^2 - 2(.06239)(.02348) \cos 110° 32'.$$

$$\begin{array}{rlrl}
\log b = & 8.7951 & \log c = & 8.3708 \\
 & \underline{\qquad\quad 2} & & \underline{\qquad\quad 2} \\
\log b^2 = & 7.5902 & \log c^2 = & 6.7416 \\
b^2 = & .003893 & c^2 = & .0005516 \\
+\,c^2 = & .0005516 & \log 2 = & 0.3010 \\
-2\,bc \cos A = & \underline{.001028} & \log b = & 8.7951 \\
a^2 = & .005473 & \log c = & 8.3708 \\
 & & \log \cos A = & \underline{9.5450} \\
 & & \log 2\,bc \cos A = & 7.0119 \\
 & & 2\,bc \cos A = & -.001028
\end{array}$$

Then $\log a = \frac{1}{2} \log a^2 = 8.8691$ and $a = .07398$.

To find B and C we have the formulæ of Art. 51.

The above computation can in some cases be done best and quickest without the use of logarithms.

Another Method of Solution for Case III, preferred by many, is as follows:

From Fig. 27 we see that

$$DC = b \sin A,\ AD = b \cos A,\ DB = c - AD.$$

Then $\tan B = \frac{DC}{DB}$, whence B is known, and $C = 180° - (A + B)$. To find a we use $a = \frac{DC}{\sin B}$.

Applying this method to the example above we have

$$\begin{array}{rl} \log b &= 8.7951 \\ \underline{\log \sin A} &\underline{= 9.9715} \\ \log DC &= 8.7666 \\ \underline{\log DB} &\underline{= 8.6567} \\ \log \tan B &= 0.1099 \\ \\ B &= 52° \, 10' \\ C &= 17° \, 18' \end{array}$$

$$\begin{array}{rl} \log b &= 8.7951 \\ \underline{\log^{-} \cos A} &\underline{= 9.5450} \\ \log^{-} AD &= 8.3401 \\ AD &= -.02188 \\ DB = .02348 + .02188 &= .04536. \\ \\ \log DC &= 8.7666 \\ \underline{\log \sin B} &\underline{= 9.8975} \\ \log a &= 8.8691 \\ a &= \ .07398 \end{array}$$

NOTE. The fundamental importance of the *law of sines* and the *law of cosines* should be noted. By their use, direct or indirect, any triangle whatever may be solved.

AREAS OF TRIANGLES

62. Right Triangles.

Case I. *Given the two legs a and b,* Fig. 28.

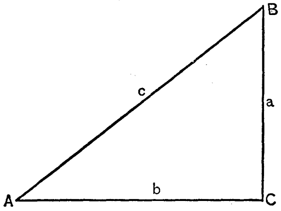


FIG. 28.

Representing the area of the triangle by K, it is obvious that

$$2K = ab. \qquad (33)$$

Case II. *Given the hypotenuse and an acute angle, c and A.*

Then $a = c \sin A, \quad b = c \cos A.$

Whence, by (33),

$$2K = c^2 \sin A \cos A = \tfrac{1}{2} c^2 \sin 2A,$$

or

$$\mathbf{4K = c^2 \sin 2A.}$$

Case III. *Given an angle and the adjacent leg, A and b.*

Then $a = b \tan A.$

Whence, by (33),

$$\mathbf{2K = b^2 \tan A.}$$

Case IV. *Given the hypotenuse and a leg, c and a.*

Then $b^2 = c^2 - a^2$ or $b = \sqrt{(c+a)(c-a)}.$

Whence, by (33),

$$\mathbf{2K = a\sqrt{(c+a)(c-a)}.}$$

63. Oblique Triangles.

Case I. *Given two sides and the included angle, a, b, C.*

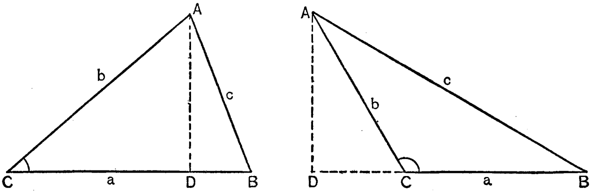


FIG. 29.

In Fig. 29 the line AD is perpendicular to BC.
It is obvious that $2K = a \times DA$.

But $\sin C = \frac{DA}{b}$, or $DA = b \sin C.$

Therefore, $$\mathbf{2K = ab \sin C.} \qquad (34)$$

Case II. *Given a side and the angles, a, A, B, C.*

By Art. 51,

$$\frac{b}{a} = \frac{\sin B}{\sin A} \quad \text{or} \quad b = \frac{a \sin B}{\sin A}.$$

Whence, by (34),

$$2\,K = \frac{a^2 \sin B \cdot \sin C}{\sin A}. \tag{35}$$

Case III. *Given the three sides, a, b, c.*

The formula (34), of Case I may be written, by Art. 42, (13),

$$2\,K = 2\,ab \sin \tfrac{1}{2}\,C \cdot \cos \tfrac{1}{2} C.$$

Substituting in this the values of $\sin \frac{1}{2}\,C$ and $\cos \frac{1}{2}\,C$ given in Arts. 57 and 58, we have

$$2\,K = 2\,ab\sqrt{\frac{(s-a)(s-b)}{ab}} \cdot \sqrt{\frac{s(s-c)}{ab}},$$

or

$$K = \sqrt{s(s-a)(s-b)(s-c)}. \tag{36}$$

EXAMPLES

Solve the following triangles, in each case obtaining also the area of the triangle :

1. $a = 1419$,
   $B = 29^\circ\, 59'$,
   $C = 16^\circ\, 1'$.
2. $a = 3.384$,
   $b = 9.828$,
   $B = 109^\circ$.
3. $a = 302$,
   $b = 427$,
   $C = 134^\circ\, 29'$.
4. $a = 56.22$,
   $b = 63.91$,
   $c = 70.54$.
5. $b = 38.65$,
   $c = 48.12$,
   $B = 34^\circ\, 32'$.
6. $c = 5141$,
   $A = 96^\circ\, 3'$,
   $C = 55^\circ\, 46'$.
7. $b = 56.2$,
   $c = 63.9$,
   $A = 71^\circ\, 33'$.
8. $b = 268.5$,
   $c = 282.9$,
   $C = 75^\circ\, 20'$.
9. $b = 6.362$,
   $A = 76^\circ\, 13'$,
   $C = 35^\circ\, 17'$.
10. $a = 5499$,
    $c = 2959$,
    $A = 133^\circ\, 34'$.
11. $b = .5042$,
    $a = .3618$,
    $B = 74^\circ\, 43'$.
12. $a = .03574$,
    $b = .02921$,
    $c = .01853$.
13. $b = .2792$,
    $a = .2271$,
    $A = 65^\circ\, 45'$.
14. $a = .01044$,
    $A = 26^\circ\, 32'$,
    $B = 146^\circ\, 26'$.
15. $a = 31.49$,
    $b = 49.88$,
    $B = 44^\circ\, 35'$.

16. $c = .0357$,
$a = .0292$,
$B = 31° 7'$.

17. $a = 32.15$,
$b = 67.54$,
$A = 28° 26'$.

18. $b = 4621$,
$a = 6473$,
$B = 31° 7'$.

19. $b = .4312$,
$c = .8901$,
$A = 29° 55'$.

20. $a = 6.743$,
$b = 3.025$,
$c = 4.271$.

21. $c = .01825$,
$b = .02893$,
$B = 83° 30'$.

PROBLEMS IN THE SOLUTION OF TRIANGLES

22. A man owns a triangular lot on the corner of two streets which do not intersect at right angles. The frontage on one street is 300 feet, on the other 250 feet. The back line of the lot is 350 feet long. If he buys land to add 275 feet to the 300-foot frontage, by how much is his lot increased in size?

23. A man owns a triangular lot on the corner of two streets which intersect at an angle of 62°. The frontage on one street is 200 feet, on the other 150 feet. If the land is worth one dollar a square foot and the man has $1200 with which to increase the size of his lot, by how much can he lengthen the 150-foot frontage?

24. The perimeter of a triangle is 100 feet, and the perpendicular from the vertex C to the base AB is 30 feet. The angle A is 50°. Find the length of the base AB.

25. What is the perpendicular height of a hill which is known to rise 72 feet for every 100 feet of length of its slope, if the angle of elevation of the hilltop from a point 100 yards from the base of the hill is 31°?

26. From where I stand, 50 feet from the bank of a stream, the angles of depression of the near and far banks of the stream are respectively 15° 37′ and 6° 24′. How wide is the stream? How far am I above the level of the stream?

27. A man 5 feet 6 inches tall, standing on a bluff 40 feet high, measures the angles of depression of the near and far shores of a bay. The angles are 46° 52′ and 5° 3′ respectively. How wide is the bay?

28. A man 5 feet tall, standing on the edge of a pond, finds the angle of elevation of the top of a tree on the other bank to be 44° 26′. The angle of depression of the reflection of the treetop is 60° 47′. Find the height of the tree.

The reflection of an object appears as far below the surface as the object is above the surface.

29. The frontage on the beach (AB) of a quadrangular lot $ABCD$ cannot be measured. The sides BC, CD, and DA are found to be 236, 155 and 105 feet respectively. The angles DAC and DBC are 32° 20′ and 29° 50′ respectively. Find the length of AB.

30. The bases of a trapezoid are 48.25 and 94.75 feet. The angles at the ends of the longer base are 63° 52′ and 70° 55′. Find the lengths of the other two sides.

31. Two sides of a triangle are 8.53 and 7.41. The difference between the angles opposite these sides is 18° 23′. Solve the triangle.

32. The area of a triangle is 979 square feet. The angle A is 56° 22′ and the side b is 44.80 feet. Solve the triangle.

33. Two sides of a parallelogram are 8005 and 5008. The included angle is 60° 53′. Find the lengths of the diagonals.

34. The diagonals of a quadrangular field $ABCD$ intersect at O at an angle of 78° 3′. The lines AO, BO, CO, and DO are 27.5, 31.8, 58.5 and 63.2 feet respectively. What is the area of the field?

35. Two sides of a triangle are $b = 302$ and $c = 40.8$. Find the angle A so that the triangle may have the same area as the triangle whose sides are 62, 51 and 30. If b were 30.2 and c were 40.8 could A be found? Why?

36. Two vessels start from the same point and sail, one northeast at the rate of 6 miles per hour, and the other east 30° south at the rate of 8 miles per hour. How far apart will the ships be after $2\frac{1}{2}$ hours?

37. A submarine in submerging drifts back 5 feet for every 20 feet it sinks. After the submarine has sunk vertically 300 yards, at what angle must a torpedo be shot from a cruiser one mile away to hit the submarine, if the latter drifts away from the cruiser?

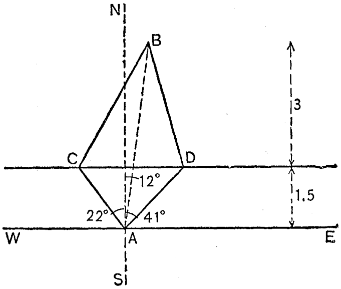
38. A post 6 feet high casts a shadow 10 feet long. What is the length of a flagpole that casts a shadow 60 feet long if the pole makes an angle of 82° with the horizontal on the side away from the sun?

39. In problem 38 find the length of the flagpole if the angle made with the horizontal is 82° on the side towards the sun.

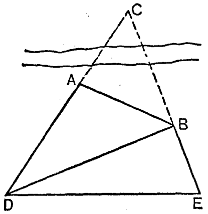
40. Two yachts begin a race by sailing from a point A, along the windward leg of the course in the direction northeast until they reach a buoy B. They then sail before the wind, east 32° south, until they reach a point C, 5 miles east along a straight coast from A. The first yacht sails to windward 5 miles per hour, and before the wind 6.5 miles per hour; the second 5.8 miles per hour to windward and 6 miles before the wind. Which yacht wins the race and by how much?

41. A triangular beach lot has a frontage on the sea of 100 yards. The boundary lines running from the beach make, on the inner side of the lot, angles of 60° and 50° respectively with the shore line. How must a line be drawn from the middle point of the shore line to form two equal lots?

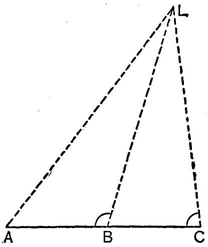
42. A point A, on the south bank of a river 1.5 miles broad and flowing due east is to be connected by bridge and road with a town B, 3 miles back in a straight line from the north bank of the river. It is found that the bridge can be built to a point, C, on the farther bank lying north 22° west from A, or to D lying north 41° east from A. The town B lies north 12° east from A. If the bridge costs \$2000 per mile to build and the road \$500 per mile, which route is the more economical and by how much? (See Fig.)



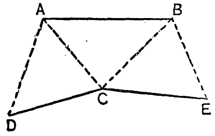
43. The distances of a point C, on the far side of a river from two points A and B on the near side, are to be found but can not be directly measured. In the direction CA a distance $AD = 150$ feet is measured, and in the direction CB a distance $BE = 250$ feet. The distance from A to B is 279.5 feet, and by measurement it is found that $BD = 315.8$ feet, $DE = 498.7$ feet. How far is C from A and B? (See Fig.)



44. The distance from a point, A, on the coast to a lighthouse, L, is to be found. A straight line is run from A along the coast, and on the line two points, B and C, are taken from which the lighthouse is visible. By measurement it is found that $AB = 236.7$ feet, $BC = 215.9$ feet, the angle $ABL = 142°\ 37'$, the angle $ACL = 76°\ 14'$. How far is the lighthouse from A? (See Fig.)



45. On the north side of a river lie two points A and B both of which can be seen from C, and from no other point, on the south side of the stream. From a point D, whose distance from C is 425.3 feet, A and C are sighted. It is found that the angle $ADC = 37°\ 15'$, and the angle $ACD = 42°\ 35'$. From another point E, whose distance from C is 405.4 feet, and from which B and C are visible, the angles $CEB = 53°\ 15'$, and $ECB = 58°\ 5'$ are measured. The angle ACB is also measured and found to be $65°\ 11'$. What is the distance from A to B? (See Fig.)



CHAPTER VII

THE SOLUTION OF TRIGONOMETRIC EQUATIONS

64. The trigonometric equations hitherto dealt with have been identical equations; equations, that is, true for any values of the variables involved. We shall now deal with trigonometric equations which are not identities, and shall examine the methods by which such equations are solved. No methods applicable to all such equations can be given, but methods applicable to several important classes will be discussed. In general it may be said that all such equations are algebraic in form, the one difference being that now the unknown quantities are the trigonometric functions, sine, tangent, etc., or, occasionally, the inverse functions. Therefore, all methods applicable to the solution of algebraic equations are applicable to the solution of trigonometric equations. Moreover, in the case of trigonometric equations we have the various fundamental identities, treated in former chapters, which being true for all values of the variables involved can be used in connection with any equation whose solution is desired.

65. For example, given the equation

$$2 \sin^2 x - \cos^2 x + \tfrac{1}{4} = 0$$

to find the value of x.

In form this is an algebraic, quadratic equation in two unknowns, $\sin x$ and $\cos x$. To find the values of two unknowns we must have two consistent and independent equations. But we also know that $\cos^2 x = 1 - \sin^2 x$. Therefore, our equation may be written

$$2\sin^2 x - (1 - \sin^2 x) = -\tfrac{1}{4},$$

$$3\sin^2 x = \tfrac{3}{4},$$

whence $$\sin x = \pm \tfrac{1}{2},$$

and $$x = \sin^{-1}(\pm \tfrac{1}{2}).$$

The principal values of x are, therefore, $\pm\frac{\pi}{6}$ and the general values are

$$x = n\pi + (-1)^n\left(\pm\frac{\pi}{6}\right) = n\pi \pm \frac{\pi}{6};$$

or we may proceed thus:

Given $$2\sin^2 x - \cos^2 x = -\tfrac{1}{4}.$$

We know $$\sin^2 x + \cos^2 x = 1.$$

Adding the two equations,

$$3\sin^2 x = \tfrac{3}{4}, \text{ etc.}$$

Example 2. Solve the equation

$$\cos x - \sqrt{3}\sin x + 1 = 0.$$

For $\sin x$ substitute $\sqrt{1 - \cos^2 x}$.

Then

$$\cos x - \sqrt{3}\cdot\sqrt{1 - \cos^2 x} + 1 = 0,$$

$$\cos x + 1 = \sqrt{3}\cdot\sqrt{1 - \cos^2 x},$$

$$\cos^2 x + 2\cos x + 1 = 3 - 3\cos^2 x,$$

$$2\cos^2 x + \cos x - 1 = 0,$$

a quadratic equation in $\cos x$ whose roots are

$$\cos x = -1 \text{ or } \tfrac{1}{2}.$$

Therefore,

$$x = \cos^{-1}(-1) = 2n\pi + \pi = (2n + 1)\pi,$$

and $$x = \cos^{-1}\left(\frac{1}{2}\right) = 2n\pi \pm \frac{\pi}{3}.$$

These roots, as in every case, must be tested by substitution in the original equation. It is found that $2n\pi - \frac{\pi}{3}$ does not satisfy the equation, while the other two values do. The roots are, therefore,

$$x = (2n+1)\pi \text{ and } 2n\pi + \frac{\pi}{3}.$$

Another method of solving the last equation is as follows:

Given $$\cos x - \sqrt{3}\sin x = -1.$$

Divide by 2,

$$\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = -\frac{1}{2}.$$

But $$\frac{1}{2} = \cos\frac{\pi}{3} \text{ and } \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}.$$

Therefore we may write,

$$\cos\frac{\pi}{3}\cos x - \sin\frac{\pi}{3}\sin x = -\frac{1}{2}$$

or $$\cos\left(x + \frac{\pi}{3}\right) = -\frac{1}{2}$$

Whence $$x + \frac{\pi}{3} = \cos^{-1}\left(-\frac{1}{2}\right) = 2n\pi \pm \frac{2\pi}{3},$$

and $$x = (2n-1)\pi \text{ or } 2n\pi + \frac{\pi}{3}.$$

Note that the two general solutions $(2n+1)\pi$ and $(2n-1)\pi$ are identical since each represents any odd multiple of π.

66. Special Types of Equations. This last solution is an example of the *type* of equation

1. $$a\cos x + b\sin x = c,\ c \leqq \sqrt{a^2+b^2}.$$

To solve equations of this type divide by $\sqrt{a^2+b^2}$.

$$\frac{a}{\sqrt{a^2+b^2}}\cos x+\frac{b}{\sqrt{a^2+b^2}}\sin x=\frac{c}{\sqrt{a^2+b^2}}.$$

Now $\quad \dfrac{a}{\sqrt{a^2+b^2}}=\cos\alpha$ and $\dfrac{b}{\sqrt{a^2+b^2}}=\sin\alpha$

since

$$\cos^2\alpha+\sin^2\alpha=\left(\frac{a}{\sqrt{a^2+b^2}}\right)^2+\left(\frac{b}{\sqrt{a^2+b^2}}\right)^2=1.$$

Therefore we may write

$$\cos\alpha\cos x+\sin\alpha\sin x=\frac{c}{\sqrt{a^2+b^2}},$$

$$\cos(x-\alpha)=\frac{c}{\sqrt{a^2+b^2}},$$

$$x-\alpha=\cos^{-1}\frac{c}{\sqrt{a^2+b^2}}=2\,n\pi\pm\cos^{-1}\frac{c}{\sqrt{a^2+b^2}},$$

and $\quad x=2\,n\pi\pm\cos^{-1}\dfrac{c}{\sqrt{a^2+b^2}}+\alpha.$

Another *type* of equation is

2. $\quad \boldsymbol{\tan a\theta=\cot b\theta}$ or $\boldsymbol{\sin a\theta=\cos b\theta}$.

We may put

$$\cot b\theta=\tan\left(\frac{\pi}{2}-b\theta\right);\qquad \sin a\theta=\cos\left(\frac{\pi}{2}-a\theta\right).$$

Therefore

$$\tan a\theta=\tan\left(\frac{\pi}{2}-b\theta\right);\qquad \cos b\theta=\cos\left(\frac{\pi}{2}-a\theta\right).$$

$$a\theta=n\pi+\left(\frac{\pi}{2}-b\theta\right);\qquad b\theta=2\,n\pi\pm\left(\frac{\pi}{2}-a\theta\right).$$

or

$$\theta = \frac{n\pi + \frac{\pi}{2}}{a+b}. \qquad \theta = \frac{2\,n\pi \pm \frac{\pi}{2}}{b \pm a}.$$

Example. Given $\tan 3\,\theta = \cot 2\,\theta$, find θ.

$$\cot 2\,\theta = \tan\left(\frac{\pi}{2} - 2\,\theta\right).$$

Therefore,

$$\tan 3\,\theta = \tan\left(\frac{\pi}{2} - 2\,\theta\right),$$

$$3\,\theta = n\pi + \left(\frac{\pi}{2} - 2\,\theta\right),$$

$$\theta = \frac{n\pi + \frac{\pi}{2}}{5} = \frac{(2\,n+1)\pi}{10}.$$

A third *type* of equation is

3.
$$\begin{aligned} \sin ax + \sin bx + \sin cx &= 0,\\ \cos ax + \cos bx + \cos cx &= 0,\\ \cos ax + \cos bx + \sin cx &= 0,\\ \sin ax + \sin bx + \cos cx &= 0. \end{aligned}$$

To solve equations of this type, formulæ (19) and (20) are used.

Example. Solve the equation $\sin 5\,x - \sin 3\,x + \sin x = 0$.

We may write

$$\begin{aligned} \sin 5\,x - \sin 3\,x &= 2\cos\tfrac{1}{2}(5\,x + 3\,x)\sin\tfrac{1}{2}(5\,x - 3\,x)\\ &= 2\cos 4\,x \sin x. \end{aligned}$$

Therefore

$$2\cos 4\,x \sin x + \sin x = 0,$$

$$\sin x\,(2\cos 4\,x + 1) = 0.$$

Whence,

$$\sin x = 0 \qquad \text{or} \qquad 2\cos 4\,x + 1 = 0$$

$$x = \sin^{-1} 0 \qquad 4\,x = \cos^{-1}(-\tfrac{1}{2})$$

$$x = \sin^{-1} 0 = n\pi \qquad 4\,x = \cos^{-1}(-\tfrac{1}{2}) = 2\,n\pi \pm \frac{2\,\pi}{3}$$

$$x = \frac{n\pi}{2} \pm \frac{\pi}{6}.$$

67. Simultaneous equations involving trigonometric functions can in many cases be solved.

Example 1. Given $y = 1 - \cos x$, $y = 1 + \sin x$, find x and y.

We have
$$1 + \sin x = 1 - \cos x,$$
$$\sin x = -\cos x,$$
$$\tan x = -1.$$
$$x = \tan^{-1}(-1) = n\pi + \frac{3\pi}{4},$$

and
$$y = 1 + \sin x = 1 - \cos x = 1 \pm \frac{1}{\sqrt{2}}.$$

Example 2. Given $r\cos\left(\theta - \frac{\pi}{3}\right) = a$, $r\cos\left(\theta - \frac{\pi}{6}\right) = a$, find r and θ.

We have
$$r\cos\left(\theta - \frac{\pi}{3}\right) = r\cos\left(\theta - \frac{\pi}{6}\right),$$
$$\cos\left(\theta - \frac{\pi}{3}\right) = \cos\left(\theta - \frac{\pi}{6}\right),$$
$$\theta - \frac{\pi}{3} = 2n\pi \pm \left(\theta - \frac{\pi}{6}\right),$$
$$\theta = n\pi + \frac{\pi}{4},$$
$$r = a\sec\left(\theta - \frac{\pi}{3}\right) = a\sec\left(\theta - \frac{\pi}{6}\right)$$
$$= a\sec\left(n\pi - \frac{\pi}{12}\right) = a\sec\left(n\pi + \frac{\pi}{12}\right).$$

When n is even, $\sec\left(n\pi - \frac{\pi}{12}\right) = +\sec\frac{\pi}{12}$,
$$\sec\left(n\pi + \frac{\pi}{12}\right) = +\sec\frac{\pi}{12}.$$

When n is odd:

$$\sec\left(n\pi - \frac{\pi}{12}\right) = \sec\left(n\pi + \frac{\pi}{12}\right) = -\csc\frac{\pi}{12}.$$

Therefore, $\quad r = -a\csc\frac{\pi}{12}$, or $a\sec\frac{\pi}{12}$

68. Equations Involving Inverse Trigonometric Functions may, in general, be solved by transforming to other, equivalent equations involving the direct functions. The method of solution is illustrated by the following examples.

Example 1. Solve the equation $2\tan^{-1}x = \cot^{-1}x$.

We have $\quad \cot(2\tan^{-1}x) = \cot(\cot^{-1}x)$

or $$\frac{[\cot(\tan^{-1}x)]^2 - 1}{2\cot(\tan^{-1}x)} = x.$$

That is, $\dfrac{\frac{1}{x^2} - 1}{\frac{2}{x}} = x$ or $3x^2 = 1$; whence $x = \pm\dfrac{1}{\sqrt{3}}$.

Example 2. Solve the equation $\cos^{-1}x + \sin^{-1}2x = 0$.

We have $$\sin(\cos^{-1}x + \sin^{-1}2x) = 0,$$
or
$$\sin(\cos^{-1}x)\cdot\cos(\sin^{-1}2x) + \cos(\cos^{-1}x)\cdot\sin(\sin^{-1}2x) = 0.$$

That is, $\pm\sqrt{1-x^2}\cdot\sqrt{1-4x^2} + x\cdot 2x = 0$.

Whence, $(1-x^2)(1-4x^2) = 4x^4$, $5x^2 = 1$, $x = \pm\dfrac{1}{\sqrt{5}}$.

A second method of solving example 2 is as follows:

$$\cos^{-1}x = -\sin^{-1}2x,$$
$$\sin(\cos^{-1}x) = \sin(-\sin^{-1}2x),$$
$$\sqrt{1-x^2} = -2x,$$
$$5x^2 = 1,$$

$$x = \pm \frac{1}{\sqrt{5}}.$$

In every case the values must be checked by substitution in the original equation.

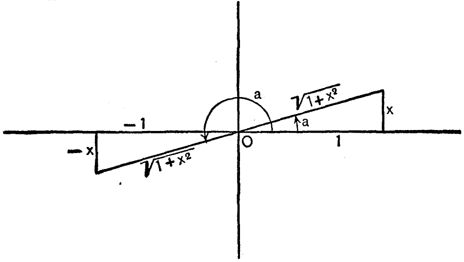


FIG. 30.

It is often convenient, in dealing with inverse functions, to assume the angle whose function is given and to construct a figure to show the values of the remaining functions. Thus, in example 1, we wish to find $\cot(\tan^{-1} x)$. Let $\tan^{-1} x = a$ and construct the angle a, Fig. 30, with ordinate equal to $\pm x$ and abscissa equal to ± 1. The distance is

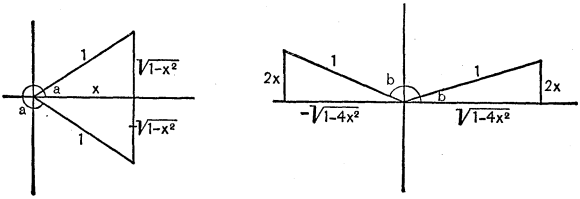


FIG. 31.

then $\sqrt{1+x^2}$, and all the functions are readily found. Thus, $\cot(\tan^{-1} x) = \frac{1}{x}$. Similarly, in example 2, we wish to find $\sin(\cos^{-1} x)$ and $\cos(\sin^{-1} 2x)$. By figure 31 we see assuming $\cos^{-1} x = a$ and $\sin^{-1} 2x = b$, that $\sin(\cos^{-1} x) = \pm\sqrt{1-x^2}$ and $\cos(\sin^{-1} 2x) = \pm\sqrt{1-4x^2}$.

EXAMPLES

Solve the following equations:

1. $\sin 5x - \sin 3x + \sin x = 0.$
2. $\cos\theta + \cos 2\theta + \cos 3\theta = 0.$
3. $\sin 4x - \sin 2x - \cos 3x = 0.$
4. $6\sin\theta + \cos\theta = 2.$
5. $2\sin\theta + \cos\theta = 2.$
6. $\sin 2x = \sin x.$
7. $\cos 2x = \sin x.$
8. $\sin 3\theta = \cos\theta.$
9. $\tan x = \cos x.$
10. $\cos 2x = \frac{1}{2}\cos x.$
11. $\sin 2\theta\cos 2\theta + 2\sin\theta = 0.$
12. $\sin 4x - 2\sin x\cos 2x = 0.$
13. $\sin 4\theta = \cos 2\theta.$
14. $\cos 2\theta = \sin 2\theta - 1.$
15. $\cos(x - a)\cos x - \sin(x - a)\sin x = 0.$
16. $\sec^2 x = 3\csc^2 x.$
17. $\sin\theta\cos\theta - \sin\left(\frac{\pi}{3} - \theta\right)\cos\left(\frac{\pi}{3} - \theta\right) = 0.$
18. $27\csc\theta\cot\theta = 8\sec\theta\tan\theta.$
19. $25\sin\theta - 12.8\csc^2\theta = 0.$
20. $\cos^3\theta - 2\sin^2\theta\cos\theta = 0.$

Solve the following simultaneous equations for x and y, or r and θ.

21. $y = 1 - \cos 2x,$
$y = 1 + \sin 2x.$

22. $r = \sec^2\frac{\theta}{2},$
$r = \csc^2\frac{\theta}{2}.$

23. $r = a\sin\theta,$
$r = a\sin 2\theta.$

24. $y = 2a\cos x,$
$y = 2a\sin x\tan x.$

25. $y\cos x = 2a,$
$y = 5a\sin x.$

26. $r^2\sin 2\theta = 1,$
$r^2\cos 2\theta = 1.$

27. $r = 3\sin\theta + 2\cos\theta,$
$r = 3\cos\theta + 2\sin\theta.$

28. $y^2 = 4\sin 2x,$
$y^2\sin 2x = 1.$

29. $y = \sin x,$
$y = \sin(x + a).$

30. $r\cos\left(\theta - \frac{\pi}{2}\right) = \frac{3a}{4},$
$r = a\sin\theta.$

31. $r^2 = \sin\theta,$
$r^2 = \sin 3\theta.$

32. $r^2\sin 2\theta = 8,$
$r = 2\sec\theta.$

33. $r^2\sin 2\theta = 4,$
$r^2 = 16\sin 2\theta.$

Solve the following equations:

34. $\sin^{-1} x = \sin^{-1} a + \sin^{-1} b.$

35. $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}.$

36. $\sin^{-1}\frac{4}{x} + \sin^{-1}\frac{8}{x} = \frac{\pi}{2}.$

37. $\sin^{-1} x = \cos^{-1}(-x).$

38. $\tan^{-1} 2x + \tan^{-1} 3x = \frac{3\pi}{4}.$

39. $\tan^{-1} x = 2\cos^{-1}\frac{x}{2}.$

40. $\tan^{-1}(x+1) - \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1} 2.$

41. $\cos^{-1}\frac{x^2-1}{x^2+1} + \tan^{-1}\frac{2x}{x^2-1} = \frac{2\pi}{3}.$

42. $\tan^{-1}\frac{x-1}{x+1} + \tan^{-1}\frac{x}{x-1} = \tan^{-1}(-2).$

Solve the following equations, finding only the principal values of angles:

43. $\cos 5x - \cos 3x + \sin x = 0.$

44. $\sin 4x + \sin 2x + 2\cos x = 0.$

45. $\cos 3x = \cos x.$

46. $\cot x = \cos x.$

47. $y = 1 + \cos 2x,$
$y = 1 - \sin 2x.$

48. $r = a\cos\theta,$
$r = a\sin 2\theta.$

49. $y = \tan 2x,$
$y = \cos 2x.$

50. $r = \sin\theta,$
$r = \sin\left(\theta + \frac{\pi}{4}\right).$

51. $\sin 4\theta = \cos\theta.$

52. $3\sin\theta + \cos\theta = 2.$

SPHERICAL TRIGONOMETRY

CHAPTER VIII

FUNDAMENTAL RELATIONS

69. Spherical trigonometry deals with the relations among the sides and angles of a spherical triangle; that is, of a portion of the surface of a sphere bounded by the intersecting arcs of three great circles. It deals also with the computation of unknown parts of such a triangle from parts which are known, the process being called, as in plane trigonometry, the solution of the triangle. The sides of a spherical triangle, being arcs of circles, are expressed in degrees, minutes, and seconds, and, as is customary, we shall consider only those triangles in which each part (angle or side) is less than one hundred and eighty degrees.

70. Law of Cosines. There is one theorem, the law of cosines, which may be called the fundamental theorem of spherical trigonometry, because by means of the theorem any spherical triangle may be solved when three of its parts are known. We shall proceed to prove the *Law of Cosines*.

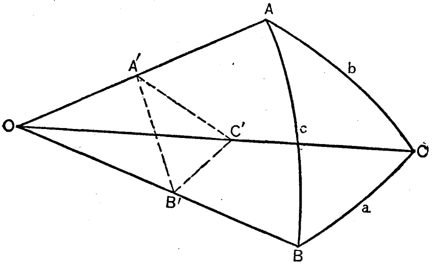


FIG. 32.

Let ABC, Fig. 32, be a spherical triangle on a sphere whose center is O, and let the sides b and c be less than 90°.

Through any point, A', on OA pass a plane perpendicular to OA cutting the planes OAC, OAB, and OBC, in $A'C'$, $A'B'$, and $B'C'$, respectively. Then the angle $B'A'C'$ is the measure of the diedral angle $B\text{–}OA\text{–}C$ and, therefore, of the spherical angle A. Also, by the construction, the angles $OA'B'$ and $OA'C'$ are right angles. In the triangle $A'B'C'$

$$\overline{B'C'}^2 = \overline{B'A'}^2 + \overline{C'A'}^2 - 2\,B'A' \cdot C'A' \cos A,$$

and in the triangle $B'OC'$

$$\overline{B'C'}^2 = \overline{B'O}^2 + \overline{C'O}^2 - 2\,B'O \cdot C'O \cos a.$$

Whence

$$\begin{aligned}&\overline{B'O}^2 + \overline{C'O}^2 - 2\,B'O \cdot C'O \cos a\\ &\quad = \overline{B'A'}^2 + \overline{C'A'}^2 - 2\,B'A' \cdot C'A' \cos A,\end{aligned}$$

or

$$\begin{aligned}&2\,B'O \cdot C'O \cos a\\ &\quad = \overline{B'O}^2 - \overline{B'A'}^2 + \overline{C'O}^2 - \overline{C'A'}^2 + 2\,B'A' \cdot C'A' \cos A.\end{aligned}$$

But $B'OA'$ and $C'OA'$ are right triangles, and therefore,

$$\overline{B'O}^2 - \overline{B'A'}^2 = \overline{OA'}^2;\ \overline{C'O}^2 - \overline{C'A'}^2 = \overline{OA'}^2.$$

We then have

$$B'O \cdot C'O \cos a = \overline{OA'}^2 + B'A' \cdot C'A' \cos A,$$

or

$$\cos a = \frac{OA'}{B'O} \cdot \frac{OA'}{C'O} + \frac{B'A'}{B'O} \cdot \frac{C'A'}{C'O} \cdot \cos A.$$

But

$$\frac{OA'}{B'O} = \cos BOA = \cos c,\quad \frac{OA'}{C'O} = \cos AOC = \cos b,$$

$$\frac{B'A'}{B'O} = \sin BOA = \sin c,\quad \frac{C'A'}{C'O} = \sin AOC = \sin b.$$

Hence

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

which is the law of cosines.

In the above demonstration the sides b and c were taken less than 90° in order that the construction of the right triangles $B'OA'$ and $C'OA'$ might be possible. The resulting theorem, however, is true in all cases. Let us assume $90° < b < 180°$ and $90° < c < 180°$. Then, Fig. 33, produce the arcs AB and AC to meet in A', thus forming a lune. In the triangle $A'BC$, b' and c' are less than 90°.

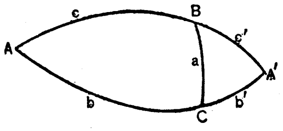


Fig. 33.

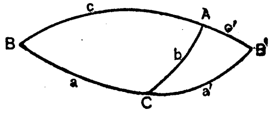


Fig. 34.

The law of cosines is, therefore, true for the triangle $A'BC$, so that, since $A' = A$,

$$\cos a = \cos b' \cos c' + \sin b' \sin c' \cos A.$$

But $$b' = 180° - b \text{ and } c' = 180° - c.$$

Whence

$$\cos a = \cos(180° - b)\cos(180° - c) + \sin(180° - b)\sin(180° - c)\cos A,$$

or, $$\cos a = \cos b \cos c + \sin b \sin c \cos A. \qquad \text{Q.E.D.}$$

Again, let $b < 90°$ and $90° < c < 180°$. Produce the arcs BA and BC, Fig. 34, to meet in B', thus forming a lune. Then, in the triangle $AB'C$, $b < 90°$ and $c' < 90°$, and, therefore,

$$\cos a' = \cos b \cos c' + \sin b \sin c' \cos CAB'.$$

But

$$a' = 180° - a,\ c' = 180° - c, \text{ and } CAB' = 180° - A.$$

Hence

$$\cos(180° - a) = \cos b \cos(180° - c) + \sin b \sin(180° - c)\cos(180° - A),$$

or, $$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

which proves the law of cosines for all cases.

We thus have the three fundamental equations, the *law of cosines:*

$$\begin{aligned}\cos a &= \cos b \cos c + \sin b \sin c \cos A,\\ \cos b &= \cos c \cos a + \sin c \sin a \cos B,\\ \cos c &= \cos a \cos b + \sin a \sin b \cos C,\end{aligned} \tag{37}$$

by means of which any spherical triangle may be solved.

For example, given $a = 60°$, $b = 70°$, $A = 65°$.

We have

$$\cos 60° = \cos 70° \cos c + \sin 70° \sin c \cos 65°,$$

or,

$$.500 = .342 \cos c + .940 \sin c \times .423,$$

$$.342 \cos c + .398 \sin c = .500,$$

$$\frac{.342 \cos c + .398 \sin c}{\sqrt{(.342)^2 + (.398)^2}} = \frac{.500}{\sqrt{(.342)^2 + (.398)^2}},$$

$$\frac{.342}{.525} \cos c + \frac{.398}{.525} \sin c = \frac{.500}{.525},$$

$$.651 \cos c + .758 \sin c = .952,$$

$$.651 = \cos 49.4°, \ .758 = \sin 49.4°.$$

Therefore,

$$\cos (c - 49.4°) = .952,$$

$$c - 49.4° = \cos^{-1} .952 = 18.2° \text{ and } c = 67.6°.$$

Similarly the other parts may be found. The equations are not, however, adapted to logarithmic computation, so that for practical use, as will presently be shown, they must be transformed in various ways.

71. Law of Cosines Applied to the Polar Triangle. The law of cosines, being true for any triangle, is true for the polar triangle of *ABC*. Therefore, denoting the six parts of the polar triangle by the same letters accented, we have

$$\cos a' = \cos b' \cos c' + \sin b' \sin c' \cos A'.$$

But

$$a' = 180^\circ - A,\ b' = 180^\circ - B,\ c' = 180^\circ - C,\ A' = 180^\circ - a.$$

Whence,

$$\cos(180^\circ - A) = \cos(180^\circ - B)\cos(180^\circ - C) + \sin(180^\circ - B)\sin(180^\circ - C)\cdot\cos(180^\circ - a),$$

or

$$-\cos A = \cos B \cos C - \sin B \sin C \cos a,$$

so that the truth of the three following equations is obvious:

$$\begin{aligned}\cos A &= -\cos B \cos C + \sin B \sin C \cos a,\\ \cos B &= -\cos C \cos A + \sin C \sin A \cos b,\\ \cos C &= -\cos A \cos B + \sin A \sin B \cos c.\end{aligned} \tag{38}$$

72. Law of Sines. Another theorem of importance in the solution of spherical triangles, known as the law of sines, is as follows: *In any spherical triangle the sines of the sides are proportional to the sines of the opposite angles.* That is,

$$\sin a : \sin b : \sin c = \sin A : \sin B : \sin C. \tag{39}$$

From equations (37) we have,

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}.$$

Whence

$$1 - \cos^2 A = 1 - \frac{(\cos a - \cos b \cos c)^2}{\sin^2 b \sin^2 c},$$

or,

$$\sin^2 A = \frac{\sin^2 b \sin^2 c - (\cos a - \cos b \cos c)^2}{\sin^2 b \sin^2 c}$$

$$= \frac{(1 - \cos^2 b)(1 - \cos^2 c) - (\cos a - \cos b \cos c)^2}{\sin^2 b \sin^2 c}$$

$$= \frac{1 - \cos^2 b - \cos^2 c - \cos^2 a + 2\cos a \cos b \cos c}{\sin^2 b \sin^2 c}.$$

Whence,

$$\frac{\sin^2 A}{\sin^2 a}=\frac{1-\cos^2 a-\cos^2 b-\cos^2 c+2\cos a\cos b\cos c}{\sin^2 a\sin^2 b\sin^2 c},$$

or,

$$\frac{\sin A}{\sin a}=\frac{\sqrt{1-\cos^2 a-\cos^2 b-\cos^2 c+2\cos a\cos b\cos c}}{\sin a\sin b\sin c}.$$

where the positive sign is taken because A and a are each less than 180°. The right-hand member of this expression is symmetric in a, b, and c, so that if we started with $\cos B$ or $\cos C$ instead of with $\cos A$, the final result for the right-hand member would be identical with that written above. Therefore, obviously, we have

$$\frac{\sin A}{\sin a}=\frac{\sin B}{\sin b}=\frac{\sin C}{\sin c},$$

the law of sines which was to be proved.

CHAPTER IX

THE SOLUTION OF RIGHT SPHERICAL TRIANGLES

73. Special Formulæ for Right Triangles. If we let C be the right angle in a right spherical triangle, and put $C = 90°$ in the third equation of (37), we have

$$\cos c = \cos a \cos b. \tag{40}$$

The third equation of (38) gives

$$0 = -\cos A \cos B + \sin A \sin B \cos c,$$

or,

$$\cos c = \cot A \cot B. \tag{41}$$

The first two equations of (38) give

$$\begin{aligned} \cos A &= \sin B \cos a, \\ \cos B &= \sin A \cos b. \end{aligned} \tag{42}$$

Using the proportions of (39) when $C = 90°$, we have

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{1}{\sin c}.$$

Whence,

$$\begin{aligned} \sin a &= \sin A \sin c, \\ \sin b &= \sin B \sin c. \end{aligned} \tag{43}$$

From (42) by (43) we have

$$\cos A = \sin B \cos a = \frac{\sin b}{\sin c} \cdot \cos a,$$

or, by (40)

$$\cos A = \frac{\sin b}{\sin c} \cdot \frac{\cos c}{\cos b} = \frac{\tan b}{\tan c}.$$

Similarly,

$$\cos B = \frac{\tan a}{\tan c}. \tag{44}$$

By (43), (44), and (40),

$$\tan A = \frac{\sin A}{\cos A} = \frac{\dfrac{\sin a}{\sin c}}{\dfrac{\tan b}{\tan c}} = \frac{\sin a}{\sin c} \cdot \frac{\tan c}{\tan b}$$

$$= \frac{\sin a}{\tan b \cos c} = \frac{\sin a}{\tan b \cdot \cos a \cos b} = \frac{\tan a}{\sin b}.$$

In the same way,

$$\tan B = \frac{\tan b}{\sin a}. \tag{45}$$

74. The formulæ (40) to (45) may be assembled, in a slightly different form, as follows:

$$\begin{aligned}
\sin A &= \frac{\sin a}{\sin c}. & \sin B &= \frac{\sin b}{\sin c}.\\
\cos A &= \frac{\tan b}{\tan c}. & \cos B &= \frac{\tan a}{\tan c}.\\
\tan A &= \frac{\tan a}{\sin b}. & \tan B &= \frac{\tan b}{\sin a}.\\
\cos A &= \cos a \sin B. & \cos B &= \cos b \sin A.\\
\cos c &= \cos a \cos b. & \cos c &= \cot A \cot B.
\end{aligned} \tag{46}$$

A device, known as Napier's Rules, was formulated by Napier to facilitate the remembering of the above formulæ. Let us take for the *five* parts of a right triangle the sides a and b, and the complements of A, B, and c. These five parts, Fig. 35, arrange themselves so that each is a *middle* to *two adjacent parts* and a *middle* to *two opposite parts.*

NAPIER'S RULES state

I. *The sine of the middle part equals the product of the tangents of the adjacent parts.*

II. *The sine of the middle part equals the product of the cosines of the opposite parts.*

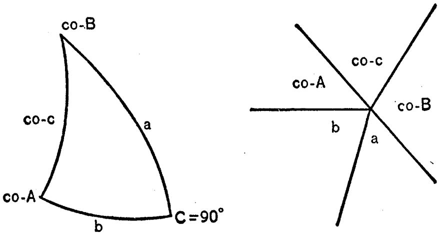


Fig. 35.

By applying these rules to the various parts all the formulæ of (46) may be obtained. Thus, for example,

$$\sin(\text{co-}A) = \tan b \cdot \tan(\text{co-}c).$$

That is,

$$\sin(90° - A) = \tan b \cdot \tan(90° - c),$$

or,

$$\cos A = \tan b \cot c = \frac{\tan b}{\tan c}.$$

75. Rules for Solution. In a right triangle, the right angle being always known, only two other parts need be known to solve the triangle. To solve a right triangle by means of the formulæ (46) we have, therefore, the general rule: *Select that equation which involves the two known parts and one unknown part.*

The algebraic signs of the functions must be carefully noted in order to determine the sign of the resulting function and thereby the angle. If the part to be found is got from a cosine, tangent, or cotangent there is no ambiguity, for if these functions are *plus* the part will have a value less than 90°. If they are *minus* the part will have for its value the supplement of the angle found from the tables of trigonometric functions.

On the other hand, if the unknown part is determined by a sine, the sine being positive for all angles between 0° and 180°, the value may be either that got from the tables or its supplement. In general both solutions must be used unless the ambiguity can be removed by the following laws:

1. *If the sides adjacent to the right angle are in the same quadrant, the hypotenuse is less than* 90° ; *if they are in different quadrants, the hypotenuse is greater than* 90°.

2. *An angle and its opposite side are in the same quadrant.*

Proof of Law 1. By (40) $\cos c = \cos a \cos b$.

Let $a \lessgtr 90°$ and $b \lessgtr 90°$. Then $\cos a = \pm$, $\cos b = \pm$, and $\cos c = (\pm)(\pm) = +$. Therefore, $c < 90°$. Again, let $a \lessgtr 90°$, and $b \gtrless 90°$. Then $\cos a = \pm$, $\cos b = \mp$, $\cos c = (\pm)(\mp) = -$, and $c > 90°$.

Proof of Law 2. By (45) $\sin b = \dfrac{\tan a}{\tan A}$.

Since $\sin b$ is necessarily positive, it follows that $\tan a$ and $\tan A$ are both *plus* or both *minus*. Therefore a and A are each less than 90° or each greater than 90°.

76. The solution of right triangles is illustrated by the following examples:

Example 1. Given $A = 33° \, 50'$, $b = 108°$, find B, a, and c.

From the formulæ (46) we select

$$\tan A = \frac{\tan a}{\sin b}, \qquad \cos A = \frac{\tan b}{\tan c}, \qquad \cos B = \cos b \sin A,$$

or,

$$\tan a = \tan A \sin b, \quad \cot c = \cos A \cot b, \quad \cos B = \sin A \cos b.$$

$$\begin{array}{ll} {}^+\log \tan A = 9.8263 & {}^+\log \cos A = 9.9194 \\ {}^+\log \sin b \;\; = 9.9782 & {}^-\log \cot b \;\; = 9.5118 \\ \hline {}^+\log \tan a \;\; = 9.8045 & {}^-\log \cot c' \;\; = 9.4312 \\ & \qquad\quad c' = 74° \, 54' \\ \qquad\quad a = 32° \, 31' & \qquad\quad c \; = 105° \, 6' \end{array}$$

$$\begin{array}{rl}
{}^{+}\log \sin A & = 9.7457 \\
{}^{-}\log \cos b & = 9.4900 \\
\hline
{}^{-}\log \cos B' & = 9.2357 \\
B' & = 80^\circ\ 6' \\
B & = 99^\circ\ 54'
\end{array}$$

To check the results we select a formula involving the three parts to be found; a, c, and B. Thus $\cos B = \dfrac{\tan a}{\tan c}$, or,

$$\log \cos B = \log \tan a + \log \cot c$$

$$9.2357 = 9.8045 + 9.4312 \qquad \text{CHECK.}$$

Example 2. Given $a = 47^\circ\ 30'$, $c = 120^\circ\ 20'$, find A, B, and b.

We have

$$\sin A = \frac{\sin a}{\sin c}, \qquad \cos B = \frac{\tan a}{\tan c}, \qquad \cos b = \frac{\cos c}{\cos a}.$$

$$\begin{array}{lll}
\log \sin a = 9.8676 & {}^{+}\log \tan a = 0.0379 & {}^{-}\log \cos c = 9.7033 \\
\log \sin c = 9.9361 & {}^{-}\log \tan c = 0.2327 & {}^{+}\log \cos a = 9.8297 \\
\hline
\log \sin A = 9.9315 & {}^{-}\log \cos B' = 9.8052 & {}^{-}\log \cos b' = 9.8736 \\
 & \qquad B' = 50^\circ\ 19' & \qquad b' = 41^\circ\ 38' \\
\quad A = 58^\circ 40' & \qquad B = 129^\circ\ 41' & \qquad b = 138^\circ\ 22
\end{array}$$

CHECK. $$\cos B = \cos b \sin A$$

$$\log \cos B = \log \cos b + \log \sin A$$

$$9.8052 = 9.8736 + 9.9315 = 9.8051.$$

NOTE. The value of A less than 90° is taken by virtue of law 2, Art. 75.

Example 3. Given $B = 105^\circ\ 59'$, $b = 128^\circ\ 33'$, find A, a, and c.

We have,

$$\cos B = \cos b \sin A, \qquad \tan B = \frac{\tan b}{\sin a}, \qquad \sin B = \frac{\sin b}{\sin c},$$

or,

$$\sin A = \frac{\cos B}{\cos b}, \qquad \sin a = \tan b \cot B, \qquad \sin c = \frac{\sin b}{\sin B}.$$

$$\begin{array}{lll}
{}^{-}\log \cos B = 9.4399 & {}^{-}\log \cot B = 9.4570 & \log \sin b = 9.8932 \\
{}^{-}\log \cos b = 9.7946 & {}^{-}\log \tan b = 0.0986 & \log \sin B = 9.9828 \\
\hline
\log \sin A = 9.6453 & \log \sin a = 9.5556 & \log \sin c = 9.9104 \\
\quad A_1 = 26^\circ\ 14' & \quad a_1 = 21^\circ\ 4' & \quad c_1 = 54^\circ\ 27' \\
\quad A_2 = 153^\circ\ 46' & \quad a_2 = 158^\circ\ 56' & \quad c_2 = 125^\circ\ 33'
\end{array}$$

CHECK. $\sin a = \sin A \sin c.$

$$\log \sin a = \log \sin A + \log \sin c$$

$$9.5556 = 9.6453 + 9.9104 = 9.5557.$$

By law 2 both sets of values must be used; but by law 1 the acute value c_1 belongs with the obtuse values of A and a, the obtuse value c_2 with the acute values of A and a. Thus the two solutions are:

1. $A = 26^\circ\ 14'$, $a = 21^\circ\ 4'$, $c = 125^\circ\ 33'$.
2. $A = 153^\circ\ 46'$, $a = 158^\circ\ 56'$, $c = 54^\circ\ 27'$.

NOTE. *A quadrantal spherical triangle* is one which has a side equal to a quadrant. The polar triangle of a quadrantal triangle is right. Therefore, to solve a quadrantal triangle solve its polar triangle and take the supplements of the parts thus found.

EXAMPLES

Solve the following triangles in which $C = 90^\circ$.

1. $A = 40^\circ\ 13'$, $a = 26^\circ\ 25'$.

2. $B = 83^\circ\ 15'$, $b = 76^\circ\ 46'$.

3. $B = 110^\circ\ 50'$, $b = 118^\circ\ 30'$.

4. $b = 127^\circ\ 36'$, $c = 94^\circ\ 52'$.

5. $a = 165^\circ\ 19'$, $c = 46^\circ\ 50'$.

6. $b = 40^\circ\ 49'$, $c = 135^\circ\ 40'$.

7. $a = 21^\circ\ 18'$, $b = 49^\circ\ 55'$.

8. $a = 78^\circ\ 32'$, $b = 132^\circ\ 25'$.

9. $a = 144^\circ\ 1'$, $b = 123^\circ\ 6'$.

10. $A = 59^\circ\ 17'$, $B = 51^\circ\ 46'$.

11. $A = 137^\circ\ 18'$, $B = 119^\circ\ 30'$.

12. $A = 71^\circ\ 46'$, $B = 148^\circ\ 3'$.

13. $A = 20^\circ\ 34'$, $c = 23^\circ\ 18'$.

14. $B = 97^\circ\ 36'$, $c = 96^\circ\ 31'$.

15. $A = 100^\circ\ 38'$, $c = 51^\circ\ 44'$.

16. $B = 59^\circ\ 54'$, $a = 6^\circ\ 50'$.

17. $B = 47^\circ\ 34'$, $a = 144^\circ\ 24'$.

18. $A = 102^\circ\ 49'$, $b = 10^\circ\ 19'$.

19. $A = 98^\circ\ 17'$, $a = 143^\circ\ 8'$.

20. $a = 172^\circ\ 28'$, $c = 124^\circ\ 39'$.

21. $a = 4^\circ\ 54'$, $b = 169^\circ\ 27'$.

22. $A = 76^\circ\ 17'$, $B = 144^\circ\ 1'$.

23. $B = 82^\circ\ 43'$, $c = 99^\circ\ 26'$.

24. $B = 99^\circ\ 47'$, $a = 26^\circ\ 43'$.

25. $A = 97^\circ\ 24'$, $a = 103^\circ\ 12'$.

26. $b = 164^\circ\ 10'$, $c = 133^\circ\ 50'$.

27. $b = 34^\circ\ 3'$, $a = 54^\circ\ 26'$.

28. $A = 156^\circ\ 30'$, $B = 104^\circ\ 50'$.

29. $A = 165^\circ\ 1'$, $c = 50^\circ\ 30'$.

30. $B = 37^\circ\ 56'$, $a = 157^\circ\ 12'$.

Solve the following quadrantal spherical triangles ($c = 90^\circ$) :

31. $A = 30^\circ\ 12'$, $a = 72^\circ\ 29'$.

32. $B = 118^\circ\ 16'$, $a = 137^\circ\ 57'$.

33. $b = 51^\circ\ 33'$, $C = 25^\circ\ 48'$.

34. $A = 141^\circ\ 13'$, $C = 49^\circ\ 35'$.

35. $A = 159^\circ\ 20'$, $a = 136^\circ\ 30'$.

36. $b = 18^\circ\ 41'$, $A = 39^\circ\ 24'$.

CHAPTER X

THE SOLUTION OF OBLIQUE SPHERICAL TRIANGLES

77. Six Cases may be enumerated in the solution of oblique spherical triangles.

1. *Given the three sides,* a, b, c.

2. *Given the three angles,* A, B, C.

3. *Given two sides and the included angle,* a, b, C.

4. *Given two angles and the included side,* A, B, c.

5. *Given two sides and the angle opposite one of them,* a, b, A.

6. *Given two angles and the side opposite one of them,* A, B, a.

We shall proceed to consider these cases in the order named.

78. Case 1. *Given the three sides.* The law of cosines is sufficient to solve this case, but the equations are not adapted to logarithmic computation. We therefore develop them as follows:

We have proved the formula

$$\tan \tfrac{1}{2} A = \sqrt{\frac{1 - \cos A}{1 + \cos A}}.$$

By the law of cosines

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}.$$

Whence

$$\frac{1-\cos A}{1+\cos A}=\frac{\sin b\sin c+\cos b\cos c-\cos a}{\sin b\sin c-\cos b\cos c+\cos a}$$

$$=\frac{\cos(b-c)-\cos a}{\cos a-\cos(b+c)}=\frac{\cos a-\cos(b-c)}{\cos(b+c)-\cos a}.$$

But

$$\cos a-\cos(b-c)=-2\sin\tfrac{1}{2}(a+b-c)\sin\tfrac{1}{2}(a-b+c),$$

$$\cos(b+c)-\cos a=-2\sin\tfrac{1}{2}(a+b+c)\sin\tfrac{1}{2}(b+c-a).$$

Hence

$$\frac{1-\cos A}{1+\cos A}=\frac{\sin\frac{1}{2}(a+b-c)\sin\frac{1}{2}(a-b+c)}{\sin\frac{1}{2}(a+b+c)\sin\frac{1}{2}(b+c-a)}.$$

Let $a+b+c=2s$; then $a+b-c=2(s-c)$,

$a-b+c=2(s-b)$, and $b+c-a=2(s-a)$.

Therefore, $$\frac{1-\cos A}{1+\cos A}=\frac{\sin(s-b)\sin(s-c)}{\sin s\sin(s-a)},$$

or $$\tan\tfrac{1}{2}A=\sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s\sin(s-a)}}. \tag{47}$$

Similarly,

$$\tan\tfrac{1}{2}B=\sqrt{\frac{\sin(s-c)\sin(s-a)}{\sin s\sin(s-b)}},$$

$$\tan\tfrac{1}{2}C=\sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin s\sin(s-c)}}.$$

We may write

$$\tan\tfrac{1}{2}A=\frac{1}{\sin(s-a)}\cdot\sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}},$$

or, putting

$$k=\sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}},$$

$$\tan\tfrac{1}{2}A=\frac{k}{\sin(s-a)}, \tag{48}$$

$$\tan\tfrac{1}{2}B=\frac{k}{\sin(s-b)},\quad\tan\tfrac{1}{2}C=\frac{k}{\sin(s-c)}.$$

Either set of formulæ (47) or (48) may be used in the solution of Case 1. If a *check* is desired in the solution the law of sines may be so used. Thus, since

$$\frac{\sin A}{\sin a}=\frac{\sin B}{\sin b}=\frac{\sin C}{\sin c},$$

it follows that

$$\begin{aligned}\log\sin A-\log\sin a&=\log\sin B-\log\sin b\\&=\log\sin C-\log\sin c.\end{aligned}$$

It must be remembered, however, that results may check and still be incorrect. If they do not check they are wrong; if they check they *may* be right, or may be wrong, since errors may compensate each other. It is important to check one's work, but far more important to learn, by careful attention, to work accurately.

Example. Given $a=103°$, $b=53°$, $c=61°$, find A, B, and C.

Using the formulæ (48) we find

$$s=\tfrac{1}{2}(103°+53°+61°)=108°\,30',$$

$$s-a=5°\,30',\ s-b=55°\,30',\ s-c=47°\,30'.$$

$$\begin{aligned}\log\sin(s-a)&=\ \ 8.9816\\\log\sin(s-b)&=\ \ 9.9160\\\log\sin(s-c)&=\ \ 9.8676\\\log\csc s&=\ \ \underline{0.0230}\\&\ 2\overline{)18.7882}\\\log k&=\ \ 9.3941\end{aligned}$$

$$\begin{aligned}\log k&=9.3941\\\log\sin(s-a)&=\underline{8.9816}\\\log\tan\tfrac{1}{2}A&=0.4125\\\tfrac{1}{2}A&=68°\,51',\\A&=137°\,42',\end{aligned}\qquad\qquad\begin{aligned}\log k&=9.3941\\\log\sin(s-b)&=\underline{9.9160}\\\log\tan\tfrac{1}{2}B&=9.4781\\\tfrac{1}{2}B&=16°\,44',\\B&=33°\,28',\end{aligned}$$

$$\begin{array}{r} \log k = 9.3941 \\ \underline{\log \sin (s - c) = 9.8676} \\ \log \tan \tfrac{1}{2} C = 9.5265 \\ \tfrac{1}{2} C = 18^\circ\ 35', \\ C = 37^\circ\ 10'. \end{array}$$

CHECK.

$$\begin{array}{rrr} \log \sin A = 9.8280 & \log \sin B = 9.7415 & \log \sin C = 9.7811 \\ \underline{\log \sin a\ \ = 9.9887} & \underline{\log \sin b\ \ = 9.9023} & \underline{\log \sin c\ \ = 9.9418} \\ 9.8393 & 9.8392 & 9.8393 \end{array}$$

79. Case 2. *Given the three angles.* This case may be solved by the same formulæ that are used in Case 1, by making use of the *principle of polar triangles.* Thus, using accented letters to represent the corresponding parts of the polar triangle, we have $a' = 180^\circ - A$, $b' = 180^\circ - B$, $c' = 180^\circ - C$. Knowing the sides a', b', c', we can find the angles A', B', C', as in Art. 78. Then the sides of the original triangle will be

$$a = 180^\circ - A',\ b = 180^\circ - B',\ c = 180^\circ - C'.$$

Example. Given $A = 123^\circ$, $B = 43^\circ$, $C = 64^\circ$, find a, b, c.

Here $\quad a' = 180^\circ - A = 57^\circ,\ b' = 137^\circ,\ c' = 116^\circ,$

$$s = \tfrac{1}{2}(57^\circ + 137^\circ + 116^\circ) = 155^\circ,$$

$$s - a' = 98^\circ,\ s - b' = 18^\circ,\ s - c' = 39^\circ.$$

$$\begin{array}{rr} \log \sin (s - a') = & 9.9958 \\ \log \sin (s - b') = & 9.4900 \\ \log \sin (s - c') = & 9.7989 \\ \log \csc s = & \underline{0.3741} \\ & 2\overline{)19.6588} \\ \log k = & 9.8294 \end{array}$$

$$\begin{array}{rr} \log k = 9.8294 & \log k = 9.8294 \\ \underline{\log \sin (s - a') = 9.9958} & \underline{\log \sin (s - b') = 9.4900} \\ \log \tan \tfrac{1}{2} A' = 9.8336 & \log \tan \tfrac{1}{2} B' = 0.3394 \\ \tfrac{1}{2} A' = 34^\circ\ 17', & \tfrac{1}{2} B' = 65^\circ\ 24', \\ A' = 68^\circ\ 34', & B' = 130^\circ\ 48', \end{array}$$

$$\begin{aligned} \log k &= 9.8294 \\ \log \sin (s - c') &= 9.7989 \\ \hline \log \tan \tfrac{1}{2} C' &= 0.0305 \\ \tfrac{1}{2} C' &= 47^\circ\ 1', \\ C' &= 94^\circ\ 2'. \end{aligned}$$

Therefore

$$a = 111^\circ\ 26', \qquad b = 49^\circ\ 12', \qquad c = 85^\circ\ 58'.$$

CHECK.

$\log \sin A = 9.9236$	$\log \sin B = 9.8338$	$\log \sin C = 9.9537$
$\log \sin a = 9.9689$	$\log \sin b = 9.8791$	$\log \sin c = 9.9989$
9.9547	9.9547	9.9548

NOTE. Using the law of cosines as stated in (38), Art. 71, whence

$$\cos a = \frac{\cos B \cos C + \cos A}{\sin B \sin C},$$

and proceeding as in Art. 78, the following formulæ may be got:

$$\cot \tfrac{1}{2} a = \sqrt{-\frac{\cos (S - B) \cos (S - C)}{\cos S \cos (S - A)}},$$

or

$$\cot \tfrac{1}{2} a = \frac{K}{\cos (S - A)},$$

where

$$K = \sqrt{-\frac{\cos (S - A) \cos (S - B) \cos (S - C)}{\cos S}},$$

and similar formulæ for $\cot \frac{1}{2} b$ and $\cot \frac{1}{2} c$. These formulæ are simple and convenient, but it is unnecessary to burden the memory with them.

80. Cases 3 and 4, *two sides and the included angle, two angles and the included side,* are solved by means of **Napier's Analogies,** which we shall proceed to derive.

From (48), Art. 78, we may write

$$\tan \frac{A}{2} \tan \frac{B}{2} = \frac{k^2}{\sin (s - a) \sin (s - b)},$$

or, since
$$k^2 = \frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s},$$

$$\frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} \cdot \frac{\sin\frac{B}{2}}{\cos\frac{B}{2}} = \frac{\sin(s-c)}{\sin s}. \qquad (\alpha)$$

Whence,
$$1 - \frac{\sin\frac{A}{2}\sin\frac{B}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}} = 1 - \frac{\sin(s-c)}{\sin s},$$

or,
$$\frac{\cos\frac{A}{2}\cos\frac{B}{2} - \sin\frac{A}{2}\sin\frac{B}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}} = \frac{\sin s - \sin(s-c)}{\sin s}.$$

That is,
$$\frac{\cos\frac{1}{2}(A+B)}{\cos\frac{A}{2}\cos\frac{B}{2}} = \frac{2\cos\frac{1}{2}(2s-c)\sin\frac{1}{2}c}{\sin s}.$$

Whence, since $2s - c = a + b + c - c = a + b$,
$$\frac{\cos\frac{1}{2}(A+B)}{\cos\frac{A}{2}\cos\frac{B}{2}} = \frac{2\cos\frac{1}{2}(a+b)\sin\frac{1}{2}c}{\sin s}. \qquad (\beta)$$

Also, from (α) above,
$$1 + \frac{\sin\frac{A}{2}\sin\frac{B}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}} = 1 + \frac{\sin(s-c)}{\sin s},$$

which being transformed in the same manner gives
$$\frac{\cos\frac{1}{2}(A-B)}{\cos\frac{A}{2}\cos\frac{B}{2}} = \frac{2\sin\frac{1}{2}(a+b)\cos\frac{1}{2}c}{\sin s}. \qquad (\gamma)$$

Dividing (β) by (γ) we have

$$\frac{\cos\frac{1}{2}(A+B)}{\cos\frac{1}{2}(A-B)}=\frac{\tan\frac{1}{2}c}{\tan\frac{1}{2}(a+b)}. \tag{49}$$

Again, from (48) Art. 78 we may write

$$\frac{\tan\dfrac{A}{2}}{\tan\dfrac{B}{2}}=\frac{\sin(s-b)}{\sin(s-a)},$$

or,

$$\frac{\sin\dfrac{A}{2}\cos\dfrac{B}{2}}{\cos\dfrac{A}{2}\sin\dfrac{B}{2}}=\frac{\sin(s-b)}{\sin(s-a)}.$$

Whence,

$$\frac{\sin\dfrac{A}{2}\cos\dfrac{B}{2}}{\cos\dfrac{A}{2}\sin\dfrac{B}{2}}\pm 1=\frac{\sin(s-b)}{\sin(s-a)}\pm 1$$

$$\frac{\sin\dfrac{A}{2}\cos\dfrac{B}{2}\pm\cos\dfrac{A}{2}\sin\dfrac{B}{2}}{\cos\dfrac{A}{2}\sin\dfrac{B}{2}}=\frac{\sin(s-b)\pm\sin(s-a)}{\sin(s-a)}.$$

Using the upper signs,

$$\frac{\sin\frac{1}{2}(A+B)}{\cos\dfrac{A}{2}\sin\dfrac{B}{2}}=\frac{2\sin\frac{1}{2}(2s-a-b)\cos\frac{1}{2}(a-b)}{\sin(s-a)}$$

$$=\frac{2\sin\frac{1}{2}c\cos\frac{1}{2}(a-b)}{\sin(s-a)}. \tag{δ}$$

Using the lower signs,

$$\frac{\sin\frac{1}{2}(A-B)}{\cos\dfrac{A}{2}\sin\dfrac{B}{2}}=\frac{2\cos\frac{1}{2}(2s-a-b)\sin\frac{1}{2}(a-b)}{\sin(s-a)}$$

$$=\frac{2\cos\frac{1}{2}c\sin\frac{1}{2}(a-b)}{\sin(s-a)}. \tag{ϵ}$$

Dividing (δ) by (ε),

$$\frac{\sin \frac{1}{2}(A+B)}{\sin \frac{1}{2}(A-B)} = \frac{\tan \frac{1}{2} c}{\tan \frac{1}{2}(a-b)}. \tag{50}$$

Applying (49) and (50) to the polar triangles we obtain

$$\frac{\cos \frac{1}{2}(A'+B')}{\cos \frac{1}{2}(A'-B')} = \frac{\tan \frac{1}{2} c'}{\tan \frac{1}{2}(a'+b')},$$

$$\frac{\sin \frac{1}{2}(A'+B')}{\sin \frac{1}{2}(A'-B')} = \frac{\tan \frac{1}{2} c'}{\tan \frac{1}{2}(a'-b')}.$$

Remembering that $A' = 180° - a$, $a' = 180° - A$, etc. these become

$$\frac{\cos \frac{1}{2}(a+b)}{\cos \frac{1}{2}(a-b)} = \frac{\cot \frac{1}{2} C}{\tan \frac{1}{2}(A+B)}, \tag{51}$$

$$\frac{\sin \frac{1}{2}(a+b)}{\sin \frac{1}{2}(a-b)} = \frac{\cot \frac{1}{2} C}{\tan \frac{1}{2}(A-B)}. \tag{52}$$

The formulæ (49), (50), (51), and (52), called **Napier's Analogies** because of their similarity to formula (28) of the plane trigonometry, can obviously be written in other forms by the cyclical interchange of the letters.

81. Case 3. Example. Given $a = 100° 30'$, $b = 40° 20'$, $C = 46° 40'$, find A, B, c. Napier's analogies (51) and (52) may be written

$$\tan \tfrac{1}{2}(A+B) = \frac{\cos \frac{1}{2}(a-b) \cot \frac{1}{2} C}{\cos \frac{1}{2}(a+b)},$$

$$\tan \tfrac{1}{2}(A-B) = \frac{\sin \frac{1}{2}(a-b) \cot \frac{1}{2} C}{\sin \frac{1}{2}(a+b)},$$

which will determine A and B. Then to find c we may use either (49) or (50). The latter may be written

$$\tan \tfrac{1}{2} c = \frac{\sin \frac{1}{2}(A+B) \tan \frac{1}{2}(a-b)}{\sin \frac{1}{2}(A-B)}.$$

We have

$$\tfrac{1}{2}(a+b)=70^\circ\,25',\ \tfrac{1}{2}(a-b)=30^\circ\,5',\ \tfrac{1}{2}C=23^\circ\,20'.$$

$$\begin{array}{rl@{\qquad}rl}
\log\cos\frac{1}{2}(a-b) &= 9.9371 & \log\sin\frac{1}{2}(a-b) &= 9.7001\\
\log\cot\frac{1}{2}C &= 0.3652 & \log\cot\frac{1}{2}C &= 0.3652\\
\log\sec\frac{1}{2}(a+b) &= 0.4748 & \log\csc\frac{1}{2}(a+b) &= 0.0259\\
\hline
\log\tan\frac{1}{2}(A+B) &= 0.7771 & \log\tan\frac{1}{2}(A-B) &= 0.0912\\
\frac{1}{2}(A+B) &= 80^\circ\,31', & \frac{1}{2}(A-B) &= 50^\circ\,59'.
\end{array}$$

Whence $A=131^\circ\,30'$, $B=29^\circ\,32'$.

$$\begin{array}{rl}
\log\sin\frac{1}{2}(A+B) &= 9.9940\\
\log\tan\frac{1}{2}(a-b) &= 9.7629\\
\log\csc\frac{1}{2}(A-B) &= 0.1096\\
\hline
\log\tan\frac{1}{2}c &= 9.8665\\
\frac{1}{2}c &= 36^\circ\,20',\\
c &= 72^\circ\,40'.
\end{array}$$

The signs are all *plus* in the above computation.

Case 4. Example. Given $B=110^\circ\,40'$, $C=100^\circ\,36'$, $a=76^\circ\,38'$, find b, c, A.

Napier's analogies (49) and (50) may be written

$$\tan\tfrac{1}{2}(b+c)=\frac{\cos\frac{1}{2}(B-C)\tan\frac{1}{2}a}{\cos\frac{1}{2}(B+C)},$$

$$\tan\tfrac{1}{2}(b-c)=\frac{\sin\frac{1}{2}(B-C)\tan\frac{1}{2}a}{\sin\frac{1}{2}(B+C)},$$

which will determine b and c. To find A either (51) or (52) may be used. The latter is

$$\cot\tfrac{1}{2}A=\frac{\sin\frac{1}{2}(b+c)\tan\frac{1}{2}(B-C)}{\sin\frac{1}{2}(b-c)}.$$

Here $\frac{1}{2}(B+C)=105^\circ\,38'$, $\frac{1}{2}(B-C)=5^\circ\,2'$, $\frac{1}{2}a=38^\circ\,19'$.

$$\begin{array}{rl@{\qquad}rl}
{}^{+}\log\cos\frac{1}{2}(B-C) &= 9.9983 & {}^{+}\log\sin\frac{1}{2}(B-C) &= 8.9432\\
{}^{+}\log\tan\frac{1}{2}a &= 9.8977 & {}^{+}\log\tan\frac{1}{2}a &= 9.8977\\
{}^{-}\log\sec\frac{1}{2}(B+C) &= 0.5695 & {}^{+}\log\csc\frac{1}{2}(B+C) &= 0.0164\\
\hline
{}^{-}\log\tan\frac{1}{2}(b+c) &= 0.4655 & {}^{+}\log\tan\frac{1}{2}(b-c) &= 8.8573\\
180^\circ-\frac{1}{2}(b+c) &= 71^\circ\,6', & \frac{1}{2}(b-c) &= 4^\circ\,7'.\\
\frac{1}{2}(b+c) &= 108^\circ\,54'. & &
\end{array}$$

Whence $b = 113° 1'$, $c = 104° 47'$.

$$\begin{aligned}
\log^{+} \sin \tfrac{1}{2}(b+c) &= 9.9759 \\
\log^{+} \tan (B-C) &= 8.9449 \\
\log^{+} \csc \tfrac{1}{2}(b-c) &= 1.1440 \\
\hline
\log^{+} \cot \tfrac{1}{2}A &= 0.0648 \\
\tfrac{1}{2}A &= 40° 44', \\
A &= 81° 28'.
\end{aligned}$$

Note that the algebraic signs are not all *plus*, and that the quadrant in which the angle lies is determined by the sign in the case of the tangent, cotangent, or cosine.

82. Cases 5 and 6, *two sides and an opposite angle* or *two angles and an opposite side*, may be solved by the law of sines together with Napier's analogies. Thus, if a, b and A are given, we may write

$$\sin B = \frac{\sin b \sin A}{\sin a}$$

which, however, does not determine B unambiguously, since B may be either acute or obtuse. In this case, indeed, there may be two solutions, one solution, or none. We know, however, that if two sides (or angles) of a spherical triangle are unequal the angles (or sides) opposite are unequal, and the greater angle (or side) lies opposite the greater side (or angle). These theorems enable us to determine which values of the angle (or side) are possible. Thus *if* $b \lessgtr a$, *then only values of* B *which are* $\left\{\begin{matrix} \textit{less} \\ \textit{greater} \end{matrix}\right\}$ *than* A *are possible;* both values of B may be so, or only one value. *If the sine of* B *is greater than unity;* that is, *if log sin* B *is positive, no solution is possible.* These same considerations obviously apply to case 6 also.

Another method of removing the ambiguity of Cases 5 and 6 is as follows: Two angles are of the *same species* when they are both acute or both obtuse. Also, since each side and angle of a spherical triangle is less than 180°, we see that $\frac{1}{2}(A+B)$ and $\frac{1}{2}(a+b)$ are each less than 180°; while $\frac{1}{2}(A-B)$ and $\frac{1}{2}c$ are each less than 90°. It follows, in Napier's first analogy,

$$\frac{\cos\frac{1}{2}(A+B)}{\cos\frac{1}{2}(A-B)}=\frac{\tan\frac{1}{2}c}{\tan\frac{1}{2}(a+b)},$$

that $\tan\frac{1}{2}c$ and $\cos\frac{1}{2}(A-B)$ are both positive. Then $\cos\frac{1}{2}(A+B)$ and $\tan\frac{1}{2}(a+b)$ must have the same algebraic sign, and, therefore, $\frac{1}{2}(A+B)$ and $\frac{1}{2}(a+b)$ *are of the same species.* Thus, when a and b are given and A or B is to be found, *if* $\frac{1}{2}(a+b)\gtrless 90°$ *must also* $\frac{1}{2}(A+B)\gtrless 90°$; and the values of A or B must be so chosen as to satisfy this condition.

Having thus found B, say, (whether there be two values or only one) we may complete the solution of the triangle by the use of Napier's analogies.

Example 1. Given $a=46°\,30'$, $b=30°\,20'$, $B=36°\,40'$, solve the triangle.

We have

$$\sin A=\frac{\sin a\sin B}{\sin b}$$

$$\begin{aligned}\log\sin a &= 9.8606\\ \log\sin B &= 9.7761\\ \underline{\log\csc b} &= \underline{0.2967}\\ \log\sin A &= 9.9334\\ A = 59°\,4' &\text{ or } 120°\,56' = A'.\end{aligned}$$

Here $a>b$, and, therefore, must $A>B$. This is *true of both values of* A found, so that there are *two possible solutions* of the triangle. To find C and c we may use (52) and (50).

$$\cot\frac{C}{2}=\frac{\sin\frac{1}{2}(a+b)\tan\frac{1}{2}(A-B)}{\sin\frac{1}{2}(a-b)}$$

$$\tan\frac{c}{2}=\frac{\sin\frac{1}{2}(A+B)\tan\frac{1}{2}(a-b)}{\sin\frac{1}{2}(A-B)}.$$

We have

First solution	*Second solution*
$\frac{1}{2}(a+b)=38^\circ 25,'$	$\frac{1}{2}(a+b)=38^\circ 25',$
$\frac{1}{2}(a-b)=8^\circ 5',$	$\frac{1}{2}(a-b)=8^\circ 5',$
$\frac{1}{2}(A+B)=47^\circ 52,'$	$\frac{1}{2}(A'+B)=78^\circ 48',$
$\frac{1}{2}(A-B)=11^\circ 12'.$	$\frac{1}{2}(A'-B)=42^\circ 8'.$

$$\begin{array}{rlll}
\log\sin\frac{1}{2}(a+b) & = 9.7934 & \text{or} & 9.7934 \\
\log\tan\frac{1}{2}(A-B) & = 9.2966 & & 9.9565 \\
\log\csc\frac{1}{2}(a-b) & = 0.8519 & & 0.8519 \\
\hline
\log\cot\frac{C}{2} & = 9.9419 & \text{or} & 0.6018
\end{array}$$

$$\frac{C}{2}=48^\circ 49',\quad \frac{C'}{2}=14^\circ 3',$$
$$C=97^\circ 38'.\quad C'=28^\circ 6'.$$

$$\begin{array}{rlll}
\log\sin\frac{1}{2}(A+B) & = 9.8701 & \text{or} & 9.9916 \\
\log\tan\frac{1}{2}(a-b) & = 9.1524 & & 9.1524 \\
\log\csc\frac{1}{2}(A-B) & = 0.7117 & & 0.1734 \\
\hline
\log\tan\frac{c}{2} & = 9.7342 & \text{or} & 9.3174
\end{array}$$

$$\frac{c}{2}=28^\circ 28',\qquad \frac{c'}{2}=11^\circ 44',$$
$$c=56^\circ 56'.\qquad c'=23^\circ 38'.$$

The two complete solutions are, therefore,

$$\begin{array}{rlll}
A & = 59^\circ 4', & \text{or} & 120^\circ 56', \\
C & = 97^\circ 38', & & 28^\circ 6', \\
c & = 56^\circ 56'. & & 23^\circ 28'.
\end{array}$$

Example 2. Given $a=126^\circ$, $c=70^\circ$, $A=56^\circ$, solve the triangle.

Using the formula

$$\sin C=\frac{\sin A\sin c}{\sin a}$$

we have

$$\begin{array}{r}
\log \sin A = 9.9186 \\
\log \sin c = 9.9730 \\
\log \csc a = 0.0920 \\
\hline
\log \sin C = 9.9836 \\
C = 74^\circ\ 20' \text{ or } 105^\circ\ 40'.
\end{array}$$

But since **$a > c$**, must **$A > C$.** Therefore, there is *no solution.* Otherwise thus:

$\frac{1}{2}(a + c) = 98^\circ$, $\frac{1}{2}(A + C) = 65^\circ\ 10'$ or $80^\circ\ 50'$, which are *not* of the same species.

Example 3. Given $A = 84^\circ$, $C = 19^\circ$, $a = 28^\circ$, solve the triangle.

Using the law of sines, $\sin c = \dfrac{\sin C \sin a}{\sin A}$

$$\begin{array}{r}
\log \sin C = 9.5126 \\
\log \sin a = 9.6716 \\
\log \csc A = 0.0024 \\
\hline
\log \sin c = 9.1866 \\
c = 8^\circ\ 50' \text{ or } 171^\circ\ 10'.
\end{array}$$

But since **$C < A$,** must **$c < a$,** and the *second value* is *impossible.*

To find b use (50).

$$\begin{array}{r}
\log \sin \frac{1}{2}(A + C) = 9.8935 \\
\log \tan \frac{1}{2}(a - c) = 9.2275 \\
\log \csc \frac{1}{2}(A - C) = 0.2698 \\
\hline
\log \tan \frac{1}{2} b = 9.3908 \\
\frac{1}{2} b = 13^\circ\ 49',\ b = 27^\circ\ 38'.
\end{array}$$

To find B we may use (52), which has the advantage of giving an unambiguous result, or the law of sines. Selecting the latter method we have

$$\begin{array}{r}
\log \sin C = 9.5126 \\
\log \sin b = 9.6663 \\
\log \csc c = 0.8137 \\
\hline
\log \sin B = 9.9926 \\
B = 79^\circ\ 27' \text{ or } 100^\circ\ 33'.
\end{array}$$

But since $b < a$, must $B < A$, and the *second value* is *impossible.* The complete solution is, therefore,

$$c = 8° \, 50', \ b = 27° \, 38', \ B = 79° \, 27'.$$

83. Delambre's Analogies or Gauss's Equations.

Using the law of cosines we may write

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}.$$

Whence

$$1 - \cos A = 2 \sin^2 \tfrac{1}{2} A = \frac{(\cos b \cos c + \sin b \sin c) - \cos a}{\sin b \sin c}$$

or,

$$2 \sin^2 \frac{A}{2} = \frac{\cos(b - c) - \cos a}{\sin b \sin c} = \frac{2 \sin \frac{1}{2}(a+b-c) \sin \frac{1}{2}(a-b+c)}{\sin b \sin c}.$$

That is,

$$\sin \frac{A}{2} = \sqrt{\frac{\sin (s - b) \sin (s - c)}{\sin b \sin c}}$$

with similar formulæ for $\sin \dfrac{B}{2}$ and $\sin \dfrac{C}{2}$.

In the same manner, by adding unity to each side of the first equation of this article, may be obtained formulæ of which the type is

$$\cos \frac{A}{2} = \sqrt{\frac{\sin s \sin (s - a)}{\sin b \sin c}}.$$

From these obviously follows

$$\sin \frac{A}{2} \cos \frac{B}{2} = \frac{\sin (s - b)}{\sin c} \sqrt{\frac{\sin s \sin (s - c)}{\sin a \sin b}}$$

$$= \frac{\sin (s - b)}{\sin c} \cos \frac{C}{2}. \qquad (\alpha)$$

In the same way we obtain

$$\cos \frac{A}{2} \sin \frac{B}{2} = \frac{\sin (s - a)}{\sin c} \cos \frac{C}{2}. \qquad (\beta)$$

Adding (α) and (β),

$$\sin\frac{A}{2}\cos\frac{B}{2}+\cos\frac{A}{2}\sin\frac{B}{2}=\frac{\sin(s-a)+\sin(s-b)}{\sin c}\cos\frac{C}{2}.$$

Whence

$$\sin\tfrac{1}{2}(A+B)=\frac{\sin\frac{1}{2}c\cos\frac{1}{2}(a-b)}{\sin\frac{1}{2}c\cos\frac{1}{2}c}\cos\frac{C}{2},$$

or

$$\sin\tfrac{1}{2}(A+B)=\frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}c}\cos\frac{C}{2}. \qquad \text{I}$$

Similarly may be obtained

$$\sin\tfrac{1}{2}(A-B)=\frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}c}\cos\frac{C}{2}, \qquad \text{II}$$

$$\cos\tfrac{1}{2}(A+B)=\frac{\cos\frac{1}{2}(a+b)}{\cos\frac{1}{2}c}\sin\frac{C}{2}, \qquad \text{III}$$

$$\cos\tfrac{1}{2}(A-B)=\frac{\sin\frac{1}{2}(a+b)}{\sin\frac{1}{2}c}\sin\frac{C}{2}, \qquad \text{IV}$$

which are the analogies or equations sought. These important equations may be conveniently used in the solution of Cases 3 and 4 of oblique triangles.

Example. Given $a = 132° 47'$, $b = 59° 50'$, $C = 56° 28'$, solve the triangle.

We have

$$\tfrac{1}{2}(a+b)=96° 19',\quad \tfrac{1}{2}(a-b)=36° 29',\quad \tfrac{1}{2}C=28° 14'.$$

$\log\sin\frac{1}{2}(a+b)=9.9973,$ $\qquad\log\sin\frac{1}{2}(a-b)=9.7742,$

$\log\cos\frac{1}{2}(a+b)=9.0414,$ $\qquad\log\cos\frac{1}{2}(a-b)=9.9053,$

$$\log\sin\frac{C}{2}=9.6749,\qquad \log\cos\frac{C}{2}=9.9450.$$

From equations II and IV,

$$\log\{\sin\tfrac{1}{2}c\sin\tfrac{1}{2}(A-B)\}=9.7742+9.9450=9.7192,$$

$$\log\{\sin\tfrac{1}{2}c\cos\tfrac{1}{2}(A-B)\}=9.9973+9.6749=9.6722.$$

Whence

$$\log \tan \tfrac{1}{2}(A-B) = 0.0470,$$

$$\tfrac{1}{2}(A-B) = 48^\circ\, 6'.$$

From I and III,

$$\log^{+} \{\cos \tfrac{1}{2} c \sin \tfrac{1}{2}(A+B)\} = 9.9053 + 9.9450 = 9.8503,$$

$$\log^{-} \{\cos \tfrac{1}{2} c \cos \tfrac{1}{2}(A+B)\} = 9.0414 + 9.6749 = 8.7163.$$

Whence $\log^{-} \tan \frac{1}{2}(A+B) = 1.1340.$

$$180^\circ - \tfrac{1}{2}(A+B) = 85^\circ\, 48',\ \tfrac{1}{2}(A+B) = 94^\circ\, 12'.$$

Therefore, $A = 142^\circ\, 18',\ B = 46^\circ\, 6'.$

Also $\log \sin \frac{1}{2}(A-B) = 9.8718.$

$$\begin{array}{r} \text{Therefore, } \log \{\sin \tfrac{1}{2} c \sin \tfrac{1}{2}(A-B)\} = 9.7192 \\ \log \sin \tfrac{1}{2}(A-B) = 9.8718 \\ \hline \log \sin \tfrac{1}{2} c = 9.8474 \end{array}$$

$$\tfrac{1}{2} c = 44^\circ\, 44',\quad c = 89^\circ\, 28'.$$

Possibility of Solution by Inspection of Data. Before attempting the solution of a spherical triangle it may be desirable to determine whether the triangle is possible with the given data.

Case 1. Given the three sides. The triangle is always possible if the sum of the sides is less than 360°, and if no one side is greater than the sum of the other two. This follows at once from well-known geometrical theorems.

Case 2. Given the three angles. This case can be readily tested by the criteria of Case 1 applied to the polar triangle. For example, the triangle $A = 78^\circ$, $B = 100^\circ$, $C = 160^\circ$ is impossible because the sides of the polar triangle, $a' = 102^\circ$, $b' = 80^\circ$, $c' = 20^\circ$, are such that $a' > b' + c'$.

Case 3, given two sides and the included angle, and **Case 4**, two angles and the included side, are always possible.

Cases 5 and 6 have been discussed in Art. 82.

EXAMPLES

1. $a = 68^\circ 25'$,
$b = 71^\circ 11'$,
$c = 56^\circ 57'$.

2. $a = 100^\circ 8'$,
$b = 50^\circ 2'$,
$c = 60^\circ 6'$.

3. $A = 51^\circ 59'$,
$B = 83^\circ 55'$,
$C = 58^\circ 54'$.

4. $A = 142^\circ 33'$,
$B = 27^\circ 53'$,
$C = 32^\circ 27'$.

5. $b = 42^\circ 10'$,
$c = 96^\circ 11'$,
$A = 110^\circ 5'$.

6. $a = 146^\circ$,
$c = 69^\circ 20'$,
$B = 125^\circ 10'$.

7. $a = 90^\circ 50'$,
$c = 117^\circ 50'$,
$B = 120^\circ 6'$.

8. $B = 41^\circ 6'$,
$C = 122^\circ 10'$,
$a = 37^\circ$.

9. $A = 135^\circ$,
$C = 50^\circ 50'$,
$b = 68^\circ 50'$.

10. $A = 147^\circ 30'$,
$C = 163^\circ 10'$,
$b = 76^\circ 25'$.

11. $a = 29^\circ 2'$,
$b = 14^\circ 3'$,
$A = 49^\circ 5'$.

12. $b = 98^\circ$,
$c = 36^\circ$,
$C = 163^\circ$.

13. $a = 132^\circ$,
$b = 56^\circ$,
$A = 116^\circ 18'$.

14. $a = 111^\circ 20'$,
$c = 41^\circ 30'$,
$C = 25^\circ 10'$.

15. $A = 159^\circ 1'$,
$C = 36^\circ$,
$a = 9^\circ 5'$.

16. $A = 25^\circ 20'$,
$C = 153^\circ 30'$,
$a = 73^\circ 33'$.

17. $B = 142^\circ 30'$,
$C = 71^\circ 20'$,
$c = 39^\circ 35'$.

18. $A = 110^\circ 5'$,
$B = 123^\circ 20'$,
$b = 126^\circ 55'$.

19. $a = 59^\circ 34'$,
$b = 136^\circ 11'$,
$c = 150^\circ 2'$.

20. $a = 109^\circ 24'$,
$c = 81^\circ 50'$,
$A = 107^\circ 40'$.

21. $a = 99^\circ 50'$,
$c = 64^\circ 10'$,
$A = 96^\circ 13'$.

22. $A = 35^\circ 31'$,
$B = 24^\circ 43'$,
$C = 138^\circ 25'$.

23. $A = 31^\circ 20'$,
$C = 122^\circ 40'$,
$b = 40^\circ 40'$.

24. $a = 120^\circ 45'$,
$c = 70^\circ 25'$,
$B = 50^\circ 16'$.

25. $A = 120^\circ 21'$,
$B = 130^\circ 22'$,
$C = 140^\circ 7'$.

26. $c = 109^\circ 20'$,
$b = 80^\circ 20'$,
$C = 106^\circ 50'$.

27. $A = 132^\circ$,
$B = 139^\circ 50'$,
$b = 127^\circ 10'$.

28. $A = 79^\circ$,
$B = 40^\circ$,
$c = 108^\circ$.

29. $a = 40^\circ$,
$b = 118^\circ 21'$,
$A = 29^\circ 25'$.

30. $C = 148^\circ$,
$B = 22^\circ 20'$,
$c = 136^\circ$.

31. $a = 114^\circ$,
$c = 148^\circ$,
$C = 135^\circ 7'$.

32. $A = 73^\circ$,
$B = 81^\circ 50'$,
$a = 122^\circ 47'$.

33. $B = 61^\circ 40'$,
$C = 140^\circ 15'$,
$c = 150^\circ 25'$.

34. $a = 125^\circ 16'$,
$b = 151^\circ 37'$,
$c = 75^\circ 55'$.

35. $A = 60^\circ 40'$,
$C = 105^\circ$,
$a = 64^\circ 30'$.

36. $a = 55^\circ 5'$,
$c = 138^\circ 5'$,
$A = 42^\circ 28'$.

37. $B = 116^\circ 6'$,
$C = 73^\circ 50'$,
$c = 80^\circ$.

38. $B = 134^\circ$,
$C = 51^\circ$,
$a = 70^\circ 20'$.

39. $b = 108^\circ$,
$c = 40^\circ$,
$C = 39^\circ$.

40. $a = 58^\circ 20'$,
$b = 138^\circ 5'$,
$c = 116^\circ 3'$.

41. $a = 61^\circ$,
$c = 97^\circ$,
$B = 110^\circ$.

42. $A = 70^\circ 5'$,
$B = 122^\circ$,
$C = 95^\circ 4'$.

43. $a = 60^\circ$,
$b = 120^\circ$,
$c = 50^\circ$.

44. $A = 115^\circ$,
$B = 60^\circ$,
$C = 135^\circ$.

45. $a = 150^\circ$,
$b = 160^\circ$,
$B = 10^\circ$.

46. $a = 112^\circ 30'$,
$b = 108^\circ 40'$,
$c = 140^\circ 10'$.

47. $A = 20^\circ 30'$,
$B = 32^\circ 30'$,
$C = 124^\circ 30'$.

CHAPTER XI

THE EARTH AS A SPHERE

84. Distances on the Earth. As we remarked in the introductory chapter, plane trigonometry is sufficient for the survey of small areas. For larger areas and in navigation, except in the most refined work, the Earth is treated as a sphere and we make use of the principles of spherical trigonometry already enunciated.

The shortest distance between two points on the Earth is the arc of a great circle joining them. If we know the number of degrees in that arc we can compute its length by the formula $s = xr$ (Art. 47), where s is the length of arc, x the angle in circular measure, and r the radius of the sphere; in this case 3960 miles, the radius of the Earth.

Example. Find the length of an arc of 26° on the Earth's surface.

$$26^\circ = \frac{26\,\pi}{180} = \frac{13\,\pi}{90} \text{ radians.}$$

Therefore,

$$s = \frac{13\,\pi}{90} \times 3960 = \frac{13}{90} \cdot \frac{22}{7} \cdot 3960 = 1798 \text{ miles.}$$

It is convenient to compute and remember the number of miles in one degree of arc for the purpose of finding lengths of arcs.

$$s = \frac{\pi}{180} \times 3960 = 69.1 \text{ miles, approximately.}$$

85. Position and direction. The *position* of a point on the Earth is determined by its *latitude* and *longitude;* that is, by the number of degrees the point lies north or south of the equator, and the number of degrees east or west of a great circle, through the Earth's axis, chosen as a reference line. We shall use the great circle, or meridian, through Greenwich.

A point moving along a great circle of the Earth, unless that circle be a meridian or the equator, is constantly changing its direction, or *course*. For example, Fig. 36, at A the compass points north along AN, and a ship at A is sailing, say, due west. When the ship has reached B the compass points north along BN and the ship is sailing west 30° south. On the other hand if a ship sailed constantly on a course, say, west 30° south it would move around the Earth in a spiral approaching continually nearer to the South Pole.

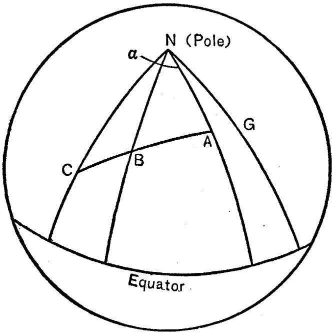


FIG. 36.

86. Bearings. To illustrate the use of spherical trigonometry in determining positions, directions, and distances on the Earth's surface, consider, Fig. 36, a ship sailing from C to A along the great circle CBA. The lines NC, NB, NA, and NG are meridians, the last being the meridian of Greenwich. Suppose the latitude and longitude of C are 44° 40′ N., 63° 35′ W.; of A, 53° 24′ N., 3° 4′ W. The longitude of G, obviously, is 0°. The positions of the points C and A being thus known, let us find the directions (called *bearings*) of A from C and of C from A, and the distance from C to A. From the meaning of longitude we have

$$GNA = 3^\circ\,4', \quad GNC = 63^\circ\,35', \text{ whence } \quad \alpha = ANC = 60^\circ\,31'.$$

Also, by the meaning of latitude,

$$AN = 90^\circ - 53^\circ\,24' = 36^\circ\,36'; \; CN = 90^\circ - 44^\circ\,40' = 45^\circ\,20'.$$

We therefore have, in the spherical triangle CNA, $CN = a = 45^\circ\,20'$, $AN = c = 36^\circ\,36'$ and the included angle $\alpha = 60^\circ\,31'$, which is case 3 in the solution of spherical triangles. The data:

$$\tfrac{1}{2}(a+c) = 40^\circ\,58',\; \tfrac{1}{2}(a-c) = 4^\circ\,22',\; \tfrac{1}{2}\alpha = 30^\circ\,15.5'.$$

$$\begin{array}{rlrl}
\log\cos\tfrac{1}{2}(a-c) &= 9.9987 & \log\sin\tfrac{1}{2}(a-c) &= 8.8816 \\
\log\cot\frac{\alpha}{2} &= 0.2340 & \log\cot\frac{\alpha}{2} &= 0.2340 \\
\log\sec\tfrac{1}{2}(a+c) &= 0.1220 & \log\csc\tfrac{1}{2}(a+c) &= 0.1834 \\
\hline
\log\tan\tfrac{1}{2}(A+C) &= 0.3547 & \log\tan\tfrac{1}{2}(A-C) &= 9.2990 \\
\tfrac{1}{2}(A+C) &= 66^\circ\;10 & \tfrac{1}{2}(A-C) &= 11^\circ\,16'. \\
\text{Whence} \quad A &= 77^\circ\;26'. & C &= 54^\circ\,54'.
\end{array}$$

Therefore the bearings of C from A are N. 77° 26′ W.; of A from C, N. 54° 54′ E.

To find the side $CA = x$ we have

$$\begin{array}{rl}
\log\sin\tfrac{1}{2}(A+C) &= 9.9613 \\
\log\tan\tfrac{1}{2}(a-c) &= 8.8829 \\
\log\csc\tfrac{1}{2}(A-C) &= 0.7092 \\
\hline
\log\tan\frac{x}{2} &= 9.5534
\end{array}$$

$$\tfrac{1}{2}x = 19^\circ\,47.5', \qquad x = 39^\circ\;35'.$$

Therefore length $CA = 39.6^\circ \times 69.1$ miles $= 2736$ miles.

If only the distance sailed is required it is simpler to use the law of cosines. Thus, $\cos x = \cos c \cos a + \sin c \sin a \cos \alpha$.

$$\begin{array}{rl}
\log\cos c = & 9.9046 \\
\log\cos a = & 9.8469 \\
\hline
 & 9.7515 \\
\text{number} = & .5643 \\
 & .2087 \\
\hline
\cos x = & 0.7730, \\
x = & 39^\circ\ 23'
\end{array}
\qquad\qquad
\begin{array}{rl}
\log\sin c = & 9.7754 \\
\log\sin a = & 9.8520 \\
\log\cos \alpha = & 9.6921 \\
\hline
 & 9.3195 \\
\text{number} = & .2087
\end{array}$$

and the distance $= 39.4^\circ \times 69.1 = 2723$ miles.

87. The course of the ship at C would be N. 54° 54′ E. To show how the ship's course changes as it sails along CA let us find the course as the ship crosses the meridian 38° W. at the point B, Fig. 35. In the triangle NCB we have $b = CN = 45^\circ\ 20'$, $C = NCB = 54^\circ\ 54'$, $N = CNB = 63^\circ\ 35' - 38^\circ = 25^\circ\ 35'$; that is, two angles and the included side.

$\frac{1}{2}(C+N) = 40^\circ\ 14.5'$, $\frac{1}{2}(C-N) = 14^\circ\ 39.5'$, $\frac{1}{2}b = 22^\circ\ 40'$.

$$\begin{array}{rl}
\log\cos\frac{1}{2}(C-N) = & 9.9856 \\
\log\tan\frac{1}{2}b = & 9.6208 \\
\log\sec\frac{1}{2}(C+N) = & 0.1173 \\
\hline
\log\tan\frac{1}{2}(c+n) = & 9.7237 \\
\frac{1}{2}(c+n) = & 27^\circ\ 53.5'
\end{array}
\qquad
\begin{array}{rl}
\log\sin\frac{1}{2}(C-N) = & 9.4033 \\
\log\tan\frac{1}{2}b = & 9.6208 \\
\log\csc\frac{1}{2}(C+N) = & 0.1897 \\
\hline
\log\tan\frac{1}{2}(c-n) = & 9.2138 \\
\frac{1}{2}(c-n) = & 9^\circ\ 17.5'.
\end{array}$$

Whence $c = BN = 37^\circ\ 11'$, $n = CB = 18^\circ\ 36'$.

The latitude of B is $90^\circ - BN = 52^\circ\ 49'$ N., and the distance sailed is

$$CB = 18.6^\circ \times 69.1 \text{ miles} = 1285 \text{ miles}.$$

To find the angle $B = CBN$ we have

$$\begin{array}{rl}
\log\sin\frac{1}{2}(c+n) = & 9.6700 \\
\log\tan\frac{1}{2}(C-N) = & 9.4176 \\
\log\csc\frac{1}{2}(c-n) = & 0.7919 \\
\hline
\log\cot\frac{1}{2}B = & 9.8795
\end{array}$$

$\frac{1}{2}B = 52^\circ\ 51'$, $B = 105^\circ\ 42'$, and $NBA = 180^\circ - B = 74^\circ\ 18'$.

Therefore the ship's course at B (the bearing of A from B) is N. 74° 18′ E.

88. The Area of a Spherical Triangle may be found as follows: The theorem has been proved that the area of a spherical triangle is equal to its spherical excess (the excess of the sum of its angles over two right angles) times the area of the tri-rectangular triangle; it being understood that the right angle is the unit of angles. Thus, using Δ to represent the area of a triangle whose angles (in degrees) are A, B, and C; and noting that the tri-rectangular triangle is one eighth of the surface of the sphere; we have

$$\Delta = \frac{A+B+C-180^\circ}{90^\circ} \cdot \frac{4\pi r^2}{8} = \frac{(A+B+C-180)\pi r^2}{180^\circ}.$$

Example. Given $A = 105^\circ$, $B = 80^\circ$, $C = 95^\circ$, and taking $r = 3960$ miles, the radius of the earth,

$$\Delta = \frac{(105^\circ + 80^\circ + 95^\circ - 180^\circ)}{180^\circ} \cdot \pi(3960)^2 = \frac{5\pi(3960)^2}{9}.$$

$$\begin{aligned} \log 5 &= 0.6990 \\ \log \pi &= 0.4971 \\ 2 \log r &= 7.1954 \\ \text{colog } 9 &= 9.0458 - 10 \\ \hline \log \Delta &= 7.4373 \\ \text{and } \Delta &= 27{,}370{,}000 \text{ square miles.} \end{aligned}$$

TABLE OF LATITUDE AND LONGITUDE

Baltimore	39° 17′ N.,	76° 37′ W.
Boston	42° 21′ N.,	71° 4′ W.
Chicago	41° 53′ N.,	87° 38′ W.
Greenwich	51° 29′ N.,	0° W.
Honolulu	21° 18′ N.,	157° 55′ W.
Liverpool	53° 24′ N.,	3° 4′ W.
New York	40° 43′ N.,	74° W.
Pernambuco	8° S.,	34° W.
Rio de Janeiro	22° 54′ S.,	43° 10′ W.
San Francisco	37° 48′ N.,	122° 24′ W.
Washington	38° 54′ N.,	77° 3′ W.

EXAMPLES

In the following problems assume that one can travel directly along the arc of a great circle between the points named.

1. A ship sails from Baltimore to Boston. Find the course of the ship as she leaves Baltimore, her course as she crosses the meridian of New York, and the entire distance she sails. What are the bearings of Baltimore from Boston, and of Boston from Baltimore?

2. Find the course at Liverpool, the course at 55° W., and the total distance sailed by a ship going from Liverpool to New York. What are the bearings of these cities from each other?

3. A ship sails from Baltimore to Rio de Janeiro. She sails first to a point off Pernambuco in latitude 8° S., longitude 34° W., and from there to Rio. How far does she sail, and what is her course off Pernambuco?

NOTE. In the Southern Hemisphere latitudes are taken as algebraically negative. Use the north-polar distances of places as sides in solving triangles.

4. In problem 3 what course will the ship be sailing after she has gone 1000 miles? What will be her latitude and longitude at that point?

5. How far is the Washington Observatory from the Greenwich Observatory? What are the bearings of the two observatories from each other?

6. A ship sails from Boston on a course East 12° South. At what distance would she be sailing due East? What are her latitude and longitude at that instant?

7. A ship sails northwest from San Francisco. What would be the highest latitude she would reach? What would be the ship's longitude at that instant?

8. Find the number of square miles in the triangle whose vertices are Baltimore, Boston, and Chicago.

9. A ship sails from Honolulu to San Francisco. Find the entire distance sailed, and the course of the ship when she has gone halfway.

10. An aëroplane sails from Washington to Chicago along a great circle arc one mile above the surface of the Earth. In what time is the flight made at a rate of 75 miles per hour?

11. Find the number of square miles in the triangle whose vertices are Baltimore, New York, and Chicago.

12. Find the face and edge angles of a regular triangular pyramid.

13. What is the latitude of three points on the Earth equally distant from each other and from the North pole?

14. Each face of a triangular pyramid is a triangle whose sides are 3, 4, and 5 respectively. Find the face and edge angles of the pyramid.

ANSWERS

CHAPTER III

Art. 28

1. $b = 340$
$c = 422$

2. $A = 43° 5$
$c = 54.9$

3. $A = 51° 47'$
$b = .433$

4. $b = 478$
$a = 154$

5. $a = 713$
$c = 823$

6. $b = 96.4$
$c = 232$

7. 70.7 ft. 8. 71.2 ft. 9. 60° 10′ 10. 260 ft. 11. 184 ft.
12. 99.5 ft. 13. 43° 36′ 14. Heights equal.

Art. 35

8. 1.913 9. $-.874$ 14. e^2 15. 10^{a+b} 16. e^5
17. 100 20. 0 21. 0 22. 1 23. Impossible.
25. $\dfrac{2(\log_e a - \log_e b)}{a-b}$ 26. $(a-b)\dfrac{b^{\frac{a}{a-b}}}{a^{\frac{b}{a-b}}}$ 27. 0 29. .6931
30. 1.099 31. 1.386 32. .2312 33. 1.058 34. $-.3088$
35. 4.409 36. 238.2 37. -358300 38. .07212 39. Impossible.
40. -311.7 41. -1747 42. 57090 43. .00003162
44. .03728 45. 100 46. 3.241 47. .001347 48. 1142
49. 2448

SOLUTIONS OF RIGHT TRIANGLES

1. $a = .04691$
$c = .05151$
$K = .0004988$

2. $a = 2316$
$b = 3402$
$K = 3{,}941{,}000$

3. $b = 24850$
$c = 36100$
$K = 325{,}400{,}000$

4. $A = 58° 10'$
$b = 48.04$
$K = 1859$

5. $a = 578.8$
$c = 2491$
$K = 701{,}000$

6. $a = .00883$
$b = .003607$
$K = .00001593$

7. $A = 63° 48'$
$c = .4694$
$K = .04364$

8. $a = .6441$
$c = .6503$
$K = .02879$

9. $A = 43° 24'$
$b = .8966$
$K = .3801$

10. $a = .003845$
$b = .006723$
$K = .00001293$

11. $b = 5091$
$c = 5268$
$K = 3{,}444{,}000$

12. $b = 99.43$
$c = 156.8$
$K = 6030$

13. $A = 79° 28'$
$a = 842$
$K = 65{,}900$

14. $b = 63{,}840$
$c = 92{,}280$
$K = 2{,}128{,}000{,}000$

15. $a = .000005737$
$c = .00002118$
$K = .0000000000585$

16. $a = .0003899$
$b = .0006772$
$K = .000{,}000{,}1321$

17. $A = 27° 17'$
$a = 4.252$
$K = 17.53$

18. $a = 18.59$
$b = 30.51$
$K = 283.7$

19. $b = 24{,}540$
$c = 30{,}010$
$K = 211{,}900{,}000$

20. $A = 43° 45'$
$c = 5280$
$K = 6{,}970{,}000$

21. First steeper by 54′. **22.** 25.6 mi. and 30.5 mi.

23. Team by 15 seconds. **24.** 3° 25′ **25.** 648 ft. **26.** 14.7 in.

27. 9 hr. 28 min. A.M. or 2 hr. 32 min. P.M.

28. Reduced by 10.1 ft. **29.** Buoy farther by 1133 ft.

30. Increase in altitude 251.4 ft. **31.** 8° 48′. **32.** $\frac{1}{5}$ ft. shorter.

33. 1575 mi. **34.** 57° 43′ N. or S. **35.** N. 58° 15′ E. 15 mi.

36. E. 62° 46′ N. 7.29 mi. per hour.

37. 112.5 mi. **38.** E. 30° N. or S. 4.33 mi. **39.** 127.9 mm.

40. 175.6 ft. **41.** 74.17 yd.

CHAPTER IV

Art. 40

5. $\frac{9 \pm 4\sqrt{7}}{20}$ 6. $\frac{1 \pm 2\sqrt{30}}{12}$ 7. $\frac{\pm\sqrt{2}}{2}$ 8. $\frac{\pm\sqrt{2}}{2}$

9. $\frac{\pm\sqrt{2}}{10}$ 10. $\frac{\pm 7\sqrt{2}}{10}$

Art. 41

4. $-\frac{6}{7}$ 5. $\frac{\pm 9\sqrt{3} \pm 8\sqrt{2}}{5}$ 6. $\frac{\pm 32\sqrt{2} \pm 9\sqrt{15}}{7}$

7. One value, 45° **8.** $\pm \frac{24}{7}$, 0 **9.** Two values, ± 90°

Art. 44

1. $\pm\frac{4\sqrt{2}}{9}$ **2.** $\frac{7}{9}$ 3. $\pm\frac{4\sqrt{2}}{7}$ **4.** $\pm\frac{\sqrt{2} \pm 1}{\sqrt{6}}$

5. $\pm\frac{\sqrt{2} \pm 1}{\sqrt{6}}$ **6.** $\pm(3 \pm 2\sqrt{2})$

CHAPTER V

Art. 47

18. Minute hand, π
Hour hand, $\frac{\pi}{12}$

19. $\frac{7\pi}{6}$

20. $\frac{\pi}{20}$

21. $\frac{20\pi}{9}$

22. 14π

23. $\frac{5\pi}{3}$ radians per second.

24. 191 revolutions per minute.

25. 5934 mi.

26. $1978\sqrt{2}$ mi.

27. .035 in.

28. 5.7 ft.

Art. 49

17. $\frac{\sqrt{2}+\sqrt{6}}{4}$

18. $\frac{\sqrt{6}-\sqrt{2}}{4}$

19. $-\frac{\sqrt{3}}{3}$

20. 0

CHAPTER VI

1. $b = 986$
$c = 544.3$
$K = 193{,}000$

2. $A = 19°$
$C = 52°$
$c = 8.19$
$K = 13.1$

3. $A = 18° 39'$
$B = 26° 52'$
$c = 673.9$
$K = 45{,}990$

4. $A = 49° 8'$
$B = 59° 18'$
$C = 71° 36'$
$K = 1705$

5. $A = 100° 35'$ or $10° 21'$
$C = 44° 53'$ $135° 7'$
$a = 67.02$ 12.25
$K = 914$ 167.1

6. $a = 6184$
$b = 2937$
$K = 7{,}510{,}000$

7. $B = 49° 8'$
$C = 59° 19'$
$a = 70.48$
$K = 1703$

8. $A = 37° 58'$
$B = 66' 42'$
$a = 179.9$
$K = 23{,}370$

9. $a = 6.64$
$c = 3.95$
$K = 12.21$

10. $B = 23° 29'$
$C = 22° 57'$
$b = 3024$
$K = 3{,}243{,}000$

11. $A = 43° 49'$
$C = 61° 28'$
$c = .4592$
$K = .0802$

12. $A = 94° 16'$
$B = 54° 36'$
$C = 31° 8'$
$K = .0002699$

13. No solution.

14. $b = .01292$
$c = .002861$
$K = .00000826$

15. $A = 26° 19'$
$C = 109° 6'$
$c = 67.14$
$K = 742$

16. $b = .0185$
$A = 54° 40'$
$C = 94° 13'$
$K = .000002694$

17. $B = 90°$
$c = 59.39$
$K = 955$

18. $A = 46^\circ\ 23'$ or $133^\circ\ 37'$
$C = 102^\circ\ 30'$ $\quad 15^\circ\ 16'$
$c = 8730$ $\quad 2354$
$K = 14{,}600{,}000$ $\quad 3{,}938{,}000$

19. $B = 22^\circ\ 37'$
$C = 127^\circ\ 28'$
$a = .5593$
$K = .0958$

20. $A = 134^\circ\ 22'$ $\quad B = 18^\circ\ 42'$ $\quad C = 26^\circ\ 56'$ $\quad K = 4.622$

21. $A = 57^\circ\ 41'$ $\quad C = 38^\circ\ 49'$ $\quad a = .02461$ $\quad K = .0002232$

22. 33,695 sq. ft. **23.** 13.6 ft. **24.** 30.4 ft. **25.** 428 ft.

26. Width 74.6 ft.; height above stream, 14 ft. If 50 ft. from bank means up the slope, the results are 71.9 ft. and 13.5 ft. **27.** 472 ft.

28. 17.1 ft. **29.** 301 ft. **30.** 61.93 and 58.81 ft.

31. $A = 57^\circ\ 21'$ $\quad B = 75^\circ\ 44'$ $\quad C = 46^\circ\ 55'$ $\quad c = 6.43$

32. $B = 53^\circ\ 26'$ $\quad a = 46.45$ ft. $\quad c = 52.48$ ft.

33. 11,320 and 7082. **34.** 3997 sq. ft. **35.** $A = 7^\circ\ 5'$; no.

36. 21.7 mi. **37.** 9° 17′ with the surface of the water.

38. 33.5 ft. **39.** 39.7 ft. **40.** Second yacht by 1 min. 12 sec. Result varies from 1 min. to 1 min. 42 sec. according to the number of figures used in computing. **41.** At 82° 33′ with shore on the side towards the 60° angle. **42.** $A\text{–}C\text{–}B$ by \$560. Result between \$555 and \$560 according to method of solution and number of figures used. **43.** $AC = 152.4$ ft. $\quad BC = 320$ ft.

44. 441 ft. **45.** 336.9 ft.

CHAPTER VII

1. $n\pi$
$\frac{n\pi}{2} \pm \frac{\pi}{6}$

2. $n\pi \pm \frac{\pi}{4}$
$2\,n\pi \pm \frac{2\,\pi}{3}$

3. $\frac{2\,n\pi}{3} \pm \frac{\pi}{6}$
$n\pi + (-)^n \frac{\pi}{6}$

4. $2\,n\pi + \cos^{-1}\frac{2 \mp 6\sqrt{33}}{37}$

5. $2\,n\pi + \frac{\pi}{2}$
$2\,n\pi + \cos^{-1}\frac{4}{5}$

6. $n\pi$
$2\,n\pi \pm \frac{\pi}{3}$

7. $2\,n\pi - \frac{\pi}{2}$
$\frac{2\,n\pi}{3} + \frac{\pi}{6}$

8. $n\pi + \frac{\pi}{4}$
$\frac{n\pi}{2} + \frac{\pi}{8}$

9. $n\pi + (-)^n \sin^{-1}\left(\frac{\sqrt{5} - 1}{2}\right)$

10. $2\,n\pi \pm \cos^{-1}\left(\frac{1 \pm \sqrt{33}}{8}\right)$.

11. $n\pi$.

12. $n\pi \pm \frac{\pi}{4}$
$n\pi$
$(2\,n + 1)\frac{\pi}{3}$

13. $n\pi + \frac{\pi}{4}$
$\frac{n\pi}{3} + \frac{\pi}{12}.$

14. $2\,n\pi \pm \frac{\pi}{2}$
$n\pi + \frac{\pi}{4}$

15. $n\pi + \frac{a}{2} \pm \frac{\pi}{4}$

16. $2\,n\pi \pm \frac{\pi}{3}$
$2\,n\pi \pm \frac{2\,\pi}{3}$

17. $\frac{n\pi}{2} + \frac{\pi}{6}$

18. $n\pi + \tan^{-1}\frac{3}{2}$

19. $n\pi + (-)^n \sin^{-1}\frac{4}{5}$

20. $2\,n\pi \pm \frac{\pi}{2}, \quad n\pi \pm \tan^{-1}\frac{\sqrt{2}}{2}$

21. $x = \frac{n\pi}{2} - \frac{\pi}{8}$
$y = 1 \pm \frac{\sqrt{2}}{2}$

22. $\theta = 2\,n\pi \pm \frac{\pi}{2}$
$r = 2$

23. $\theta = n\pi,\ 2\,n\pi \pm \frac{\pi}{3}$
$r = 0, \ \frac{\pm\, a\sqrt{3}}{2}$

24. $x = \frac{n\pi}{2} + \frac{\pi}{4}$
$y = \pm\, a\sqrt{2}$

25. $x = \frac{1}{2}\{n\pi + (-)^n \sin^{-1}\frac{4}{5}\}$
$y = -\,a\sqrt{5},\ -\,2\,a\sqrt{5},\ n$ even
$y = a\sqrt{5},\ 2\,a\sqrt{5},\ n$ odd

26. $\theta = \frac{n\pi}{2} + \frac{\pi}{8}$
$r = \pm\sqrt[4]{2}$

27. $\theta = n\pi + \frac{\pi}{4}$
$r = \pm\,\frac{5\sqrt{2}}{2}$

28. $x = \frac{n\pi}{2} + (-)^n\frac{\pi}{12}$
$y = \pm\sqrt{2}$

29. $x = \frac{(2\,n+1)\pi - a}{2}$
$y = \pm\cos\frac{a}{2}$

30. $\theta = n\pi \pm \frac{\pi}{3}$
$r = \frac{\pm\, a\sqrt{3}}{2}$

31. $\theta = \frac{n\pi}{2}$, $r = 0$, n even. $\theta = (8n+1)\frac{\pi}{4}$ or $(8n+3)\frac{\pi}{4}$, $r = \pm\,\frac{1}{\sqrt[4]{2}}$, n odd

32. $\theta = n\pi + \frac{\pi}{4}$
$r = \pm\,2\sqrt{2}$

33. $\theta = \frac{n\pi}{2} + (-1)^n\frac{\pi}{12}$
$r = \pm\,2\sqrt{2}$

34. $a\sqrt{1-b^2} + b\sqrt{1-a^2}$

35. $\frac{\pm\sqrt{21}}{14}$

36. $\pm\,4\sqrt{5}$

37. $\pm\,\frac{\sqrt{2}}{2}$

38. 1 or $-\frac{1}{6}$

39. 0, $\pm\sqrt{3}$

40. $\pm\,1$

41. $\pm\sqrt{3},\ \frac{\pm\sqrt{3}}{3}$

42. $-\,1$ or $\frac{1}{2}$

43. 0, $\frac{\pi}{24}$

44. $\frac{\pi}{2},\ -\frac{\pi}{6}$

45. 0.

46. $\frac{\pi}{2}$

47. $x = -\frac{\pi}{8}$
$y = \frac{2+\sqrt{2}}{2}$

48. $\theta = \frac{\pi}{2},\ \frac{\pi}{6}$
$r = 0,\ \frac{a\sqrt{3}}{2}$

49. $x = \frac{1}{2}\sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$
$y = \frac{\sqrt{2\sqrt{5}-2}}{2}$

50. $\theta = \frac{3\pi}{8}$
$r = \sin\frac{3\pi}{8}$

51. $\frac{\pi}{10}$

52. $\cos^{-1}\frac{2+3\sqrt{6}}{10}$

CHAPTER IX

1. $B = 58^\circ 30'$
$121^\circ 30'$
$b = 35^\circ 59'$
$144^\circ 1'$
$c = 43^\circ 33'$
$136^\circ 27'$

2. $A = 30^\circ 53'$
$149^\circ 7'$
$a = 30^\circ 13'$
$149^\circ 47'$
$c = 78^\circ 35'$
$101^\circ 25'$

3. $A = 48^\circ 11'$
$131^\circ 49'$
$a = 44^\circ 29'$
$135^\circ 31'$
$c = 109^\circ 52'$
$70^\circ 8'$

4. $A = 83^\circ 39'$
$B = 127^\circ 20'$
$a = 82^\circ 1'$

5. $A = 159^\circ 39'$
$B = 104^\circ 14'$
$b = 135^\circ$

6. $A = 147^\circ 34'$
$B = 66^\circ 3'$
$a = 157^\circ 26'$

7. $A = 27^\circ$
$B = 73^\circ$
$c = 53^\circ 8'$

8. $A = 81^\circ 29'$
$B = 131^\circ 50'$
$c = 97^\circ 42'$

9. $A = 139^\circ 5'$
$B = 110^\circ 57'$
$c = 63^\circ 47'$

10. $a = 49^\circ 26'$
$b = 43^\circ 58'$
$c = 62^\circ 5'$

11. $a = 147^\circ 37'$
$b = 136^\circ 32'$
$c = 52^\circ 11'$

12. $a = 53^\circ 45'$
$b = 153^\circ 17'$
$c = 121^\circ 53'$

13. $a = 7^\circ 59'$
$b = 21^\circ 58'$
$B = 70^\circ 59'$

14. $a = 49^\circ 11'$
$b = 100^\circ$
$A = 49^\circ 37'$

15. $a = 129^\circ 30'$
$b = 166^\circ 50'$
$B = 163^\circ 8'$

16. $A = 30^\circ 47'$
$b = 11^\circ 36'$
$c = 13^\circ 26'$

17. $A = 126^\circ 53'$
$b = 32^\circ 29'$
$c = 133^\circ 18'$

18. $a = 141^\circ 47'$
$c = 140^\circ 37'$
$B = 16^\circ 25'$

19. $B = 10^\circ 23'$
$169^\circ 37'$
$b = 6^\circ 16'$
$173^\circ 44'$
$c = 142^\circ 41'$
$37^\circ 19'$

20. $A = 170^\circ 50'$
$B = 84^\circ 45'$
$b = 55^\circ 1'$

21. $A = 25^\circ 5'$
$B = 114^\circ 38'$
$c = 168^\circ 23'$

22. $a = 66^\circ 12'$
$b = 146^\circ 25'$
$c = 109^\circ 39'$

23. $a = 142^\circ 40'$
$b = 78^\circ 7'$
$A = 142^\circ 3'$

24. $A = 28^\circ 19'$
$b = 110^\circ 59'$
$c = 108^\circ 39'$

25. $b = 33^\circ 37'$
$146^\circ 23'$
$c = 101^\circ$
79°
$B = 34^\circ 20'$
$145^\circ 40'$

26. $A = 74^\circ 12'$
$B = 157^\circ 47'$
$a = 43^\circ 57'$

27. $A = 68^\circ 11'$
$B = 39^\circ 43'$
$c = 61^\circ 11'$

28. $a = 161^\circ 32'$
$b = 129^\circ 57'$
$c = 52^\circ 28'$

29. $a = 168^\circ\ 30'$
$b = 130^\circ\ 29'$
$B = 99^\circ\ 40'$

30. $A = 124^\circ\ 32'$
$b = 16^\circ\ 48'$
$c = 151^\circ\ 57'$

31. $b = 20^\circ\ 23'$ $B = 10^\circ\ 35'$ $C = 148^\circ\ 10$
$159^\circ\ 37'$ $169^\circ\ 25'$ $31^\circ\ 50'$

32. $A = 141^\circ\ 32'$
$C = 111^\circ\ 46'$
$b = 108^\circ\ 29'$

33. $A = 163^\circ\ 16'$
$B = 19^\circ\ 55'$
$a = 138^\circ\ 36'$

34. $a = 124^\circ\ 38'$
$b = 46^\circ\ 49'$
$B = 33^\circ\ 43'$

35. $B = 23^\circ\ 26'$ $C = 30^\circ\ 51'$ $b = 50^\circ\ 50'$
$156^\circ\ 34'$ $149^\circ\ 9'$ $129^\circ\ 10'$

36. $B = 12^\circ\ 7'$
$C = 139^\circ\ 5'$
$a = 75^\circ\ 40'$

CHAPTER X

1. $A = 76^\circ$
$B = 81^\circ$
$C = 61^\circ$

2. $A = 138^\circ\ 18'$
$B = 31^\circ\ 12'$
$C = 35^\circ\ 52'$

3. $a = 38^\circ\ 2'$
$b = 51^\circ\ 2'$
$c = 42^\circ\ 2'$

4. $a = 101^\circ\ 2'$
$b = 49^\circ$
$c = 60^\circ$

5. $B = 41^\circ\ 32'$
$C = 79^\circ\ 2'$
$a = 108^\circ\ 10'$

6. $A = 145^\circ\ 23'$
$C = 108^\circ\ 3'$
$b = 126^\circ\ 24'$

7. $A = 105^\circ\ 57'$
$C = 121^\circ\ 45'$
$b = 115^\circ\ 56'$

8. $b = 47^\circ\ 49'$
$c = 72^\circ\ 37'$
$A = 32^\circ\ 16'$

9. $a = 120^\circ\ 25'$
$c = 71^\circ\ 1'$
$B = 49^\circ\ 52'$

10. $a = 124^\circ\ 57'$
$c = 153^\circ\ 47'$
$B = 140^\circ\ 26'$

11. $B = 22^\circ\ 13'$
$C = 112^\circ\ 8'$
$c = 36^\circ\ 30'$

12. No solution

13. $B = 90^\circ$
$C = 138^\circ\ 32'$
$c = 146^\circ\ 42'$

14. $A = 36^\circ\ 42'$ $B = 160^\circ\ 32'$ $b = 148^\circ\ 44$
$143^\circ\ 18'$ $38^\circ\ 52'$ 78°

15. No solution

16. $c = 90^\circ$
$b = 18^\circ\ 15'$
$B = 8^\circ\ 2'$

17. $a = 138^\circ\ 34'$
$b = 155^\circ\ 50'$
$A = 100^\circ\ 16'$

18. $a = 63^\circ\ 59'$ $c = 156^\circ\ 10'$ $C = 155^\circ\ 2'$
$116^\circ\ 1'$ $72^\circ\ 54'$ $87^\circ\ 36'$

19. $A = 110^\circ\ 4'$
$B = 131^\circ\ 2'$
$C = 147^\circ\ 2'$

20. $b = 115^\circ\ 19'$
$B = 114^\circ\ 2'$
$C = 90^\circ$

21. $B = 96^\circ\ 16'$
$C = 65^\circ\ 14'$
$b = 99^\circ\ 52'$

22. $a = 61^\circ 2'$
$b = 39^\circ 2'$
$c = 92^\circ 2'$

23. $B = 37^\circ 30'$
$a = 33^\circ 49'$
$c = 64^\circ 15'$

24. $b = 69^\circ 46'$
$A = 67^\circ 37'$
$C = 25^\circ 17'$

25. $a = 90^\circ 58'$
$b = 118^\circ$
$c = 132^\circ 2'$

26. $a = 120^\circ 29'$
$A = 119^\circ 3'$
$B = 90^\circ$

27. $a = 66^\circ 40'$
$113^\circ 20'$
$c = 160^\circ 54'$
$102^\circ 2'$
$C = 164^\circ 38'$
$127^\circ 42'$

28. $C = 109^\circ 58'$
$a = 96^\circ 42'$
$b = 40^\circ 34'$

29. $B = 42^\circ 15'$
$137^\circ 45'$
$C = 160^\circ 10'$
$49^\circ 50'$
$c = 153^\circ 38'$
$89^\circ 56'$

30. $A = 44^\circ 54'$
$a = 112^\circ 14'$
$b = 29^\circ 53'$

31. No solution.

32. No solution

33. $A = 89^\circ 36'$
$26^\circ 48'$
$a = 138^\circ 54'$
$25^\circ 28'$
$b = 42^\circ 48'$
$137^\circ 12'$

34. $A = 142^\circ$
$B = 159^\circ$
$C = 133^\circ$

35. $B = 58^\circ 6'$
$b = 61^\circ 31'$
$c = 90^\circ$

36. $b = 96^\circ 24'$
$B = 54^\circ 54'$
$C = 146^\circ 38'$

37. $A = 72^\circ 54'$
$a = 78^\circ 30'$
$b = 112^\circ 57'$

38. $A = 51^\circ 16'$
$b = 119^\circ 47'$
$c = 69^\circ 41'$

39. $a = 129^\circ 44'$
$95^\circ 40'$
$A = 131^\circ 8'$
$76^\circ 58'$
$B = 68^\circ 36'$
$111^\circ 24'$

40. $A = 70^\circ 44'$
$B = 132^\circ 12'$
$C = 94^\circ 54'$

41. $b = 110^\circ 50'$
$A = 61^\circ 35'$
$C = 86^\circ 27'$

42. $a = 62^\circ 42'$.
$b = 126^\circ 44'$
$c = 109^\circ 42'$

CHAPTER XI

1. Course at Baltimore : E. 37° 55′ N.
Course at meridian of New York : E. 36° 14′ N.
Bearings Boston from Baltimore : E. 37° 55′ N.
Bearings Baltimore from Boston : W. 34° 17′ S.
Distance sailed : 359 mi.

2. Course at Liverpool : W. 14° 51′ N.
Course at 55° W. : W. 26° 56′ S.
Bearings New York from Liverpool : W. 14° 51′ N.
Bearings Liverpool from New York : E. 40° 31′ N.
Distance sailed : 3303 mi.

3. Course at Pernambuco: S. 36° 35′ E. arrives.
Course at Pernambuco: S. 29° 33′ W. departs.
Distance sailed: 5141 mi.

4. Course: S. 42° 32′ E.
Position: 29° 12′ N., 64° 1′ W.

5. Bearings Greenwich from Washington: E. 40° 41′ N.
Bearings Washington from Greenwich: W. 18° 33′ N.
Distance: 3669 mi.

6. Distance: 11,550 mi.
Position: 43° 42′ S., 91° 25′ E.

7. Position: 56° 2′ N., 153° 54′ W.

8. 117,700 sq. mi.

9. Distance: 2398 mi.
Course: E. 29° 6′ N.

10. 8 hours, nearly.

11. 35,580 sq. mi.

12. Face angle, 60°; edge angle, 70° 32′.

13. 19° 28′ S.

14. Face: 36° 52′, 53° 8′, 90°.
Edge: 180°, 0°, 0°.

ANSWERS TO FIVE PLACES

AUTHOR'S NOTE

In response to requests that have amounted, practically, to a demand, the author is glad to supply the following answers to problems in the Plane Trigonometry worked with five place tables. It is very important, however, that the student, in using five place tables, should not be permitted to have an erroneous idea as to the accuracy of his results. No matter how many places his tables supply his results *cannot be accurate to a greater degree of precision than the data on which they are based.*

For example, in problem 14, art. 28, Chap. III, the nearer tree is said, in the new answer, to be taller by two-thousandths of a foot (one-fortieth of an inch). If the distances given are supposed to be correct to thousandths of a foot and the angles to tenths of a minute, then this result may be accepted. But with the data as given (assumed correct to feet and minutes only) the new answer is misleading. In this case the two trees, *to the degree of accuracy of the measurements*, are of *exactly the same height.*

The student's judgment should be trained to select tables of accuracy in accordance with the data given. It should be noted that both four and five place tables are included in this volume.

CHAPTER III

Art. 28

1. $b = 339.5$
   $c = 421.62$
2. $A = 43° 4.9'$
   $c = 54.897$
3. $A = 51° 47.2'$
   $b = .43302$
4. $b = 477.7$
   $a = 154.29$
5. $a = 712.63$
   $c = 823.18$
6. $b = 96.381$
   $c = 231.97$
7. 70.692 ft.
8. 71.214 ft.
9. 60° 10.4′
10. 259.81 ft.
11. 183.72 ft.
12. 99.452 ft.
13. 43° 36.2′
14. The nearer tree; .002 ft.

Art. 35

8. 1.9124
9. $-.87384$
29. .69315
30. 1.0986
31. 1.3863
32. .23121
33. 1.0286
34. $-.30878$
35. 4.4087
36. 238.27
37. $-358,380$
38. .072108
40. -311.74
41. -1746.2
42. 57,096
43. .000031623
44. .037276
45. 100
46. 3.2418
47. .0013466
48. 1142.2
49. 2447.7

SOLUTIONS OF RIGHT TRIANGLES

1. $a = .046903$
   $c = .051497$
   $K = .00049858$
2. $a = 2316.5$
   $b = 3402.3$
   $K = 3,940,800$
3. $b = 24,849$
   $c = 36,099$
   $K = 325,350,000$
4. $A = 58° 10'$
   $b = 48.04$
   $K = 1858.7$
5. $a = 578.73$
   $c = 2491.2$
   $K = 701,100$
6. $a = .0088318$
   $b = .0036072$
   $K = .000015929$

7. $A = 63^\circ\ 48.4'$
$c = .46941$
$K = .043637$

8. $a = .64405$
$c = .65022$
$K = .02878$

9. $A = 43^\circ\ 23.7'$
$b = .89666$
$K = .38009$

10. $a = .0038451$
$b = .0067231$
$K = .000012926$

11. $b = 5091$
$c = 5267.7$
$K = 3{,}444{,}100$

12. $b = 99.435$
$c = 156.77$
$K = 6026$

13. $A = 79^\circ\ 27.8'$
$a = 841.98$
$K = 65{,}930$

14. $b = 63{,}841$
$c = 92{,}292$
$K = 2{,}127{,}500{,}000$

15. $a = .0000057367$
$c = .000021178$
$K = .0000000000584$

16. $a = .00038991$
$b = .00067716$
$K = .00000013202$

17. $A = 27^\circ\ 17'$
$a = 4.2516$
$K = 17.523$

18. $a = 18.589$
$b = 30.514$
$K = 283.6$

19. $b = 24{,}542$
$c = 30{,}009$
$K = 211{,}920{,}000$

20. $A = 43^\circ\ 45'$
$c = 5279.8$
$K = 6{,}963{,}000$

21. First steeper by 53.5′

22. 25.551 mi. and 30.466 mi.

23. Man's time, 1.68 min. ; team's time, 1.44 min.

24. 3° 25′

25. 647.68 ft.

26. 14.708 in.

27. 9 hr. 28.3 min. A.M., or 2 hr. 31.7 min. P.M.

28. By 10.075 ft.

29. Buoy farther by 1133.3 ft.

30. 251.39 ft.

31. 8° 47.7′

32. $\frac{1}{5}$ ft. shorter.

33. 1575.4 mi.

34. 57° 42.9′ N. or S.

35. N. 58° 15.1′ E. 14.993 mi.

36. N. 27° 13.5′ E. 7.2888 mi. per hr.

37. 112.53 mi.

38. 4.3301 mi. E. 30 N. or S.

39. 127.86 mm.

40. 175.65 ft.

41. 74.162 yd.

CHAPTER VI

1. $b = 985.83$
$c = 544.29$
$K = 192{,}990$

2. $A = 18^\circ\ 59.9'$
$C = 52^\circ\ 0.1'$
$c = 8.191$
$K = 13.104$

3. $B = 26^\circ\ 52.4'$
$A = 18^\circ\ 38.6$
$c = 673.97$
$K = 46{,}002$

4. $A = 49^\circ\ 7.8$
$B = 59^\circ\ 17.6'$
$C = 71^\circ\ 35.4'$
$K = 1704.6$

5. $A = 100^\circ\ 34.4'$ or $10^\circ\ 21.6'$
$C = 44^\circ\ 53.6'$ $\quad 135^\circ\ 6.4'$
$a = 67.021$ $\quad 12.261$
$K = 914.2$ $\quad 167.23$

6. $a = 6183.6$
$b = 2936.9$
$K = 7{,}507{,}000$

7. $B = 49^\circ\ 8.4'$
$C = 59^\circ\ 18.6'$
$a = 70.488$
$K = 1703.3$

8. $B = 66^\circ\ 39.6$
$A = 38^\circ\ 0.4'$
$a = 180.06$
$K = 23{,}385$

9. $a = 6.6408$
$c = 3.9495$
$K = 12.202$

10. $C = 22^\circ\ 56.9'$
$B = 23^\circ\ 29.1'$
$b = 3024.4$
$K = 3{,}242{,}200$

11. $A = 43^\circ\ 48.2$
$C = 61^\circ\ 28.8'$
$c = .45926$
$K = .08015$

12. $A = 94^\circ\ 16.8'$
$B = 54^\circ\ 35.4'$
$C = 31^\circ\ 8'$
$K = .00026988$

14. $b = .012922$
$c = .0028617$
$K = .000008259$

15. $A = 26^\circ\ 18.3'$
$C = 109^\circ\ 6.7'$
$c = 67.143$
$K = 742.1$

16. $A = 54^\circ\ 39.3'$
$C = 94^\circ\ 13.7'$
$b = .0185$
$K = .0000026936$

17. No solution

18. $A = 46^\circ\ 22.7'$ or $133^\circ\ 37.3'$
$C = 102^\circ\ 30.3'$ $15^\circ\ 15.7'$
$c = 8729.8$ 2353.7
$K = 14{,}602{,}000$ 3,936,900

19. $B = 22^\circ\ 36.7'$
$C = 127^\circ\ 28.3'$
$a = .55934$
$K = .09571$

20. $A = 134^\circ\ 23.4'$
$B = 18^\circ\ 41.8'$
$C = 26^\circ\ 54.8'$
$K = 4.6162$

21. $C = 38^\circ\ 48.8'$
$A = 57^\circ\ 41.2'$
$a = .024608$
$K = .00022311$

22. 33,679 sq. ft.

23. 13.591 ft.

24. 30.388 ft.

25. 198.29 yd.

26. Width 124.6 ft.; height above stream 13.976 ft.

27. 472.26 ft.

28. 17.139 ft.

29. 300.61 ft.

30. 61.913 and 58.816 ft.

31. $A = 57^\circ\ 20.2'$ $B = 75^\circ\ 43.2'$ $C = 46^\circ\ 56.6'$ $c = 6.4314$

32. $B = 53^\circ\ 25.5'$ $a = 46.447$ ft. $c = 52.491$ ft.

33. 11,321 and 7081.4.

34. 3996.4 sq. ft.

35. $A = 7^\circ\ 5.3'$; no.

36. 21.674 mi.

37. 9° 17.1′ with the surface of the water.

38. 33.526 ft.

39. 39.701 ft.

40. Second yacht by 1 min. 42 sec.

41. At 82° 32.6′ with shore on the side towards the 60° angle.

42. *A–C–B* by $ 558 plus.

43. $AC = 152.27$ ft. $BC = 319.7$ ft.

44. 441.03 ft.

45. 336.83 ft.

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REVISED EDITION

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EARLE RAYMOND HEDRICK

New York
THE MACMILLAN COMPANY
1923

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Set up and electrotyped. Revised edition published August, 1920.

Norwood Press
J. S. Cushing Co. — Berwick & Smith Co.
Norwood, Mass., U.S.A.

PREFACE

THE present edition of this book contains several tables not contained in the previous editions. The probability of the occurrence of errors has been minimized by using electrotype reproductions of the tables previously included, even when changes were made. Remarkably few errors existed in the original edition ; what few have been discovered have been corrected.

Minor changes only occur in the earlier pages. Care has been taken to preserve the page numbers of the principal tables up to page 114, so that older editions may be used in class-work without confusion, and texts which contain the principal tables may be used in the same class.

Among the minor changes are the insertion of a condensed table of logarithms and antilogarithms (Table I*a*, p. 20), the insertion of a table of values of S and T for interpolation in logarithmic trigonometric functions (Table III*a*, p. 45), and the insertion on pages 1–19 of the logarithms of a few important numbers at appropriate points.

The principal changes follow page 114. Tables VIII and IX (pp. 115–122) make reasonably complete the tables of hyperbolic functions formerly represented only by Table XII (pp. 112–114): These functions are of increasing importance, notably in Electrical Engineering.

The table of haversines (Table X, pp. 123–125) will be welcomed particularly by those interested in navigation.

The table of factors of composite numbers and logarithms of primes (Table XI, pp. 126–127) has obvious uses.

Tables XII *a*, *b*, *c*, *d*, *e*, *f*, pages 128–132, are intended for work involving compound interest, annuities, depreciation, etc. They will be useful for statistics, insurance, accounting, and the mathematics of business.

The same care has been exercised to eliminate errors in the new tables that resulted in so great a degree of reliability in the original edition of these tables.

E. R. HEDRICK.

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EXPLANATION OF THE TABLES *

TABLE I. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS FROM 1 TO 10 000

1. Powers of 10. Consider the following table of values of powers of 10:

COLUMN *A*		COLUMN *B*	COLUMN *A*		COLUMN *B*
10^1	=	10	10^0	=	1.
10^2	=	100	10^{-1}	=	.1
10^3	=	1000	10^{-2}	=	.01
10^4	=	10000	10^{-3}	=	.001
10^5	=	100000	10^{-4}	=	.0001
10^6	=	1000000	10^{-5}	=	.00001
10^7	=	10000000	10^{-6}	=	.000001
10^8	=	100000000	10^{-7}	=	.0000001
10^9	=	1000000000	10^{-8}	=	.00000001
10^{10}	=	10000000000	10^{-9}	=	.000000001

This table may be used for multiplying or dividing powers of 10, by means of the rules $10^a \cdot 10^b = 10^{a+b}$, $10^a \div 10^b = 10^{a-b}$. Thus, to multiply 1000 by 100,000, add the exponent of 10 in column *A* opposite 1000 to the exponent of 10 opposite 100,000: $3 + 5 = 8$; and look for the number in column *B* opposite 10^8, *i.e.* 100,000,000. Similarly $1,000,000 \times .0001 = 100$, since $6 + (-4) = 2$.

To divide 1,000,000 by 100, from the exponent of 10 opposite 1,000,000 subtract the exponent of 10 opposite 100; $6 - 2 = 4$; and look for the number opposite 10^4, *i.e.* 10,000. Similarly $.001 \div 1,000,000 = .000000001$, since $-3 - 6 = -9$. To find the 4th power of 100, multiply the exponent of 10 opposite 100 by 4: $4 \times 2 = 8$, and look for the number opposite 10^8, *i.e.* 100,000,000. Likewise $(.001)^3 = .000000001$, since $3 \times (-3) = -9$. To find the cube root of 1,000,000,000, divide the exponent of 10 opposite 1,000,000,000 by 3, $9 \div 3 = 3$, and look for the number opposite 10^3.

---

* This Explanation, written to accompany the five-place tables, may be used also for the four-place tables by omitting the last figure in each example in a manner obvious to the teacher.

2. Common Logarithms. The exponent of 10 in any row of column A is called the common logarithm* of the number opposite in column B; thus $\log 10 = 1$, $\log 100 = 2$, $\log 1000 = 3$, etc.; $\log 1 = 0$, $\log .1 = -1$; $\log .01 = -2$, $\log .001 = -3$, etc. In general, if $10^l = n$, l is called the *common logarithm of* n, and is denoted by *log n*.

3. Fundamental Principles. Logarithms are useful in reducing the labor of performing a series of operations of multiplication, division, raising to powers, extracting roots, as above; they have no necessary connection with trigonometry, since all the operations could be performed without them; but they are a great labor-saving device in arithmetical computations. They do not apply to addition and subtraction.

The principles of their application are stated as follows:

I. *The logarithm of a product is equal to the sum of the logarithms of the factors:* $\log ab = \log a + \log b$. This follows from the fact that if $10^l = a$ and $10^L = b$, $10^{l+L} = a \cdot b$. In brief: *to multiply, add logarithms.*

II. *The logarithm of a fraction is equal to the difference obtained by subtracting the logarithm of the denominator from the logarithm of the numerator:* $\log (a/b) = \log a - \log b$. For, if $10^l = a$ and $10^L = b$, then $10^{l-L} = a \div b$. In brief: *to divide, subtract logarithms.*

III. *The logarithm of a power is equal to the logarithm of the base multiplied by the exponent of the power:* $\log a^b = b \log a$. This follows from the fact that if $10^l = a$, then $10^{lb} = a^b$.

IV. *The logarithm of a root of a number is found by dividing the logarithm of the number by the index of the root:* $\log \sqrt[b]{a} = (\log a)/b$. This follows from the fact that if $10^l = a$, then $10^{l/b} = a^{1/b} = \sqrt[b]{a}$.

Corollary of II. *The logarithm of the reciprocal of a number is the negative of the logarithm of the number:* $\log (1/a) = -\log a$, since $\log 1 = 0$.

4. Characteristic and Mantissa. It is shown in algebra that every real positive number has a real common logarithm, and that if a and b are any two real positive numbers such that $a < b$, then $\log a < \log b$. Neither zero nor any negative number has a real logarithm.

An inspection of the following table, which is a restatement of a part

a	1	10	100	1000	10000	100000	1000000	10000000
$\log a$	0	1	2	3	4	5	6	7

---

* Common logarithms are exponents of the base 10; other systems of logarithms have bases different from 10; Napierian logarithms (see Table VII, p. 112) have a base denoted by e, an irrational number whose value is approximately 2.71828. When it is necessary to call attention to the base, the expression $\log_{10} n$ will mean common logarithm of n; $\log_e n$ will mean the Napierian logarithm, etc.; but in this book $\log n$ denotes $\log_{10} n$ unless otherwise explicitly stated.

of the table of § 1, p. v, shows that

the logarithm of every number between 1 and 10 is a proper fraction,

the logarithm of every number between 10 and 100 is 1 + a fraction,

the logarithm of every number between 100 and 1000 is 2 + a fraction; and so on. It is evident that the logarithm of every number (not an exact power of 10) consists of a whole number + a fraction (usually written as a decimal). The whole number is called the **characteristic**; the decimal is called the **mantissa**. The characteristic of the logarithm of any number greater than 1 may be determined as follows:

RULE I. *The characteristic of any number greater than 1 is one less than the number of digits before the decimal point.*

The following table, which is taken from § 1, p. v, shows that

a	.0000001	.000001	.00001	.0001	.001	.01	.1	1
$\log a$	− 7	− 6	− 5	− 4	− 3	− 2	− 1	0

the logarithm of every number between .1 and 1 is − 1 + a fraction,

the logarithm of every number between .01 and .1 is − 2 + a fraction,

the logarithm of every number between .001 and .01 is − 3 + a fraction; and so on.

Thus the characteristic of every number between 0 and 1 is a negative whole number; there is a great practical advantage, however, in computing, to write these characteristics as follows: $-1 = 9 - 10$, $-2 = 8 - 10$, $-3 = 7 - 10$, etc. *E.g.* the logarithm of .562 is $-1 + .74974$, but this should be written $9.74974 - 10$; and similarly for all numbers less than 1.

RULE II. *The characteristic of a number less than 1 is found by subtracting from 9 the number of ciphers between the decimal point and the first significant digit, and writing − 10 after the result.*

Thus, the characteristic of log 845 is 2 by Rule I; the characteristic of log 84.5 is 1 by (I); of log 8.45 is 0 by (I); of log .845 is 9 − 10 by (II); of log .0845 is 8 − 10 by (II).

An important consequence of what precedes is the following:

To move the decimal point in a given number one place to the right is equivalent to adding one unit to its logarithm, because this is equivalent to multiplying the given number by 10. Likewise, to move the decimal point one place to the left is equivalent to subtracting one unit from the logarithm. Hence, moving the decimal point any number of places to the right or left does not change the mantissa but only the characteristic.*

Thus, 5345, 5.345, 534.5, .05345, 534500 all have the same mantissa.

---

* Another rule for finding the characteristic, based on this property, is often useful: if the decimal point were just after the first significant figure, the characteristic would be zero; start at this point and count the digits passed over to the left or right to the actual decimal point; the number obtained is the characteristic, except for sign; the sign is negative if the movement was to the left, positive if the movement was to the right.

5. Use of the Table. To use logarithms in computation we need a table arranged so as to enable us to find, with as little effort and time as possible, the logarithms of given numbers and, vice versa, to find numbers when their logarithms are known. Since the characteristics may be found by means of Rules I and II, p. ix, only mantissas are given. This is done in Table I. Most of the numbers in this table are irrational, and must be represented in the decimal system by approximations. A five-place table is one which gives the values correct to five places of decimals.

Problem 1. *To find the logarithm of a given number.* First, determine the characteristic, then look in the table for the mantissa.

To find the mantissa in the table when the given number (neglecting the decimal point) consists of four, or less, digits (exclusive of ciphers at the beginning or end), look in the column marked N for the first three digits and select the column headed by the fourth digit: the mantissa will be found at the intersection of this row and this column. Thus to find the logarithm of 72050, observe first (Rule I) that the characteristic is 4. To find the mantissa, fix attention on the digits 7205; find 720 in column N, and opposite it in column 5 is the desired mantissa, .85763; hence log 72050 = 4.85763. The mantissa of .007826 is found opposite 782 in column 6 and is .89354; hence log .007826 = 7.89354 − 10.

6. Interpolation. If there are more than four significant figures in the given number, its mantissa is not printed in the table; but it can be found approximately by assuming that the mantissa varies as the number varies in the small interval not tabulated; while this assumption is not strictly correct, it is sufficiently accurate for use with this table.

Thus, to find the logarithm of 72054 we observe that log 72050 = 4.85763 and that log 72060 = 4.85769. Hence a change of 10 in the number causes a change of .00006 in the mantissa; we assume therefore that a change of 4 in the number will cause, approximately, a change of $.4 \times .00006 = .00002$ (dropping the sixth place) in the mantissa; and we write log 72054 = 4.85763 + .00002 = 4.85765.

The difference between two successive values printed in the table is called a **tabular difference** (.00006, above). The proportional part of this difference to be added to one of the tabular values is called the **correction** (.000002, above), and is found by multiplying the tabular difference by the appropriate fraction (.4, above). These proportional parts are usually written *without the zeros*, and are printed at the right-hand side of each page, to be used when mental multiplications seem uncertain.

Example 1. Find the logarithm of .0012647. Opposite 126 in column 4 find .10175; the tabular difference is 34 (zeros dropped); $.7 \times 34$ is given in the margin as 24; this correction added gives .10199 as the mantissa of .0012647; hence log .0012647 = 7.10199 − 10.

Example 2. Find the logarithm of 1.85643. Opposite 185 in column 6 find .26858; tabular difference 23; $.43 \times 23$ is given in the margin as 10; this correction added gives .26868 as the mantissa of 1.85643; hence log 1.85643 = 0.26868.

7. Reverse Reading of the Table. PROBLEM 2. *To find the number when its logarithm is known.** First, fixing attention on the mantissa only, find from the table the number having this mantissa, then place the decimal point by means of the two following rules: †

RULE III. *If the characteristic of the logarithm is positive (in which case the mantissa is not followed by* -10*), begin at the left, count digits one more than the characteristic, and place the decimal point to the right of the last digit counted.*

RULE IV. *If the characteristic is negative (in which case the mantissa will be preceded by a number* n *and followed by* -10*), prefix* $9-n$ *ciphers, and place the decimal point to the left of these ciphers.*

Example 1. Given $\log x = 1.22737$, to find x.

Since the mantissa is 22737, we look for 22 in the first column and to the right and below for 737, which we find in column 8 opposite 168. The number is therefore 1688. Since the characteristic is $+1$, we begin at the left, count 2 places, and place the point; hence $x = 16.88$.

Example 2. Given $\log x = 2.24912$, to find x.

This mantissa is not found in the table; in such cases we interpolate as follows: select the mantissa in the table next less than the given mantissa, and write down the corresponding number; here, 1774; the tabular difference is 25; the actual difference (found by subtracting the mantissa of 1774 from the given mantissa) is 17; hence the proportionality factor is $17/25 = .68$ or $.7$ (to the nearest tenth). Since moving the decimal point does not affect the mantissa, it follows that the digits in the required number are 17747 (to five places). The characteristic 2 directs to count 3 places from the left; hence $x = 177.47$.

RULE. *In general, when the given mantissa is not found in the table, write down four digits of the number corresponding to the mantissa in the table next less than the given mantissa, determine a fifth figure by dividing the actual difference by the tabular difference, and locate the decimal point by means of the characteristic.*

8. Illustrations of the Use of Logarithms in Computation.

Example 1. To find $832.43 \times 302.43 \times 16.725 \times .000178$.

$$\begin{aligned} \log 832.43 &= 2.92034 \\ \log 302.43 &= 2.48062 \\ \log 16.725 &= 1.22337 \\ \log .000178 &= \underline{6.25042 - 10} \text{ (add)} \\ \log x &= 2.87475 \qquad \text{whence } x = 749.47. \end{aligned}$$

Example 2. To find $461.29 \div 21.4$.

$$\begin{aligned} \log 461.29 &= 2.66397 \\ \log 21.4 &= \underline{1.33041} \qquad \text{(subtract)} \\ \log x &= 1.33356 \qquad \text{whence } x = 21.556. \end{aligned}$$

---

* The number whose logarithm is k is often called the **antilogarithm** of k.

† Another convenient form of these rules is as follows: if the characteristic were zero, the decimal point would fall just after the first significant figure; move the decimal point one place to the right for each positive unit in the characteristic, one place to the left for each negative unit in the characteristic.

Illustration of Cologarithms

Example 3. To find $\frac{48.25 \times 132.76 \times .1745}{1415.3}$.

We might add the logarithms of the factors in the numerator and from this sum subtract the logarithm of the denominator; but we can shorten the operation by *adding* the negative of the logarithm of the denominator instead of subtracting the logarithm itself. The negative of the logarithm of a number (when written in convenient form for computation) is called the **cologarithm** of the number. We may find the negative of any number by subtracting it from zero, and it is convenient in logarithmic computation to write zero in the form 10.00000 − 10. Thus the negative of 2.17 is 7.83 − 10; the negative of 1.1432 − 10 is 8.8568. Remembering that the cologarithm of a number is its negative we have the following rule:

To find the cologarithm of a number begin at the left of its logarithm (including the characteristic) and subtract each digit from 9, except the last, which subtract from 10; if the logarithm has not − 10 after the mantissa, write − 10 after the result; if the logarithm has − 10 after the mantissa, do not write − 10 after the result.*

By this rule the cologarithm of a number can be read directly out of the table without taking the trouble to write down the logarithm. Attention must be given not to forget the characteristic. The use of the cologarithm is governed by the principle:

Adding the cologarithm is equivalent to subtracting the logarithm.

Returning to the computation of the given problem we should write:

$$\begin{aligned}
\log 48.25 &= 1.68350 \\
\log 132.76 &= 2.12307 \\
\log .1745 &= 9.24180 - 10 \\
\operatorname{colog} 1415.3 &= \underline{6.84915 - 10} \quad \text{(add)} \\
\log x &= 9.89752 - 10 \quad \text{whence } x = .7898
\end{aligned}$$

Example 4. Find the 5th power of 7.26842

$$\begin{aligned}
\log 7.26842 &= 0.86144 \\
&\quad \underline{5} \quad \text{(multiply)} \\
\log x &= 4.30720 \quad \text{whence } x = 20286.
\end{aligned}$$

Example 5. Find the 4th root of .007564

$$\log .007564 = 7.87875 - 10.$$

(It is convenient to have, after the division by 4, − 10 after the mantissa; hence before the division we add 30.00000 − 30.)

$$\begin{aligned}
\log .007564 &= 37.87875 - 40 \quad \text{(divide by 4),} \\
\log x &= 9.46969 - 10 \quad \text{whence } x = .2949
\end{aligned}$$

Example 6. Find the value of $\sqrt[3]{\frac{(34.55)(-856.7)(-43.5)}{(98.75)(-186.3)}}$.

We have no logarithms of negative numbers, but an inspection of this problem shows that the result will be negative and numerically the same as though all the factors were positive; hence we proceed as follows:

$$\begin{aligned}
\log 34.55 &= 1.53845 \\
\log 856.7 &= 2.93283 \\
\log 43.5 &= 1.63849 \\
\operatorname{colog} 98.75 &= 8.00546 - 10 \\
\operatorname{colog} 186.3 &= \underline{7.72979} - 10 \qquad \text{(add)} \\
& 1.84502 \qquad\qquad \text{(divide by 3)} \\
\log(-x) &= 0.61501 \qquad\qquad \text{whence } x = -4.121
\end{aligned}$$

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* If the logarithm ends in one or more ciphers, the last significant digit is to be understood here.

9. The Slide Rule. A slide rule consists of two pieces of the shape of a ruler, one of which slides in grooves in the other; each is marked

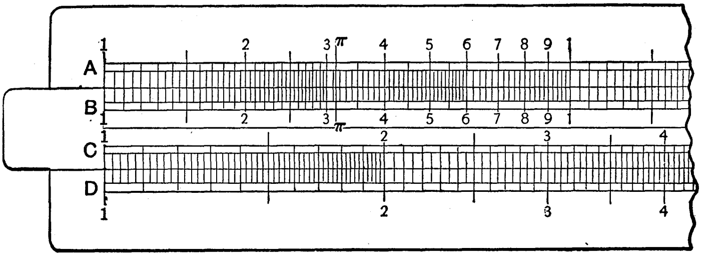

FIG. 1

(Fig. 1) in divisions (scale *A* and scale *B*) whose distances from one end are proportional to the logarithms of the numbers marked on them.

It follows that the sum of two logarithms can be obtained by simply

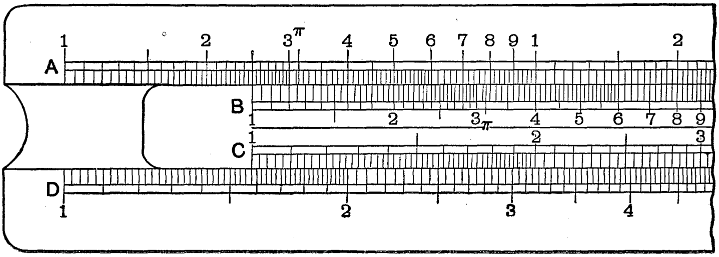

FIG. 2

sliding one rule along the other; thus if (see Fig. 2) the point marked 1 on scale *B* is set opposite the point marked 2.5 on scale *A*, the point on scale *B* marked 2 will be opposite the point on scale *A* marked 5, since $\log 2.5 + \log 2 = \log 5$. Likewise, opposite 3 (scale *B*) read 7.5 (scale *A*); opposite 2.5 (*B*) read 6.25 (*A*), *i.e.* $2.5 \times 2.5 = 6.25$.

Other multiplications can be performed in an analogous manner. Divisions can be performed by reversing the operation. Thus, if 4.5 (*B*) be set on 11.25 (*A*), then 1 (*B*) will be opposite 2.5 (*A*), as in Fig. 2.

Scales *C* and *D* are made just twice as large as scales *A* and *B*. It follows that the numbers marked on *C* and *D* are the square roots of the numbers marked opposite them on scales *A* and *B*.

For a description of more elaborate slide rules, and full directions for use, see the catalogues of instrument makers.

A slide rule for practice may be made from the cut printed on one of the fly-leaves in the back of this book.

I*a*. CONDENSED LOGARITHMS AND ANTILOGARITHMS

10. Method of Computing Logarithms. This table is a rearrangement of the condensed table given by Hoüel.[*] From it, the logarithm of any number whatever may be obtained to within 5 in the fifteenth place; or to any desired degree of accuracy less than this.

To illustrate the process, we shall compute $\log \pi$ to nine places. Taking $\pi = 3.14159\,26535\,8979$, we divide it by 3, the first significant digit, obtaining $\pi/3 = 1.04719\,755 \cdots$. We then divide this quotient by 1.04, etc., obtaining finally

$$\pi = 3(1.04)(1.006)(1.0009)(1.00001\,52172\,25).$$

We can obtain the logarithm of each of the first four factors from this table. The logarithm of the last factor can be obtained by multiplying its decimal part by $M = .43429\,44819$; for the error made in writing

$$\log(1 + x) = Mx$$

is less than $Mx^2/2$. We find Mx either by using the fact that the last column in this table gives multiples of M, or (preferably) by Table VIII, page 115. Adding the five logarithms just mentioned, we find

$$\log \pi = .49714\,98727\,4,$$

which is surely correct to within 1 in the tenth place. The correct value is $.49714\,98726\,9 \cdots$.

The process may be applied to any other number in an analogous manner. Such high-place logarithms are occasionally needed in statistical work and in the preparation of tables.

11. Method of Computing Antilogarithms. The condensed table of antilogarithms gives eleven significant figures (ten decimal places). From it, the antilogarithm of any number can be computed to within 5 in the tenth significant digit.

Thus, to compute the antilogarithm of .4342944819 to 8 significant figures, we may write

$$10^{.43429\,44819} = (10^{.4})(10^{.03})(10^{.004})(10^{.0002})(10^{.00009})(10^{.00000\,44819}).$$

The first five factors may be obtained directly from the table. The last factor may be calculated from the formula $10^x = 1 + (1/M)x$. The error in this formula is less than 3 in the $(2\,k)$th decimal place if x is less than $(.1)^k$, where $k > 1$.

However, a much more rapid process depends on the use of Tables I and XI with this table. Thus, by Table I, $10^{.43429} = 2.718$, nearly. By Table XI, $\log 2.718 = .43424\,94524 \cdots$. Hence $10^{.43429\,44819} = (2.718)(10^{.00004\,50295}) = (2.718)(10^{.00004})(10^{.00000\,50295})$. Obtaining the second factor from this table, and the last factor from the formula $10^x = 1 + (1/M)x$, by Table VIII, we find $10^{.43429\,44819} = 2.71828\,1826$; while the correct value is $2.718281828 \cdots$. This process requires only *two* long multiplications.

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[*] Hoüel, *Recueil de Formules et de Tables numériques.*

II. FIVE-PLACE TABLE OF THE ACTUAL VALUES OF THE TRIGONOMETRIC FUNCTIONS OF ANGLES

12. Direct Readings. This table gives the sines, cosines, tangents, and cotangents of the angles from $0°$ to $45°$; and by a simple device, indicated by the printing, the values of these functions for angles from $45°$ to $90°$ may be read directly from the same table. For angles less than $45°$ read down the page, the degrees being found at the top and the minutes on the left; for angles greater than $45°$ read up the page, the degrees being found at the bottom and the minutes on the right.

To find a function of an angle (such as $15° \, 27'.6$, for example) which does not reduce to an integral number of minutes, we employ the process of interpolation. To illustrate, let us find $\tan 15° \, 27'.6$. In the table we find $\tan 15° \, 27' = .27638$ and $\tan 15° \, 28' = .27670$; we know that $\tan 15° \, 27'.6$ lies between these two numbers. The process of interpolation depends on the assumption that between $15° \, 27'$ and $15° \, 28'$ the tangent of the angle varies directly as the angle; while this assumption is not strictly true, it gives an approximation sufficiently accurate for a five-place table. Thus we should assume that $\tan 15° \, 27'.5$ is halfway between .27638 and .27670. We may state the problem as follows: An increase of $1'$ in the angle increases the tangent .00032; assuming that the tangent varies as the angle, an increase of $0'.6$ in the angle will increase the tangent by $.6 \times .00032 = .00019$ (retaining only five places); hence

$$\tan 15° \, 27'.6 = .27638 + .00019 = .27657.$$

The difference between two successive values in the table is called, as in Table I, the *tabular difference* (.00032 above). The proportional part of the tabular difference which is used is called the *correction* (.00019 above), and is found by multiplying the tabular difference by the appropriate fraction of the smallest unit given in the table.

Example 1. Find $\sin 63° \, 52'.8$.

We find $\sin 63° \, 52' = .89777$;

tabular difference $= .00013$ (subtracted mentally from the table),

correction $= .8 \times .00013 = .00010$ (to be added).

Hence $\sin 63° \, 52'.8 = .89787$.

Example 2. Find $\cos 65° \, 24'.8$.

$\cos 65° \, 24' = .41628$;

tabular difference $= 26$; $.8 \times 26 = 21$

(to be subtracted because the cosine decreases as the angle increases).

Hence $\cos 65° \, 24'.8 = .41607$.

Rule. *To find a trigonometric function of an angle by interpolation: select the angle in the table which is next smaller than the given angle, and read its sine (cosine or tangent or cotangent as the case may be) and the tabular difference. Compute the correction as the proper proportional part of the tabular difference. In case of sines or tangents* **add** *the correction; in case of cosines or cotangents,* **subtract** *it.*

13. Reverse Readings. Interpolation is also used in finding the angle when one of its functions is given.

Example 1. Given $\sin x = .32845$, to find x.

Looking in the table we find the sine which is next less than the given sine to be .32832, and this belongs to 19° 10′. Subtract the value of the sine selected from the given sine to obtain the actual difference = .00013; note that the tabular difference = .00027. The actual difference divided by the tabular difference gives the correction = 13/27 = .5 as the decimal of a minute (to be added). Hence $x = 19° 10'.5$.

Example 2. Given $\cos x = .28432$, to find x.

The cosine in the table next less than this is .28429 and belongs to 73° 29′; the tabular difference is 28; the actual difference is 3; correction = 3/28 = .1 (to be subtracted). Hence $x = 73° 28'.9$.

Rule. *To find an angle when one of its trigonometric functions is given: select from the table the same named function which is next less than the given function, noting the corresponding angle and the tabular difference; compute the actual difference (between the selected value of the function and the given value) and divide it by the tabular difference; this gives the correction which is to be added if the given function is sine or tangent, and to be subtracted if the given function is cosine or cotangent.*

III. FIVE-PLACE COMMON LOGARITHMS OF THE TRIGONOMETRIC FUNCTIONS

14. Use of the Table. If it is required to find the numerical value of $x = 27.85 \times \sin 51° 27'$, we may apply logarithms as follows:

$$\begin{aligned} \log 27.85 &= 1.44483. \\ \log \sin 51° 27' &= \underline{9.89324 - 10} \text{ (add)}. \\ \log x &= 1.33807 \qquad x = 21.78 \end{aligned}$$

The only new idea here is the method of finding log sin 51° 27′, which means the logarithm of the sine of 51° 27′. The most obvious way is to find in Table I, sin 51° 27′ = .78206, and then to find in Table II, log .78206 = 9.89324 − 10, but this involves consulting two tables. To avoid the necessity of doing this, Table III gives the logarithms of the sines, cosines, tangents, and cotangents. The arrangement and the principles of interpolation are similar to those given on p. viii for Table I. The sines and cosines of all acute angles, the tangents of all acute angles less than 45° and the cotangents of all acute angles greater than 45° are proper fractions, and their logarithms end with − 10, which is not printed in the table, but which should be written down whenever such a logarithm is used.

Example 1. Find log sin 68° 25′.4.

On the page having 68° at the bottom, and in the row having 25′ on the right find log sin 68° 25′ = 9.96843 − 10; the tabular difference is 5; $.4 \times 5$ is given in the margin as 2; this is the correction to be added, giving log sin 68° 25′.4 = 9.96845 − 10.

(In case of sine and tangent *add* the correction. In case of cosine and cotangent, subtract the correction.)

Example 2. Given $\log \cos x = 9.72581 - 10$, to find x.

The logarithmic cosine next less than the given one is $9.72562 - 10$ and belongs to 57° 53′; the actual difference is 19; the tabular difference is 20; hence the correction is $19/20 = 1.0$ (to the nearest tenth); (subtract); hence $x = 57° \, 52'.0$.

In finding $\log \operatorname{ctn} \alpha$ for any angle α, note that $\log \operatorname{ctn} \alpha = -\log \tan \alpha$, since $\operatorname{ctn} \alpha = 1/\tan \alpha$. Hence *the tabular differences for* log ctn *are precisely the same as those for* log tan *throughout the table, but taken in reversed order.* Likewise, $\log \sec \alpha = -\log \cos \alpha$, $\log \csc \alpha = -\log \sin \alpha$; hence $\log \sec \alpha$ and $\log \csc \alpha$ are omitted.

For angles near 0° or near 90°, the interpolations are not very accurate if the differences are large. For the calculation of sine or tangent near 0°, Table III*a*, page 45, gives the values of

$$S = \log \sin A - \log A' \qquad \text{and} \qquad T = \log \tan A - \log A',$$

where A is the given angle and A' is the number of minutes in A, for values of A between 0° and 3°. Then

$$\log \sin A = \log A' + S \qquad \text{and} \qquad \log \tan A = \log A' + T,$$

for small angles. Moreover, since we have $\cos A = \sin(90° - A)$ and $\operatorname{ctn} A = \tan(90° - A)$,

$$\log \cos A = \log (90° - A)' + S \qquad \text{and} \qquad \log \operatorname{ctn} A = \log (90° - A)' + T,$$

when A is near 90°.

Another method practically equivalent to the preceding is to use the approximate relations

$$\log \sin A - \log \sin B = \log A' - \log B'$$

and

$$\log \tan A - \log \tan B = \log A' - \log B',$$

where A is the given angle and B is the nearest angle to A that is given in the table. If $A < 3°$ and $|A - B| < 1'$, these formulas give $\log \sin A$ and $\log \tan A$ to five decimal places.

IV–V. RADIAN MEASURE

15. Computations in Radian Measure. The reduction of degrees to radians is facilitated by Table IV — *Conversion of Degrees to Radians.* Since π radians = 180°, this table may be regarded as a table of multiples of $\pi/180$.

The values of $\sin x$, $\cos x$, $\tan x$, are stated for every angle x from 0.00 radians to 1.60 radians at intervals of .01 radian in Table V — *Trigonometric Functions in Radian Measure.* The values of any of these functions for larger values of x may be computed by first converting the value of the angle in radian measure to degree measure, by Table V*a*, and then finding the value of the function from Table II.

The reduction of radians to degrees can be performed directly by Table V; or, for greater accuracy, by the supplementary Table V*a*.

VI. POWERS — ROOTS — RECIPROCALS

16. Arrangement. This table is arranged so that the square, cube, square root, cube root, or reciprocal can be read directly to five decimal places for any number n of three significant figures. To attain this, not only n^2, n^3, $\sqrt{n}$, $\sqrt[3]{n}$, $1/n$, but also $\sqrt{10\,n}$, $\sqrt[3]{10\,n}$, $\sqrt[3]{100\,n}$ are printed on every page. All values have been carefully recomputed and checked.

Thus to find $\sqrt{1.17}$, read in $\sqrt{n}$ column the result: 1.08167. To find $\sqrt{11.7}$, read *in the same line*, in $\sqrt{10\,n}$ column the result: 3.42053. To find $\sqrt{117}$, read 10 times the entry in $\sqrt{n}$ column, since $\sqrt{117} = 10\sqrt{1.17}$.

Similarly, $\sqrt[3]{1.17} = 1.05373$ from $\sqrt[3]{n}$ column; $\sqrt[3]{11.7} = 2.27019$ from *the same line* in $\sqrt[3]{10\,n}$ column; $\sqrt[3]{117} = 4.89097$ from *the same line* in $\sqrt[3]{100\,n}$ column.

The effect of a change in the decimal point in n^2, n^3, and $1/n$ is only to shift the decimal point in the result, without altering the digits printed.

VII. NAPIERIAN OR NATURAL LOGARITHMS

17. The Base e. — Natural Logarithms. The number $e = 2.7182818 \cdots$ is called the **natural base** of logarithms. The logarithms of numbers to this base are given in Table VII at intervals of .01 from 0.01 to 10.09, and at unit intervals from 10 to 409. The fundamental relation $\log_e n = \log_e 10 \times \log_{10} n$ enables us to transfer from the base 10 to the base e, or conversely; where $\log_e 10 = 2.30258509$.

VIII. MULTIPLES OF M AND OF $1/M$

18. Multiples of M and $1/M$. This table is convenient whenever a number is to be multiplied by M or by $1/M$. This occurs whenever it is desired to change from common logarithms to natural logarithms, or conversely, since $M = \log_{10} e$ and since we have

$$\log_{10} x = (\log_e x)(\log_{10} e) = M \log_e x \quad \text{and} \quad \log_e x = (1/M) \log_{10} x.$$

Other formulas that require these multiples are

$$\log_{10} e^x = x \log_{10} e = x \cdot M \quad \text{and} \quad \log_e(10^n \cdot x) = \log_e x + n(1/M);$$

and the approximate formulas (see §§ 10, 11, p. xiv)

$$\log_{10}(1 \pm x) = \pm x \cdot M \quad \text{and} \quad 10^{\pm x} = 1 \pm (1/M)x.$$

IX. VALUES AND LOGARITHMS OF HYPERBOLIC FUNCTIONS

19. Hyperbolic Functions. This table gives the values of e^x, e^{-x}, $\sinh x$, $\cosh x$, $\tanh x$; and the logarithms of e^x, $\sinh x$, $\cosh x$, at varying intervals from $x = 0$ to $x = 10$. It is to be noted that $\log e^{-x} = -\log e^x$ and $\log \tanh x = \log \sinh x - \log \cosh x$. The table may be extended indefinitely by means of Table VIII, since $\log_{10} e^x = x \cdot M$; for this reason Table VIII may be regarded as a table of values of $\log_{10} e^x$.

X. VALUES AND LOGARITHMS OF HAVERSINES

20. Haversines. This table gives the values and the logarithms of the haversines of angles from 0° to 180° at intervals of $10'$. The haversine, which means *half of the versed sine*, is

$$\text{hav}\, A = (1/2)\ \text{vers}\, A = (1/2)(1 - \cos A)\,;$$

hence its values to five places may be computed from the table of cosines. It is used extensively in navigation, and it may be used to advantage in the solution of ordinary oblique triangles.

XI. FACTOR TABLE — LOGARITHMS OF PRIMES

21. Factors of Composite Numbers. Logarithms of Primes. The uses of this table are evident in questions involving factoring, and for finding high-place logarithms of numbers whose prime factors are less than 2018.

We shall illustrate the finding of logarithms of other numbers by finding $\log \pi$. Taking $\pi = 3.14159\,26536$, divide by 3 (the first digit), obtaining $1.04719\,75512 \cdots$. Divide this quotient by 1.047 (in general, by the nearest first four digits), obtaining $1.00018\,8683 \cdots$. By Table VIII, the approximate formula $\log(1 \pm x) = \pm x \cdot M$ gives

$$\begin{aligned} \log 1.00018\,8683 &= .00008\,1943 && \text{(Table VIII)}\\ \log 3 &= .47712\,12547 && \text{(Table XI)}\\ \log 1.047 = \log 3 + \log .349 &= \underline{.01994\,66817} && \text{(Table XI)}\\ \log \pi &= .49714\,9879 \end{aligned}$$

while the true value of $\log \pi$ is $.49714\,98726\,9$, so that the error is less than 1 in the eighth place. In general, this process will give the logarithm of *any* number to within 6 in the eighth decimal place, and the *probable error* is less than 1.5 in the eighth place. For still greater accuracy, see Table I*a* and § 10.

XII. INTEREST TABLES

22. Interest Tables. Tables XII *a*, *b*, *c*, *d* give compound interest and annuity data for various per cents up to fifty years. Aside from the obvious uses, formulas involving this data will be found in works on statistics, accounting, and the mathematics of business.

Table XII*e* gives the logarithms of $(1 + r)$ to fifteen places, for all ordinary values of r from 1/2 % to 10 %. For other values of r, $\log(1 + r)$ may be computed from Table I*a* (see § 10). The final result in interest calculations may be obtained to nine significant figures by the antilogarithms of Table I*a* (see § 11).

Table XII*f* is the American Experience Mortality Table.

XIV. FOUR-PLACE TABLES

23. Four-place Tables. These are duplicates of the preceding five-place tables, reduced to four places, and with larger intervals between the tabulations. The value of such four-place tables consists in the greater speed with which they can be used, in case the degree of accuracy they afford is sufficient for the purpose in hand.

XIV*a*. Logarithms of Numbers. The only special feature of this table is that *the proportional parts are printed for every tenth in every row;* hence the logarithm of any number of *four* significant figures can be read directly.

XIV*b*. Antilogarithms. This table will be found to facilitate approximate calculations to a marked degree. The proportional parts are stated in the right-hand margin for each row separately. This arrangement, with the corresponding one in Table XIV*a*, makes the tables *effectively* four-place each way.

XIV*c*. Values and Logarithms of Trigonometric Functions. In this table, the values of sin α, cos α, tan α, ctn α, and their common logarithms, are stated for each 10-minute interval in α. The characteristics of the logarithms are omitted, since they can be supplied readily from the value.

24. Sources and Checks used. In arranging all of these tables, several extant tables have been used as sources; and the proofs have been read against the standard seven-place tables of Vega, and at least one other table, or against at least two independent sources when the figures are not given by Vega. In all cases, the stereotyped plates have been proof-read five times, by three different persons.

In case of apparent doubt, especially in the last place of decimals, the values have been recomputed, either by series or by the condensed fifteen-place tables of Hoüel.

While errors may occur, it is believed that they must be purely typographical; in most cases such an error is revealed by the unreasonable differences it creates.

Greek Alphabet

Letters	Names	Letters	Names	Letters	Names	Letters	Names
Α α	Alpha	Η η	Eta	Ν ν	Nu	Τ τ	Tau
Β β	Beta	Θ θ	Theta	Ξ ξ	Xi	Υ υ	Upsilon
Γ γ	Gamma	Ι ι	Iota	Ο ο	Omicron	Φ φ	Phi
Δ δ	Delta	Κ κ	Kappa	Π π	Pi	Χ χ	Chi
Ε ϵ	Epsilon	Λ λ	Lambda	Ρ ρ	Rho	Ψ ψ	Psi
Ζ ζ	Zeta	Μ μ	Mu	Σ σ ς	Sigma	Ω ω	Omega

LOGARITHMIC AND TRIGONOMETRIC TABLES

TABLE I

COMMON LOGARITHMS OF NUMBERS

FROM

1 TO 10 000

TO

FIVE DECIMAL PLACES

1—100

N	Log	N	Log	N	Log	N	Log	N	Log
0	———	**20**	1.30 103	**40**	1.60 206	**60**	1.77 815	**80**	1.90 309
1	0.00 000	21	1.32 222	41	1.61 278	61	1.78 533	81	1.90 849
2	0.30 103	22	1.34 242	42	1.62 325	62	1.79 239	82	1.91 381
3	0.47 712	23	1.36 173	43	1.63 347	63	1.79 934	83	1.91 908
4	0.60 206	24	1.38 021	44	1.64 345	64	1.80 618	84	1.92 428
5	0.69 897	25	1.39 794	45	1.65 321	65	1.81 291	85	1.92 942
6	0.77 815	26	1.41 497	46	1.66 276	66	1.81 954	86	1.93 450
7	0.84 510	27	1.43 136	47	1.67 210	67	1.82 607	87	1.93 952
8	0.90 309	28	1.44 716	48	1.68 124	68	1.83 251	88	1.94 448
9	0.95 424	29	1.46 240	49	1.69 020	69	1.83 885	89	1.94 939
10	1.00 000	**30**	1.47 712	**50**	1.69 897	**70**	1.84 510	**90**	1.95 424
11	1.04 139	31	1.49 136	51	1.70 757	71	1.85 126	91	1.95 904
12	1.07 918	32	1.50 515	52	1.71 600	72	1.85 733	92	1.96 379
13	1.11 394	33	1.51 851	53	1.72 428	73	1.86 332	93	1.96 848
14	1.14 613	34	1.53 148	54	1.73 239	74	1.86 923	94	1.97 313
15	1.17 609	35	1.54 407	55	1.74 036	75	1.87 506	95	1.97 772
16	1.20 412	36	1.55 630	56	1.74 819	76	1.88 081	96	1.98 227
17	1.23 045	37	1.56 820	57	1.75 587	77	1.88 649	97	1.98 677
18	1.25 527	38	1.57 978	58	1.76 343	78	1.89 209	98	1.99 123
19	1.27 875	39	1.59 106	59	1.77 085	79	1.89 763	99	1.99 564
N	Log	N	Log	N	Log	N	Log	N	Log

N.	0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
01	432	475	518	561	604	647	689	732	775	817
02	860	903	945	988	*030	*072	*115	*157	*199	*242
03	01 284	326	368	410	452	494	536	578	620	662
04	703	745	787	828	870	912	953	995	*036	*078
05	02 119	160	202	243	284	325	366	407	449	490
06	531	572	612	653	694	735	776	816	857	898
07	938	979	*019	*060	*100	*141	*181	*222	*262	*302
08	03 342	383	423	463	503	543	583	623	663	703
09	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
11	532	571	610	650	689	727	766	805	844	883
12	922	961	999	*038	*077	*115	*154	*192	*231	*269
13	05 308	346	385	423	461	500	538	576	614	652
14	690	729	767	805	843	881	918	956	994	*032
15	06 070	108	145	183	221	258	296	333	371	408
16	446	483	521	558	595	633	670	707	744	781
17	819	856	893	930	967	*004	*041	*078	*115	*151
18	07 188	225	262	298	335	372	408	445	482	518
19	555	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*135	*171	*207	*243
21	08 279	314	350	386	422	458	493	529	565	600
22	636	672	707	743	778	814	849	884	920	955
23	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
24	09 342	377	412	447	482	517	552	587	621	656
25	691	726	760	795	830	864	899	934	968	*003
26	10 037	072	106	140	175	209	243	278	312	346
27	380	415	449	483	517	551	585	619	653	687
28	721	755	789	823	857	890	924	958	992	*025
29	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
31	727	760	793	826	860	893	926	959	992	*024
32	12 057	090	123	156	189	222	254	287	320	352
33	385	418	450	483	516	548	581	613	646	678
34	710	743	775	808	840	872	905	937	969	*001
35	13 033	066	098	130	162	194	226	258	290	322
36	354	386	418	450	481	513	545	577	609	640
37	672	704	735	767	799	830	862	893	925	956
38	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
39	14 301	333	364	395	426	457	489	520	551	582
140	613	644	675	706	737	768	799	829	860	891
41	922	953	983	*014	*045	*076	*106	*137	*168	*198
42	15 229	259	290	320	351	381	412	442	473	503
43	534	564	594	625	655	685	715	746	776	806
44	836	866	897	927	957	987	*017	*047	*077	*107
45	16 137	167	197	227	256	286	316	346	376	406
46	435	465	495	524	554	584	613	643	673	702
47	732	761	791	820	850	879	909	938	967	997
48	17 026	056	085	114	143	173	202	231	260	289
49	319	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	754	782	811	840	869
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

	44	43	42
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8

	41	40	39
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	38	37	36
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	35	34	33
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

Prop. Pts.

150 — Logarithms of Numbers — 200

N.	0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	754	782	811	840	869
51	898	926	955	984	*013	*041	*070	*099	*127	*156
52	18 184	213	241	270	298	327	355	384	412	441
53	469	498	526	554	583	611	639	667	696	724
54	752	780	808	837	865	893	921	949	977	*005
55	19 033	061	089	117	145	173	201	229	257	285
56	312	340	368	396	424	451	479	507	535	562
57	590	618	645	673	700	728	756	783	811	838
58	866	893	921	948	976	*003	*030	*058	*085	*112
59	20 140	167	194	222	249	276	303	330	358	385
160	412	439	466	493	520	548	575	602	629	656
61	683	710	737	763	790	817	844	871	898	925
62	952	978	*005	*032	*059	*085	*112	*139	*165	*192
63	21 219	245	272	299	325	352	378	405	431	458
64	484	511	537	564	590	617	643	669	696	722
65	748	775	801	827	854	880	906	932	958	985
66	22 011	037	063	089	115	141	167	194	220	246
67	272	298	324	350	376	401	427	453	479	505
68	531	557	583	608	634	660	686	712	737	763
69	789	814	840	866	891	917	943	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
71	300	325	350	376	401	426	452	477	502	528
72	553	578	603	629	654	679	704	729	754	779
73	805	830	855	880	905	930	955	980	*005	*030
74	24 055	080	105	130	155	180	204	229	254	279
75	304	329	353	378	403	428	452	477	502	527
76	551	576	601	625	650	674	699	724	748	773
77	797	822	846	871	895	920	944	969	993	*018
78	25 042	066	091	115	139	164	188	212	237	261
79	285	310	334	358	382	406	431	455	479	503
180	527	551	575	600	624	648	672	696	720	744
81	768	792	816	840	864	888	912	935	959	983
82	26 007	031	055	079	102	126	150	174	198	221
83	245	269	293	316	340	364	387	411	435	458
84	482	505	529	553	576	600	623	647	670	694
85	717	741	764	788	811	834	858	881	905	928
86	951	975	998	*021	*045	*068	*091	*114	*138	*161
87	27 184	207	231	254	277	300	323	346	370	393
88	416	439	462	485	508	531	554	577	600	623
89	646	669	692	715	738	761	784	807	830	852
190	875	898	921	944	967	989	*012	*035	*058	*081
91	28 103	126	149	171	194	217	240	262	285	307
92	330	353	375	398	421	443	466	488	511	533
93	556	578	601	623	646	668	691	713	735	758
94	780	803	825	847	870	892	914	937	959	981
95	29 003	026	048	070	092	115	137	159	181	203
96	226	248	270	292	314	336	358	380	403	425
97	447	469	491	513	535	557	579	601	623	645
98	667	688	710	732	754	776	798	820	842	863
99	885	907	929	951	973	994	*016	*038	*060	*081
200	30 103	125	146	168	190	211	233	255	276	298
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

	29	**28**	**27**
1	2.9	2.8	2.7
2	5.8	5.6	5.4
3	8.7	8.4	8.1
4	11.6	11.2	10.8
5	14.5	14.0	13.5
6	17.4	16.8	16.2
7	20.3	19.6	18.9
8	23.2	22.4	21.6
9	26.1	25.2	24.3

	26	**25**	**24**
1	2.6	2.5	2.4
2	5.2	5.0	4.8
3	7.8	7.5	7.2
4	10.4	10.0	9.6
5	13.0	12.5	12.0
6	15.6	15.0	14.4
7	18.2	17.5	16.8
8	20.8	20.0	19.2
9	23.4	22.5	21.6

	23	**22**	**21**
1	2.3	2.2	2.1
2	4.6	4.4	4.2
3	6.9	6.6	6.3
4	9.2	8.8	8.4
5	11.5	11.0	10.5
6	13.8	13.2	12.6
7	16.1	15.4	14.7
8	18.4	17.6	16.8
9	20.7	19.8	18.9

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
200	30 103	125	146	168	190	211	233	255	276	298
01	320	341	363	384	406	428	449	471	492	514
02	535	557	578	600	621	643	664	685	707	728
03	750	771	792	814	835	856	878	899	920	942
04	963	984	*006	*027	*048	*069	*091	*112	*133	*154
05	31 175	197	218	239	260	281	302	323	345	366
06	387	408	429	450	471	492	513	534	555	576
07	597	618	639	660	681	702	723	744	765	785
08	806	827	848	869	890	911	931	952	973	994
09	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
11	428	449	469	490	510	531	552	572	593	613
12	634	654	675	695	715	736	756	777	797	818
13	838	858	879	899	919	940	960	980	*001	*021
14	33 041	062	082	102	122	143	163	183	203	224
15	244	264	284	304	325	345	365	385	405	425
16	445	465	486	506	526	546	566	586	606	626
17	646	666	686	706	726	746	766	786	806	826
18	846	866	885	905	925	945	965	985	*005	*025
19	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
21	439	459	479	498	518	537	557	577	596	616
22	635	655	674	694	713	733	753	772	792	811
23	830	850	869	889	908	928	947	967	986	*005
24	35 025	044	064	083	102	122	141	160	180	199
25	218	238	257	276	295	315	334	353	372	392
26	411	430	449	468	488	507	526	545	564	583
27	603	622	641	660	679	698	717	736	755	774
28	793	813	832	851	870	889	908	927	946	965
29	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
31	361	380	399	418	436	455	474	493	511	530
32	549	568	586	605	624	642	661	680	698	717
33	736	754	773	791	810	829	847	866	884	903
34	922	940	959	977	996	*014	*033	*051	*070	*088
35	37 107	125	144	162	181	199	218	236	254	273
36	291	310	328	346	365	383	401	420	438	457
37	475	493	511	530	548	566	585	603	621	639
38	658	676	694	712	731	749	767	785	803	822
39	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
41	202	220	238	256	274	292	310	328	346	364
42	382	399	417	435	453	471	489	507	525	543
43	561	578	596	614	632	650	668	686	703	721
44	739	757	775	792	810	828	846	863	881	899
45	917	934	952	970	987	*005	*023	*041	*058	*076
46	39 094	111	129	146	164	182	199	217	235	252
47	270	287	305	322	340	358	375	393	410	428
48	445	463	480	498	515	533	550	568	585	602
49	620	637	655	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

log 2 = .30102 99566

	22	21	20
1	2.2	2.1	2.0
2	4.4	4.2	4.0
3	6.6	6.3	6.0
4	8.8	8.4	8.0
5	11.0	10.5	10.0
6	13.2	12.6	12.0
7	15.4	14.7	14.0
8	17.6	16.8	16.0
9	19.8	18.9	18.0

	19	18	17
1	1.9	1.8	1.7
2	3.8	3.6	3.4
3	5.7	5.4	5.1
4	7.6	7.2	6.8
5	9.5	9.0	8.5
6	11.4	10.8	10.2
7	13.3	12.6	11.9
8	15.2	14.4	13.6
9	17.1	16.2	15.3

Prop. Pts.

250 — Logarithms of Numbers — 300

N.	0	1	2	3	4	5	6	7	8	9
250	39 794	811	829	846	863	881	898	915	933	950
51	967	985	*002	*019	*037	*054	*071	*088	*106	*123
52	40 140	157	175	192	209	226	243	261	278	295
53	312	329	346	364	381	398	415	432	449	466
54	483	500	518	535	552	569	586	603	620	637
55	654	671	688	705	722	739	756	773	790	807
56	824	841	858	875	892	909	926	943	960	976
57	993	*010	*027	*044	*061	*078	*095	*111	*128	*145
58	41 162	179	196	212	229	246	263	280	296	313
59	330	347	363	380	397	414	430	447	464	481
260	497	514	531	547	564	581	597	614	631	647
61	664	681	697	714	731	747	764	780	797	814
62	830	847	863	880	896	913	929	946	963	979
63	996	*012	*029	*045	*062	*078	*095	*111	*127	*144
64	42 160	177	193	210	226	243	259	275	292	308
65	325	341	357	374	390	406	423	439	455	472
66	488	504	521	537	553	570	586	602	619	635
67	651	667	684	700	716	732	749	765	781	797
68	813	830	846	862	878	894	911	927	943	959
69	975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43 136	152	169	185	201	217	233	249	265	281
71	297	313	329	345	361	377	393	409	425	441
72	457	473	489	505	521	537	553	569	584	600
73	616	632	648	664	680	696	712	727	743	759
74	775	791	807	823	838	854	870	886	902	917
75	933	949	965	981	996	*012	*028	*044	*059	*075
76	44 091	107	122	138	154	170	185	201	217	232
77	248	264	279	295	311	326	342	358	373	389
78	404	420	436	451	467	483	498	514	529	545
79	560	576	592	607	623	638	654	669	685	700
280	716	731	747	762	778	793	809	824	840	855
81	871	886	902	917	932	948	963	979	994	*010
82	45 025	040	056	071	086	102	117	133	148	163
83	179	194	209	225	240	255	271	286	301	317
84	332	347	362	378	393	408	423	439	454	469
85	484	500	515	530	545	561	576	591	606	621
86	637	652	667	682	697	712	728	743	758	773
87	788	803	818	834	849	864	879	894	909	924
88	939	954	969	984	*000	*015	*030	*045	*060	*075
89	46 090	105	120	135	150	165	180	195	210	225
290	240	255	270	285	300	315	330	345	359	374
91	389	404	419	434	449	464	479	494	509	523
92	538	553	568	583	598	613	627	642	657	672
93	687	702	716	731	746	761	776	790	805	820
94	835	850	864	879	894	909	923	938	953	967
95	982	997	*012	*026	*041	*056	*070	*085	*100	*114
96	47 129	144	159	173	188	202	217	232	246	261
97	276	290	305	319	334	349	363	378	392	407
98	422	436	451	465	480	494	509	524	538	553
99	567	582	596	611	625	640	654	669	683	698
300	712	727	741	756	770	784	799	813	828	842
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	18	17	16
1	1.8	1.7	1.6
2	3.6	3.4	3.2
3	5.4	5.1	4.8
4	7.2	6.8	6.4
5	9.0	8.5	8.0
6	10.8	10.2	9.6
7	12.6	11.9	11.2
8	14.4	13.6	12.8
9	16.2	15.3	14.4

$M = \log_{10} e$
$= \log_{10} 2.718\cdots$
$= .43429\,44819$

	15	14
1	1.5	1.4
2	3.0	2.8
3	4.5	4.2
4	6.0	5.6
5	7.5	7.0
6	9.0	8.4
7	10.5	9.8
8	12.0	11.2
9	13.5	12.6

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
300	47 712	727	741	756	770	784	799	813	828	842
01	857	871	885	900	914	929	943	958	972	986
02	48 001	015	029	044	058	073	087	101	116	130
03	144	159	173	187	202	216	230	244	259	273
04	287	302	316	330	344	359	373	387	401	416
05	430	444	458	473	487	501	515	530	544	558
06	572	586	601	615	629	643	657	671	686	700
07	714	728	742	756	770	785	799	813	827	841
08	855	869	883	897	911	926	940	954	968	982
09	996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49 136	150	164	178	192	206	220	234	248	262
11	276	290	304	318	332	346	360	374	388	402
12	415	429	443	457	471	485	499	513	527	541
13	554	568	582	596	610	624	638	651	665	679
14	693	707	721	734	748	762	776	790	803	817
15	831	845	859	872	886	900	914	927	941	955
16	969	982	996	*010	*024	*037	*051	*065	*079	*092
17	50 106	120	133	147	161	174	188	202	215	229
18	243	256	270	284	297	311	325	338	352	365
19	379	393	406	420	433	447	461	474	488	501
320	515	529	542	556	569	583	596	610	623	637
21	651	664	678	691	705	718	732	745	759	772
22	786	799	813	826	840	853	866	880	893	907
23	920	934	947	961	974	987	*001	*014	*028	*041
24	51 055	068	081	095	108	121	135	148	162	175
25	188	202	215	228	242	255	268	282	295	308
26	322	335	348	362	375	388	402	415	428	441
27	455	468	481	495	508	521	534	548	561	574
28	587	601	614	627	640	654	667	680	693	706
29	720	733	746	759	772	786	799	812	825	838
330	851	865	878	891	904	917	930	943	957	970
31	983	996	*009	*022	*035	*048	*061	*075	*088	*101
32	52 114	127	140	153	166	179	192	205	218	231
33	244	257	270	284	297	310	323	336	349	362
34	375	388	401	414	427	440	453	466	479	492
35	504	517	530	543	556	569	582	595	608	621
36	634	647	660	673	686	699	711	724	737	750
37	763	776	789	802	815	827	840	853	866	879
38	892	905	917	930	943	956	969	982	994	*007
39	53 020	033	046	058	071	084	097	110	122	135
340	148	161	173	186	199	212	224	237	250	263
41	275	288	301	314	326	339	352	364	377	390
42	403	415	428	441	453	466	479	491	504	517
43	529	542	555	567	580	593	605	618	631	643
44	656	668	681	694	706	719	732	744	757	769
45	782	794	807	820	832	845	857	870	882	895
46	908	920	933	945	958	970	983	995	*008	*020
47	54 033	045	058	070	083	095	108	120	133	145
48	158	170	183	195	208	220	233	245	258	270
49	283	295	307	320	332	345	357	370	382	394
350	407	419	432	444	456	469	481	494	506	518
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

log 3 = .47712 12547
log π = .49714 98727

	15	**14**
1	1.5	1.4
2	3.0	2.8
3	4.5	4.2
4	6.0	5.6
5	7.5	7.0
6	9.0	8.4
7	10.5	9.8
8	12.0	11.2
9	13.5	12.6

	13	**12**
1	1.3	1.2
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

Prop. Pts.

350 — Logarithms of Numbers — 400

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
350	54 407	419	432	444	456	469	481	494	506	518	
51	531	543	555	568	580	593	605	617	630	642	
52	654	667	679	691	704	716	728	741	753	765	
53	777	790	802	814	827	839	851	864	876	888	
54	900	913	925	937	949	962	974	986	998	*011	
55	55 023	035	047	060	072	084	096	108	121	133	
56	145	157	169	182	194	206	218	230	242	255	
57	267	279	291	303	315	328	340	352	364	376	
58	388	400	413	425	437	449	461	473	485	497	
59	509	522	534	546	558	570	582	594	606	618	
360	630	642	654	666	678	691	703	715	727	739	
61	751	763	775	787	799	811	823	835	847	859	
62	871	883	895	907	919	931	943	955	967	979	
63	991	*003	*015	*027	*038	*050	*062	*074	*086	*098	
64	56 110	122	134	146	158	170	182	194	205	217	
65	229	241	253	265	277	289	301	312	324	336	
66	348	360	372	384	396	407	419	431	443	455	
67	467	478	490	502	514	526	538	549	561	573	
68	585	597	608	620	632	644	656	667	679	691	
69	703	714	726	738	750	761	773	785	797	808	
370	820	832	844	855	867	879	891	902	914	926	
71	937	949	961	972	984	996	*008	*019	*031	*043	
72	57 054	066	078	089	101	113	124	136	148	159	
73	171	183	194	206	217	229	241	252	264	276	
74	287	299	310	322	334	345	357	368	380	392	
75	403	415	426	438	449	461	473	484	496	507	
76	519	530	542	553	565	576	588	600	611	623	
77	634	646	657	669	680	692	703	715	726	738	
78	749	761	772	784	795	807	818	830	841	852	
79	864	875	887	898	910	921	933	944	955	967	
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081	
81	58 092	104	115	127	138	149	161	172	184	195	
82	206	218	229	240	252	263	274	286	297	309	
83	320	331	343	354	365	377	388	399	410	422	
84	433	444	456	467	478	490	501	512	524	535	
85	546	557	569	580	591	602	614	625	636	647	
86	659	670	681	692	704	715	726	737	749	760	
87	771	782	794	805	816	827	838	850	861	872	
88	883	894	906	917	928	939	950	961	973	984	
89	995	*006	*017	*028	*040	*051	*062	*073	*084	*095	
390	59 106	118	129	140	151	162	173	184	195	207	
91	218	229	240	251	262	273	284	295	306	318	
92	329	340	351	362	373	384	395	406	417	428	
93	439	450	461	472	483	494	506	517	528	539	
94	550	561	572	583	594	605	616	627	638	649	
95	660	671	682	693	704	715	726	737	748	759	
96	770	780	791	802	813	824	835	846	857	868	
97	879	890	901	912	923	934	945	956	966	977	
98	988	999	*010	*021	*032	*043	*054	*065	*076	*086	
99	60 097	108	119	130	141	152	163	173	184	195	
400	206	217	228	239	249	260	271	282	293	304	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

	13	12
1	1.3	1.2
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

	11	10
1	1.1	1.0
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
400	60 206	217	228	239	249	260	271	282	293	304	
01	314	325	336	347	358	369	379	390	401	412	
02	423	433	444	455	466	477	487	498	509	520	
03	531	541	552	563	574	584	595	606	617	627	
04	638	649	660	670	681	692	703	713	724	735	
05	746	756	767	778	788	799	810	821	831	842	
06	853	863	874	885	895	906	917	927	938	949	
07	959	970	981	991	*002	*013	*023	*034	*045	*055	
08	61 066	077	087	098	109	119	130	140	151	162	
09	172	183	194	204	215	225	236	247	257	268	
410	278	289	300	310	321	331	342	352	363	374	
11	384	395	405	416	426	437	448	458	469	479	
12	490	500	511	521	532	542	553	563	574	584	
13	595	606	616	627	637	648	658	669	679	690	
14	700	711	721	731	742	752	763	773	784	794	
15	805	815	826	836	847	857	868	878	888	899	
16	909	920	930	941	951	962	972	982	993	*003	
17	62 014	024	034	045	055	066	076	086	097	107	
18	118	128	138	149	159	170	180	190	201	211	
19	221	232	242	252	263	273	284	294	304	315	
420	325	335	346	356	366	377	387	397	408	418	
21	428	439	449	459	469	480	490	500	511	521	
22	531	542	552	562	572	583	593	603	613	624	
23	634	644	655	665	675	685	696	706	716	726	
24	737	747	757	767	778	788	798	808	818	829	
25	839	849	859	870	880	890	900	910	921	931	
26	941	951	961	972	982	992	*002	*012	*022	*033	
27	63 043	053	063	073	083	094	104	114	124	134	
28	144	155	165	175	185	195	205	215	225	236	
29	246	256	266	276	286	296	306	317	327	337	
430	347	357	367	377	387	397	407	417	428	438	
31	448	458	468	478	488	498	508	518	528	538	
32	548	558	568	579	589	599	609	619	629	639	
33	649	659	669	679	689	699	709	719	729	739	
34	749	759	769	779	789	799	809	819	829	839	
35	849	859	869	879	889	899	909	919	929	939	
36	949	959	969	979	988	998	*008	*018	*028	*038	
37	64 048	058	068	078	088	098	108	118	128	137	
38	147	157	167	177	187	197	207	217	227	237	
39	246	256	266	276	286	296	306	316	326	335	
440	345	355	365	375	385	395	404	414	424	434	
41	444	454	464	473	483	493	503	513	523	532	
42	542	552	562	572	582	591	601	611	621	631	
43	640	650	660	670	680	689	699	709	719	729	
44	738	748	758	768	777	787	797	807	816	826	
45	836	846	856	865	875	885	895	904	914	924	
46	933	943	953	963	972	982	992	*002	*011	*021	
47	65 031	040	050	060	070	079	089	099	108	118	
48	128	137	147	157	167	176	186	196	205	215	
49	225	234	244	254	263	273	283	292	302	312	
450	321	331	341	350	360	369	379	389	398	408	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

	11	**10**	**9**
1	1.1	1.0	0.9
2	2.2	2.0	1.8
3	3.3	3.0	2.7
4	4.4	4.0	3.6
5	5.5	5.0	4.5
6	6.6	6.0	5.4
7	7.7	7.0	6.3
8	8.8	8.0	7.2
9	9.9	9.0	8.1

$\log M = \log [\log e]$
$= 9.63778\,431 - 10$

450 — Logarithms of Numbers — 500

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
450	65 321	331	341	350	360	369	379	389	398	408	
51	418	427	437	447	456	466	475	485	495	504	
52	514	523	533	543	552	562	571	581	591	600	
53	610	619	629	639	648	658	667	677	686	696	
54	706	715	725	734	744	753	763	772	782	792	
55	801	811	820	830	839	849	858	868	877	887	
56	896	906	916	925	935	944	954	963	973	982	
57	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	
58	66 087	096	106	115	124	134	143	153	162	172	
59	181	191	200	210	219	229	238	247	257	266	
460	276	285	295	304	314	323	332	342	351	361	
61	370	380	389	398	408	417	427	436	445	455	
62	464	474	483	492	502	511	521	530	539	549	
63	558	567	577	586	596	605	614	624	633	642	
64	652	661	671	680	689	699	708	717	727	736	
65	745	755	764	773	783	792	801	811	820	829	
66	839	848	857	867	876	885	894	904	913	922	
67	932	941	950	960	969	978	987	997	*006	*015	
68	67 025	034	043	052	062	071	080	089	099	108	
69	117	127	136	145	154	164	173	182	191	201	
470	210	219	228	237	247	256	265	274	284	293	
71	302	311	321	330	339	348	357	367	376	385	
72	394	403	413	422	431	440	449	459	468	477	
73	486	495	504	514	523	532	541	550	560	569	
74	578	587	596	605	614	624	633	642	651	660	
75	669	679	688	697	706	715	724	733	742	752	
76	761	770	779	788	797	806	815	825	834	843	
77	852	861	870	879	888	897	906	916	925	934	
78	943	952	961	970	979	988	997	*006	*015	*024	
79	68 034	043	052	061	070	079	088	097	106	115	
480	124	133	142	151	160	169	178	187	196	205	
81	215	224	233	242	251	260	269	278	287	296	
82	305	314	323	332	341	350	359	368	377	386	
83	395	404	413	422	431	440	449	458	467	476	
84	485	494	502	511	520	529	538	547	556	565	
85	574	583	592	601	610	619	628	637	646	655	
86	664	673	681	690	699	708	717	726	735	744	
87	753	762	771	780	789	797	806	815	824	833	
88	842	851	860	869	878	886	895	904	913	922	
89	931	940	949	958	966	975	984	993	*002	*011	
490	69 020	028	037	046	055	064	073	082	090	099	
91	108	117	126	135	144	152	161	170	179	188	
92	197	205	214	223	232	241	249	258	267	276	
93	285	294	302	311	320	329	338	346	355	364	
94	373	381	390	399	408	417	425	434	443	452	
95	461	469	478	487	496	504	513	522	531	539	
96	548	557	566	574	583	592	601	609	618	627	
97	636	644	653	662	671	679	688	697	705	714	
98	723	732	740	749	758	767	775	784	793	801	
99	810	819	827	836	845	854	862	871	880	888	
500	897	906	914	923	932	940	949	958	966	975	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

	10	**9**	**8**
1	1.0	0.9	0.8
2	2.0	1.8	1.6
3	3.0	2.7	2.4
4	4.0	3.6	3.2
5	5.0	4.5	4.0
6	6.0	5.4	4.8
7	7.0	6.3	5.6
8	8.0	7.2	6.4
9	9.0	8.1	7.2

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
500	69 897	906	914	923	932	940	949	958	966	975	
01	984	992	*001	*010	*018	*027	*036	*044	*053	*062	
02	70 070	079	088	096	105	114	122	131	140	148	
03	157	165	174	183	191	200	209	217	226	234	log 5 = .69897 00043
04	243	252	260	269	278	286	295	303	312	321	
05	329	338	346	355	364	372	381	389	398	406	
06	415	424	432	441	449	458	467	475	484	492	
07	501	509	518	526	535	544	552	561	569	578	
08	586	595	603	612	621	629	638	646	655	663	
09	672	680	689	697	706	714	723	731	740	749	
510	757	766	774	783	791	800	808	817	825	834	
11	842	851	859	868	876	885	893	902	910	919	
12	927	935	944	952	961	969	978	986	995	*003	
13	71 012	020	029	037	046	054	063	071	079	088	
14	096	105	113	122	130	139	147	155	164	172	
15	181	189	198	206	214	223	231	240	248	257	
16	265	273	282	290	299	307	315	324	332	341	
17	349	357	366	374	383	391	399	408	416	425	
18	433	441	450	458	466	475	483	492	500	508	
19	517	525	533	542	550	559	567	575	584	592	
520	600	609	617	625	634	642	650	659	667	675	
21	684	692	700	709	717	725	734	742	750	759	
22	767	775	784	792	800	809	817	825	834	842	
23	850	858	867	875	883	892	900	908	917	925	
24	933	941	950	958	966	975	983	991	999	*008	
25	72 016	024	032	041	049	057	066	074	082	090	
26	099	107	115	123	132	140	148	156	165	173	
27	181	189	198	206	214	222	230	239	247	255	
28	263	272	280	288	296	304	313	321	329	337	
29	346	354	362	370	378	387	395	403	411	419	
530	428	436	444	452	460	469	477	485	493	501	
31	509	518	526	534	542	550	558	567	575	583	
32	591	599	607	616	624	632	640	648	656	665	
33	673	681	689	697	705	713	722	730	738	746	
34	754	762	770	779	787	795	803	811	819	827	
35	835	843	852	860	868	876	884	892	900	908	
36	916	925	933	941	949	957	965	973	981	989	
37	997	*006	*014	*022	*030	*038	*046	*054	*062	*070	
38	73 078	086	094	102	111	119	127	135	143	151	
39	159	167	175	183	191	199	207	215	223	231	
540	239	247	255	263	272	280	288	296	304	312	
41	320	328	336	344	352	360	368	376	384	392	
42	400	408	416	424	432	440	448	456	464	472	
43	480	488	496	504	512	520	528	536	544	552	
44	560	568	576	584	592	600	608	616	624	632	
45	640	648	656	664	672	679	687	695	703	711	
46	719	727	735	743	751	759	767	775	783	791	
47	799	807	815	823	830	838	846	854	862	870	
48	878	886	894	902	910	918	926	933	941	949	
49	957	965	973	981	989	997	*005	*013	*020	*028	
550	74 036	044	052	060	068	076	084	092	099	107	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

	9	8	7
1	0.9	0.8	0.7
2	1.8	1.6	1.4
3	2.7	2.4	2.1
4	3.6	3.2	2.8
5	4.5	4.0	3.5
6	5.4	4.8	4.2
7	6.3	5.6	4.9
8	7.2	6.4	5.6
9	8.1	7.2	6.3

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
550	74 036	044	052	060	068	076	084	092	099	107	
51	115	123	131	139	147	155	162	170	178	186	
52	194	202	210	218	225	233	241	249	257	265	
53	273	280	288	296	304	312	320	327	335	343	
54	351	359	367	374	382	390	398	406	414	421	
55	429	437	445	453	461	468	476	484	492	500	
56	507	515	523	531	539	547	554	562	570	578	
57	586	593	601	609	617	624	632	640	648	656	
58	663	671	679	687	695	702	710	718	726	733	
59	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	858	865	873	881	889	
61	896	904	912	920	927	935	943	950	958	966	
62	974	981	989	997	*005	*012	*020	*028	*035	*043	
63	75 051	059	066	074	082	089	097	105	113	120	
64	128	136	143	151	159	166	174	182	189	197	
65	205	213	220	228	236	243	251	259	266	274	
66	282	289	297	305	312	320	328	335	343	351	
67	358	366	374	381	389	397	404	412	420	427	
68	435	442	450	458	465	473	481	488	496	504	
69	511	519	526	534	542	549	557	565	572	580	
570	587	595	603	610	618	626	633	641	648	656	
71	664	671	679	686	694	702	709	717	724	732	
72	740	747	755	762	770	778	785	793	800	808	
73	815	823	831	838	846	853	861	868	876	884	
74	891	899	906	914	921	929	937	944	952	959	
75	967	974	982	989	997	*005	*012	*020	*027	*035	
76	76 042	050	057	065	072	080	087	095	103	110	
77	118	125	133	140	148	155	163	170	178	185	
78	193	200	208	215	223	230	238	245	253	260	
79	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	
81	418	425	433	440	448	455	462	470	477	485	
82	492	500	507	515	522	530	537	545	552	559	
83	567	574	582	589	597	604	612	619	626	634	
84	641	649	656	664	671	678	686	693	701	708	
85	716	723	730	738	745	753	760	768	775	782	
86	790	797	805	812	819	827	834	842	849	856	
87	864	871	879	886	893	901	908	916	923	930	
88	938	945	953	960	967	975	982	989	997	*004	
89	77 012	019	026	034	041	048	056	063	070	078	
590	085	093	100	107	115	122	129	137	144	151	
91	159	166	173	181	188	195	203	210	217	225	
92	232	240	247	254	262	269	276	283	291	298	
93	305	313	320	327	335	342	349	357	364	371	
94	379	386	393	401	408	415	422	430	437	444	
95	452	459	466	474	481	488	495	503	510	517	
96	525	532	539	546	554	561	568	576	583	590	
97	597	605	612	619	627	634	641	648	656	663	
98	670	677	685	692	699	706	714	721	728	735	
99	743	750	757	764	772	779	786	793	801	808	
600	815	822	830	837	844	851	859	866	873	880	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

	8	7
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
600	77 815	822	830	837	844	851	859	866	873	880	
01	887	895	902	909	916	924	931	938	945	952	
02	960	967	974	981	988	996	*003	*010	*017	*025	
03	78 032	039	046	053	061	068	075	082	089	097	
04	104	111	118	125	132	140	147	154	161	168	
05	176	183	190	197	204	211	219	226	233	240	
06	247	254	262	269	276	283	290	297	305	312	
07	319	326	333	340	347	355	362	369	376	383	
08	390	398	405	412	419	426	433	440	447	455	
09	462	469	476	483	490	497	504	512	519	526	
610	533	540	547	554	561	569	576	583	590	597	
11	604	611	618	625	633	640	647	654	661	668	
12	675	682	689	696	704	711	718	725	732	739	
13	746	753	760	767	774	781	789	796	803	810	
14	817	824	831	838	845	852	859	866	873	880	
15	888	895	902	909	916	923	930	937	944	951	
16	958	965	972	979	986	993	*000	*007	*014	*021	
17	79 029	036	043	050	057	064	071	078	085	092	
18	099	106	113	120	127	134	141	148	155	162	
19	169	176	183	190	197	204	211	218	225	232	
620	239	246	253	260	267	274	281	288	295	302	
21	309	316	323	330	337	344	351	358	365	372	
22	379	386	393	400	407	414	421	428	435	442	
23	449	456	463	470	477	484	491	498	505	511	
24	518	525	532	539	546	553	560	567	574	581	
25	588	595	602	609	616	623	630	637	644	650	
26	657	664	671	678	685	692	699	706	713	720	
27	727	734	741	748	754	761	768	775	782	789	
28	796	803	810	817	824	831	837	844	851	858	
29	865	872	879	886	893	900	906	913	920	927	
630	934	941	948	955	962	969	975	982	989	996	
31	80 003	010	017	024	030	037	044	051	058	065	
32	072	079	085	092	099	106	113	120	127	134	
33	140	147	154	161	168	175	182	188	195	202	
34	209	216	223	229	236	243	250	257	264	271	
35	277	284	291	298	305	312	318	325	332	339	
36	346	353	359	366	373	380	387	393	400	407	
37	414	421	428	434	441	448	455	462	468	475	
38	482	489	496	502	509	516	523	530	536	543	
39	550	557	564	570	577	584	591	598	604	611	
640	618	625	632	638	645	652	659	665	672	679	
41	686	693	699	706	713	720	726	733	740	747	
42	754	760	767	774	781	787	794	801	808	814	
43	821	828	835	841	848	855	862	868	875	882	
44	889	895	902	909	916	922	929	936	943	949	
45	956	963	969	976	983	990	996	*003	*010	*017	
46	81 023	030	037	043	050	057	064	070	077	084	
47	090	097	104	111	117	124	131	137	144	151	
48	158	164	171	178	184	191	198	204	211	218	
49	224	231	238	245	251	258	265	271	278	285	
650	291	298	305	311	318	325	331	338	345	351	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

	8	7	6
1	0.8	0.7	0.6
2	1.6	1.4	1.2
3	2.4	2.1	1.8
4	3.2	2.8	2.4
5	4.0	3.5	3.0
6	4.8	4.2	3.6
7	5.6	4.9	4.2
8	6.4	5.6	4.8
9	7.2	6.3	5.4

650 — Logarithms of Numbers — 700

N.	0	1	2	3	4	5	6	7	8	9
650	81 291	298	305	311	318	325	331	338	345	351
51	358	365	371	378	385	391	398	405	411	418
52	425	431	438	445	451	458	465	471	478	485
53	491	498	505	511	518	525	531	538	544	551
54	558	564	571	578	584	591	598	604	611	617
55	624	631	637	644	651	657	664	671	677	684
56	690	697	704	710	717	723	730	737	743	750
57	757	763	770	776	783	790	796	803	809	816
58	823	829	836	842	849	856	862	869	875	882
59	889	895	902	908	915	921	928	935	941	948
660	954	961	968	974	981	987	994	*000	*007	*014
61	82 020	027	033	040	046	053	060	066	073	079
62	086	092	099	105	112	119	125	132	138	145
63	151	158	164	171	178	184	191	197	204	210
64	217	223	230	236	243	249	256	263	269	276
65	282	289	295	302	308	315	321	328	334	341
66	347	354	360	367	373	380	387	393	400	406
67	413	419	426	432	439	445	452	458	465	471
68	478	484	491	497	504	510	517	523	530	536
69	543	549	556	562	569	575	582	588	595	601
670	607	614	620	627	633	640	646	653	659	666
71	672	679	685	692	698	705	711	718	724	730
72	737	743	750	756	763	769	776	782	789	795
73	802	808	814	821	827	834	840	847	853	860
74	866	872	879	885	892	898	905	911	918	924
75	930	937	943	950	956	963	969	975	982	988
76	995	*001	*008	*014	*020	*027	*033	*040	*046	*052
77	83 059	065	072	078	085	091	097	104	110	117
78	123	129	136	142	149	155	161	168	174	181
79	187	193	200	206	213	219	225	232	238	245
680	251	257	264	270	276	283	289	296	302	308
81	315	321	327	334	340	347	353	359	366	372
82	378	385	391	398	404	410	417	423	429	436
83	442	448	455	461	467	474	480	487	493	499
84	506	512	518	525	531	537	544	550	556	563
85	569	575	582	588	594	601	607	613	620	626
86	632	639	645	651	658	664	670	677	683	689
87	696	702	708	715	721	727	734	740	746	753
88	759	765	771	778	784	790	797	803	809	816
89	822	828	835	841	847	853	860	866	872	879
690	885	891	897	904	910	916	923	929	935	942
91	948	954	960	967	973	979	985	992	998	*004
92	84 011	017	023	029	036	042	048	055	061	067
93	073	080	086	092	098	105	111	117	123	130
94	136	142	148	155	161	167	173	180	186	192
95	198	205	211	217	223	230	236	242	248	255
96	261	267	273	280	286	292	298	305	311	317
97	323	330	336	342	348	354	361	367	373	379
98	386	392	398	404	410	417	423	429	435	442
99	448	454	460	466	473	479	485	491	497	504
700	510	516	522	528	535	541	547	553	559	566
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
700	84 510	516	522	528	535	541	547	553	559	566	
01	572	578	584	590	597	603	609	615	621	628	
02	634	640	646	652	658	665	671	677	683	689	
03	696	702	708	714	720	726	733	739	745	751	log 7 = .84509 80400
04	757	763	770	776	782	788	794	800	807	813	
05	819	825	831	837	844	850	856	862	868	874	
06	880	887	893	899	905	911	917	924	930	936	
07	942	948	954	960	967	973	979	985	991	997	
08	85 003	009	016	022	028	034	040	046	052	058	
09	065	071	077	083	089	095	101	107	114	120	
710	126	132	138	144	150	156	163	169	175	181	
11	187	193	199	205	211	217	224	230	236	242	
12	248	254	260	266	272	278	285	291	297	303	
13	309	315	321	327	333	339	345	352	358	364	
14	370	376	382	388	394	400	406	412	418	425	
15	431	437	443	449	455	461	467	473	479	485	
16	491	497	503	509	516	522	528	534	540	546	
17	552	558	564	570	576	582	588	594	600	606	
18	612	618	625	631	637	643	649	655	661	667	
19	673	679	685	691	697	703	709	715	721	727	
720	733	739	745	751	757	763	769	775	781	788	
21	794	800	806	812	818	824	830	836	842	848	
22	854	860	866	872	878	884	890	896	902	908	
23	914	920	926	932	938	944	950	956	962	968	
24	974	980	986	992	998	*004	*010	*016	*022	*028	
25	86 034	040	046	052	058	064	070	076	082	088	
26	094	100	106	112	118	124	130	136	141	147	
27	153	159	165	171	177	183	189	195	201	207	
28	213	219	225	231	237	243	249	255	261	267	
29	273	279	285	291	297	303	308	314	320	326	
730	332	338	344	350	356	362	368	374	380	386	
31	392	398	404	410	415	421	427	433	439	445	
32	451	457	463	469	475	481	487	493	499	504	
33	510	516	522	528	534	540	546	552	558	564	
34	570	576	581	587	593	599	605	611	617	623	
35	629	635	641	646	652	658	664	670	676	682	
36	688	694	700	705	711	717	723	729	735	741	
37	747	753	759	764	770	776	782	788	794	800	
38	806	812	817	823	829	835	841	847	853	859	
39	864	870	876	882	888	894	900	906	911	917	
740	923	929	935	941	947	953	958	964	970	976	
41	982	988	994	999	*005	*011	*017	*023	*029	*035	
42	87 040	046	052	058	064	070	075	081	087	093	
43	099	105	111	116	122	128	134	140	146	151	
44	157	163	169	175	181	186	192	198	204	210	
45	216	221	227	233	239	245	251	256	262	268	
46	274	280	286	291	297	303	309	315	320	326	
47	332	338	344	349	355	361	367	373	379	384	
48	390	396	402	408	413	419	425	431	437	442	
49	448	454	460	466	471	477	483	489	495	500	
750	506	512	518	523	529	535	541	547	552	558	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

	7	6	5
1	0.7	0.6	0.5
2	1.4	1.2	1.0
3	2.1	1.8	1.5
4	2.8	2.4	2.0
5	3.5	3.0	2.5
6	4.2	3.6	3.0
7	4.9	4.2	3.5
8	5.6	4.8	4.0
9	6.3	5.4	4.5

N.	0	1	2	3	4	5	6	7	8	9
750	87 506	512	518	523	529	535	541	547	552	558
51	564	570	576	581	587	593	599	604	610	616
52	622	628	633	639	645	651	656	662	668	674
53	679	685	691	697	703	708	714	720	726	731
54	737	743	749	754	760	766	772	777	783	789
55	795	800	806	812	818	823	829	835	841	846
56	852	858	864	869	875	881	887	892	898	904
57	910	915	921	927	933	938	944	950	955	961
58	967	973	978	984	990	996	*001	*007	*013	*018
59	88 024	030	036	041	047	053	058	064	070	076
760	081	087	093	098	104	110	116	121	127	133
61	138	144	150	156	161	167	173	178	184	190
62	195	201	207	213	218	224	230	235	241	247
63	252	258	264	270	275	281	287	292	298	304
64	309	315	321	326	332	338	343	349	355	360
65	366	372	377	383	389	395	400	406	412	417
66	423	429	434	440	446	451	457	463	468	474
67	480	485	491	497	502	508	513	519	525	530
68	536	542	547	553	559	564	570	576	581	587
69	593	598	604	610	615	621	627	632	638	643
770	649	655	660	666	672	677	683	689	694	700
71	705	711	717	722	728	734	739	745	750	756
72	762	767	773	779	784	790	795	801	807	812
73	818	824	829	835	840	846	852	857	863	868
74	874	880	885	891	897	902	908	913	919	925
75	930	936	941	947	953	958	964	969	975	981
76	986	992	997	*003	*009	*014	*020	*025	*031	*037
77	89 042	048	053	059	064	070	076	081	087	092
78	098	104	109	115	120	126	131	137	143	148
79	154	159	165	170	176	182	187	193	198	204
780	209	215	221	226	232	237	243	248	254	260
81	265	271	276	282	287	293	298	304	310	315
82	321	326	332	337	343	348	354	360	365	371
83	376	382	387	393	398	404	409	415	421	426
84	432	437	443	448	454	459	465	470	476	481
85	487	492	498	504	509	515	520	526	531	537
86	542	548	553	559	564	570	575	581	586	592
87	597	603	609	614	620	625	631	636	642	647
88	653	658	664	669	675	680	686	691	697	702
89	708	713	719	724	730	735	741	746	752	757
790	763	768	774	779	785	790	796	801	807	812
91	818	823	829	834	840	845	851	856	862	867
92	873	878	883	889	894	900	905	911	916	922
93	927	933	938	944	949	955	960	966	971	977
94	982	988	993	998	*004	*009	*015	*020	*026	*031
95	90 037	042	048	053	059	064	069	075	080	086
96	091	097	102	108	113	119	124	129	135	140
97	146	151	157	162	168	173	179	184	189	195
98	200	206	211	217	222	227	233	238	244	249
99	255	260	266	271	276	282	287	293	298	304
800	309	314	320	325	331	336	342	347	352	358
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
800	90 309	314	320	325	331	336	342	347	352	358
01	363	369	374	380	385	390	396	401	407	412
02	417	423	428	434	439	445	450	455	461	466
03	472	477	482	488	493	499	504	509	515	520
04	526	531	536	542	547	553	558	563	569	574
05	580	585	590	596	601	607	612	617	623	628
06	634	639	644	650	655	660	666	671	677	682
07	687	693	698	703	709	714	720	725	730	736
08	741	747	752	757	763	768	773	779	784	789
09	795	800	806	811	816	822	827	832	838	843
810	849	854	859	865	870	875	881	886	891	897
11	902	907	913	918	924	929	934	940	945	950
12	956	961	966	972	977	982	988	993	998	*004
13	91 009	014	020	025	030	036	041	046	052	057
14	062	068	073	078	084	089	094	100	105	110
15	116	121	126	132	137	142	148	153	158	164
16	169	174	180	185	190	196	201	206	212	217
17	222	228	233	238	243	249	254	259	265	270
18	275	281	286	291	297	302	307	312	318	323
19	328	334	339	344	350	355	360	365	371	376
820	381	387	392	397	403	408	413	418	424	429
21	434	440	445	450	455	461	466	471	477	482
22	487	492	498	503	508	514	519	524	529	535
23	540	545	551	556	561	566	572	577	582	587
24	593	598	603	609	614	619	624	630	635	640
25	645	651	656	661	666	672	677	682	687	693
26	698	703	709	714	719	724	730	735	740	745
27	751	756	761	766	772	777	782	787	793	798
28	803	808	814	819	824	829	834	840	845	850
29	855	861	866	871	876	882	887	892	897	903
830	908	913	918	924	929	934	939	944	950	955
31	960	965	971	976	981	986	991	997	*002	*007
32	92 012	018	023	028	033	038	044	049	054	059
33	065	070	075	080	085	091	096	101	106	111
34	117	122	127	132	137	143	148	153	158	163
35	169	174	179	184	189	195	200	205	210	215
36	221	226	231	236	241	247	252	257	262	267
37	273	278	283	288	293	298	304	309	314	319
38	324	330	335	340	345	350	355	361	366	371
39	376	381	387	392	397	402	407	412	418	423
840	428	433	438	443	449	454	459	464	469	474
41	480	485	490	495	500	505	511	516	521	526
42	531	536	542	547	552	557	562	567	572	578
43	583	588	593	598	603	609	614	619	624	629
44	634	639	645	650	655	660	665	670	675	681
45	686	691	696	701	706	711	716	722	727	732
46	737	742	747	752	758	763	768	773	778	783
47	788	793	799	804	809	814	819	824	829	834
48	840	845	850	855	860	865	870	875	881	886
49	891	896	901	906	911	916	921	927	932	937
850	942	947	952	957	962	967	973	978	983	988
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
850	92 942	947	952	957	962	967	973	978	983	988	
51	993	998	*003	*008	*013	*018	*024	*029	*034	*039	
52	93 044	049	054	059	064	069	075	080	085	090	
53	095	100	105	110	115	120	125	131	136	141	
54	146	151	156	161	166	171	176	181	186	192	
55	197	202	207	212	217	222	227	232	237	242	
56	247	252	258	263	268	273	278	283	288	293	
57	298	303	308	313	318	323	328	334	339	344	
58	349	354	359	364	369	374	379	384	389	394	
59	399	404	409	414	420	425	430	435	440	445	
860	450	455	460	465	470	475	480	485	490	495	
61	500	505	510	515	520	526	531	536	541	546	
62	551	556	561	566	571	576	581	586	591	596	
63	601	606	611	616	621	626	631	636	641	646	
64	651	656	661	666	671	676	682	687	692	697	
65	702	707	712	717	722	727	732	737	742	747	
66	752	757	762	767	772	777	782	787	792	797	
67	802	807	812	817	822	827	832	837	842	847	
68	852	857	862	867	872	877	882	887	892	897	
69	902	907	912	917	922	927	932	937	942	947	
870	952	957	962	967	972	977	982	987	992	997	
71	94 002	007	012	017	022	027	032	037	042	047	
72	052	057	062	067	072	077	082	086	091	096	
73	101	106	111	116	121	126	131	136	141	146	
74	151	156	161	166	171	176	181	186	191	196	
75	201	206	211	216	221	226	231	236	240	245	
76	250	255	260	265	270	275	280	285	290	295	
77	300	305	310	315	320	325	330	335	340	345	
78	349	354	359	364	369	374	379	384	389	394	
79	399	404	409	414	419	424	429	433	438	443	
880	448	453	458	463	468	473	478	483	488	493	
81	498	503	507	512	517	522	527	532	537	542	
82	547	552	557	562	567	571	576	581	586	591	
83	596	601	606	611	616	621	626	630	635	640	
84	645	650	655	660	665	670	675	680	685	689	
85	694	699	704	709	714	719	724	729	734	738	
86	743	748	753	758	763	768	773	778	783	787	
87	792	797	802	807	812	817	822	827	832	836	
88	841	846	851	856	861	866	871	876	880	885	
89	890	895	900	905	910	915	919	924	929	934	
890	939	944	949	954	959	963	968	973	978	983	
91	988	993	998	*002	*007	*012	*017	*022	*027	*032	
92	95 036	041	046	051	056	061	066	071	075	080	
93	085	090	095	100	105	109	114	119	124	129	
94	134	139	143	148	153	158	163	168	173	177	
95	182	187	192	197	202	207	211	216	221	226	
96	231	236	240	245	250	255	260	265	270	274	
97	279	284	289	294	299	303	308	313	318	323	
98	328	332	337	342	347	352	357	361	366	371	
99	376	381	386	390	395	400	405	410	415	419	
900	424	429	434	439	444	448	453	458	463	468	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

	6	5	4
1	0.6	0.5	0.4
2	1.2	1.0	0.8
3	1.8	1.5	1.2
4	2.4	2.0	1.6
5	3.0	2.5	2.0
6	3.6	3.0	2.4
7	4.2	3.5	2.8
8	4.8	4.0	3.2
9	5.4	4.5	3.6

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
900	95 424	429	434	439	444	448	453	458	463	468	
01	472	477	482	487	492	497	501	506	511	516	
02	521	525	530	535	540	545	550	554	559	564	
03	569	574	578	583	588	593	598	602	607	612	
04	617	622	626	631	636	641	646	650	655	660	
05	665	670	674	679	684	689	694	698	703	708	
06	713	718	722	727	732	737	742	746	751	756	
07	761	766	770	775	780	785	789	794	799	804	
08	809	813	818	823	828	832	837	842	847	852	
09	856	861	866	871	875	880	885	890	895	899	
910	904	909	914	918	923	928	933	938	942	947	
11	952	957	961	966	971	976	980	985	990	995	
12	999	*004	*009	*014	*019	*023	*028	*033	*038	*042	
13	96 047	052	057	061	066	071	076	080	085	090	
14	095	099	104	109	114	118	123	128	133	137	
15	142	147	152	156	161	166	171	175	180	185	
16	190	194	199	204	209	213	218	223	227	232	
17	237	242	246	251	256	261	265	270	275	280	
18	284	289	294	298	303	308	313	317	322	327	
19	332	336	341	346	350	355	360	365	369	374	
920	379	384	388	393	398	402	407	412	417	421	
21	426	431	435	440	445	450	454	459	464	468	
22	473	478	483	487	492	497	501	506	511	515	
23	520	525	530	534	539	544	548	553	558	562	
24	567	572	577	581	586	591	595	600	605	609	
25	614	619	624	628	633	638	642	647	652	656	
26	661	666	670	675	680	685	689	694	699	703	
27	708	713	717	722	727	731	736	741	745	750	
28	755	759	764	769	774	778	783	788	792	797	
29	802	806	811	816	820	825	830	834	839	844	
930	848	853	858	862	867	872	876	881	886	890	
31	895	900	904	909	914	918	923	928	932	937	
32	942	946	951	956	960	965	970	974	979	984	
33	988	993	997	*002	*007	*011	*016	*021	*025	*030	
34	97 035	039	044	049	053	058	063	067	072	077	
35	081	086	090	095	100	104	109	114	118	123	
36	128	132	137	142	146	151	155	160	165	169	
37	174	179	183	188	192	197	202	206	211	216	
38	220	225	230	234	239	243	248	253	257	262	
39	267	271	276	280	285	290	294	299	304	308	
940	313	317	322	327	331	336	340	345	350	354	
41	359	364	368	373	377	382	387	391	396	400	
42	405	410	414	419	424	428	433	437	442	447	
43	451	456	460	465	470	474	479	483	488	493	
44	497	502	506	511	516	520	525	529	534	539	
45	543	548	552	557	562	566	571	575	580	585	
46	589	594	598	603	607	612	617	621	626	630	
47	635	640	644	649	653	658	663	667	672	676	
48	681	685	690	695	699	704	708	713	717	722	
49	727	731	736	740	745	749	754	759	763	768	
950	772	777	782	786	791	795	800	804	809	813	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

	5	4
1	0.5	0.4
2	1.0	0.8
3	1.5	1.2
4	2.0	1.6
5	2.5	2.0
6	3.0	2.4
7	3.5	2.8
8	4.0	3.2
9	4.5	3.6

950 — Logarithms of Numbers — 1000

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
950	97 772	777	782	786	791	795	800	804	809	813	
51	818	823	827	832	836	841	845	850	855	859	
52	864	868	873	877	882	886	891	896	900	905	
53	909	914	918	923	928	932	937	941	946	950	
54	955	959	964	968	973	978	982	987	991	996	
55	98 000	005	009	014	019	023	028	032	037	041	
56	046	050	055	059	064	068	073	078	082	087	
57	091	096	100	105	109	114	118	123	127	132	
58	137	141	146	150	155	159	164	168	173	177	
59	182	186	191	195	200	204	209	214	218	223	
960	227	232	236	241	245	250	254	259	263	268	
61	272	277	281	286	290	295	299	304	308	313	
62	318	322	327	331	336	340	345	349	354	358	
63	363	367	372	376	381	385	390	394	399	403	
64	408	412	417	421	426	430	435	439	444	448	
65	453	457	462	466	471	475	480	484	489	493	
66	498	502	507	511	516	520	525	529	534	538	
67	543	547	552	556	561	565	570	574	579	583	
68	588	592	597	601	605	610	614	619	623	628	
69	632	637	641	646	650	655	659	664	668	673	
970	677	682	686	691	695	700	704	709	713	717	
71	722	726	731	735	740	744	749	753	758	762	
72	767	771	776	780	784	789	793	798	802	807	
73	811	816	820	825	829	834	838	843	847	851	
74	856	860	865	869	874	878	883	887	892	896	
75	900	905	909	914	918	923	927	932	936	941	
76	945	949	954	958	963	967	972	976	981	985	
77	989	994	998	*003	*007	*012	*016	*021	*025	*029	
78	99 034	038	043	047	052	056	061	065	069	074	
79	078	083	087	092	096	100	105	109	114	118	
980	123	127	131	136	140	145	149	154	158	162	
81	167	171	176	180	185	189	193	198	202	207	
82	211	216	220	224	229	233	238	242	247	251	
83	255	260	264	269	273	277	282	286	291	295	
84	300	304	308	313	317	322	326	330	335	339	
85	344	348	352	357	361	366	370	374	379	383	
86	388	392	396	401	405	410	414	419	423	427	
87	432	436	441	445	449	454	458	463	467	471	
88	476	480	484	489	493	498	502	506	511	515	
89	520	524	528	533	537	542	546	550	555	559	
990	564	568	572	577	581	585	590	594	599	603	
91	607	612	616	621	625	629	634	638	642	647	
92	651	656	660	664	669	673	677	682	686	691	
93	695	699	704	708	712	717	721	726	730	734	
94	739	743	747	752	756	760	765	769	774	778	
95	782	787	791	795	800	804	808	813	817	822	
96	826	830	835	839	843	848	852	856	861	865	
97	870	874	878	883	887	891	896	900	904	909	
98	913	917	922	926	930	935	939	944	948	952	
99	957	961	965	970	974	978	983	987	991	996	
1000	00 000	004	009	013	017	022	026	030	035	039	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

	5	4
1	0.5	0.4
2	1.0	0.8
3	1.5	1.2
4	2.0	1.6
5	2.5	2.0
6	3.0	2.4
7	3.5	2.8
8	4.0	3.2
9	4.5	3.6

CONDENSED LOGARITHMS TO FIFTEEN DECIMAL PLACES.

[The first digits of n are given in the first row at the top; the last digit of n in the left-hand column. The first column of logarithms are those of 1, 2, 3, · · · , 9. The remaining columns give log $(1+x)$, where $x = (.1)^k$ times 1, 2, · · · , 9.]

Last Digit	First Digit of $n \rightarrow$	1.	1.0	1.00
	Log n	First Digits of log $n \rightarrow$	.0	.00
1	00000 00000 00000	04139 26851 58225	0432 13737 82643	043 40774 79319
2	30102 99956 63981	07918 12460 47625	0860 01717 61918	086 77215 31227
3	47712 12547 19662	11394 33523 06837	1283 72247 05172	130 09330 20418
4	60205 99913 27962	14612 80356 78238	1703 33392 98780	173 37128 09001
5	69897 00043 36019	17609 12590 55681	2118 92990 69938	216 60617 56508
6	77815 12503 83644	20411 99826 55925	2530 58652 64770	259 79807 19909
7	84509 80400 14257	23044 89213 78274	2938 37776 85210	302 94705 53618
8	90308 99869 91944	25527 25051 03306	3342 37554 86950	346 05321 09506
9	95424 25094 39325	27875 36009 52829	3742 64979 40624	389 11662 36911

(*continuation*)

	1.000	1.0000	1.00000	1.000000	1.0000000	1.00000000
	.000	.0000	.00000	.000000	.0000000	.00000000
1	04 34272 76863	0 43429 23104	04342 94265	0434 29446	043 42945	04 34294
2	08 68502 11649	0 86858 02780	08685 88095	0868 58888	086 85890	08 68589
3	13 02688 05227	1 30286 39028	13028 81491	1302 88325	130 28834	13 02883
4	17 36830 58465	1 73714 31850	17371 74453	1737 17758	173 71779	17 37178
5	21 70929 72230	2 17141 81245	21714 66981	2171 47187	217 14724	21 71472
6	26 04985 47390	2 60568 87215	26057 59074	2605 76611	260 57668	26 05767
7	30 38997 84812	3 03995 49761	30400 50733	3040 06031	304 00613	30 40061
8	34 72966 85364	3 47421 68884	34743 41958	3474 35447	347 43557	34 74356
9	39 06892 49910	3 90847 44584	39086 32748	3908 64858	390 86502	39 08650

[For $x < .00000001$, log $(1+x) = x \cdot M$, to within 3 in the 17th place, where $M =$ .43429448 · · · . Hence the last column gives multiples of M except for the decimal place. All the columns that would follow have the same significant digits displaced each time one place.]

CONDENSED ANTILOGARITHMS TO TEN DECIMAL PLACES.

[The first digits of n are given in the first row at the top; $n = (0.1)^k x$; $x = 1, 2, 3,$ · · · , 9 are given in the left-hand column. The first digits in 10^n are given in the second row at the top.]

x	$n = .1x$	$.01x$	$.001x$	$.0001x$	$(.1)^5x$	$(.1)^6x$	$(.1)^7x$
	10^n	1.	1.0	1.00	1.000	1.0000	1.00000
1	1.25892 54118	02329 29923	0230 52381	023 02850	02 30261	0 23026	02303
2	1.58489 31925	04712 85481	0461 57903	046 06231	04 60528	0 46052	04605
3	1.99526 23150	07151 93052	0693 16689	069 10142	06 90799	0 69078	06908
4	2.51188 64315	09647 81961	0925 28861	092 14583	09 21076	0 92104	09210
5	3.16227 76602	12201 84543	1157 94543	115 19555	11 51359	1 15130	11513
6	3.98107 17055	14815 36215	1391 13857	138 25058	13 81646	1 38156	13816
7	5.01187 23363	17489 75549	1624 86929	161 31092	16 11939	1 61182	16118
8	6.30957 34448	20226 44346	1859 13881	184 37657	18 42238	1 84209	18421
9	7.94328 23472	23026 87708	2093 94837	207 44753	20 72541	2 07235	20723

[For $n < .000001$, $10^n = 1 + n \cdot (1/M)$ to within 3 in the 12th decimal place, where $(1/M) = 2.302585$ · · · . Hence the last column gives multiples of $(1/M)$ except for the decimal place. All the columns that would follow contain the same significant digits displaced one place for each new column.]

TABLE II

ACTUAL VALUES

OF THE

TRIGONOMETRIC FUNCTIONS

FROM

0° TO 90° AT INTERVALS OF ONE MINUTE

TO

FIVE DECIMAL PLACES

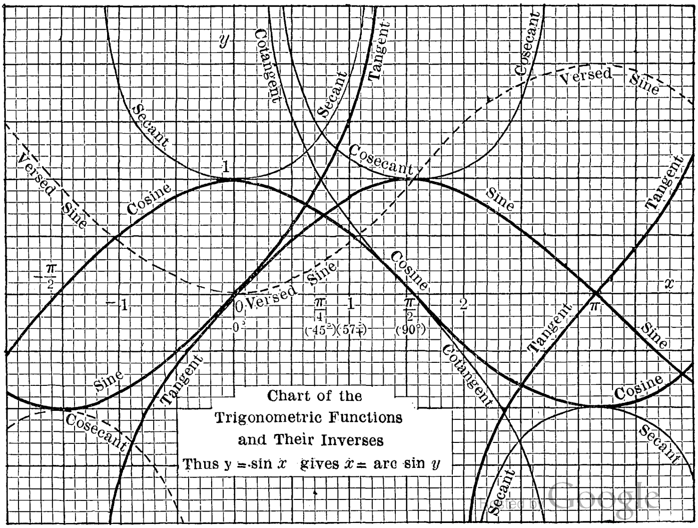

Chart of the
Trigonometric Functions
and Their Inverses
Thus y = sin x gives x = arc sin y

′	Sin	Tan	Ctn	Cos	
0	.00000	.00000	——	1.0000	**60**
1	029	029	3437.7	000	59
2	058	058	1718.9	000	58
3	087	087	1145.9	000	57
4	116	116	859.44	000	56
5	.00145	.00145	687.55	1.0000	**55**
6	175	175	572.96	000	54
7	204	204	491.11	000	53
8	233	233	429.72	000	52
9	262	262	381.97	000	51
10	.00291	.00291	343.77	1.0000	**50**
11	320	320	312.52	.99999	49
12	349	349	286.48	999	48
13	378	378	264.44	999	47
14	407	407	245.55	999	46
15	.00436	.00436	229.18	.99999	**45**
16	465	465	214.86	999	44
17	495	495	202.22	999	43
18	524	524	190.98	999	42
19	553	553	180.93	998	41
20	.00582	.00582	171.89	.99998	**40**
21	611	611	163.70	998	39
22	640	640	156.26	998	38
23	669	669	149.47	998	37
24	698	698	143.24	998	36
25	.00727	.00727	137.51	.99997	**35**
26	756	756	132.22	997	34
27	785	785	127.32	997	33
28	814	815	122.77	997	32
29	844	844	118.54	996	31
30	.00873	.00873	114.59	.99996	**30**
31	902	902	110.89	996	29
32	931	931	107.43	996	28
33	960	960	104.17	995	27
34	.00989	.00989	101.11	995	26
35	.01018	.01018	98.218	.99995	**25**
36	047	047	95.489	995	24
37	076	076	92.908	994	23
38	105	105	90.463	994	22
39	134	135	88.144	994	21
40	.01164	.01164	85.940	.99993	**20**
41	193	193	83.844	993	19
42	222	222	81.847	993	18
43	251	251	79.943	992	17
44	280	280	78.126	992	16
45	.01309	.01309	76.390	.99991	**15**
46	338	338	74.729	991	14
47	367	367	73.139	991	13
48	396	396	71.615	990	12
49	425	425	70.153	990	11
50	.01454	.01455	68.750	.99989	**10**
51	483	484	67.402	989	9
52	513	513	66.105	989	8
53	542	542	64.858	988	7
54	571	571	63.657	988	6
55	.01600	.01600	62.499	.99987	**5**
56	629	629	61.383	987	4
57	658	658	60.306	986	3
58	687	687	59.266	986	2
59	716	716	58.261	985	1
60	.01745	.01746	57.290	.99985	**0**
	Cos	Ctn	Tan	Sin	′

89°

′	Sin	Tan	Ctn	Cos	
0	.01745	.01746	57.290	.99985	**60**
1	774	775	56.351	984	59
2	803	804	55.442	984	58
3	832	833	54.561	983	57
4	862	862	53.709	983	56
5	.01891	.01891	52.882	.99982	**55**
6	920	920	52.081	982	54
7	949	949	51.303	981	53
8	.01978	.01978	50.549	980	52
9	.02007	.02007	49.816	980	51
10	.02036	.02036	49.104	.99979	**50**
11	065	066	48.412	979	49
12	094	095	47.740	978	48
13	123	124	47.085	977	47
14	152	153	46.449	977	46
15	.02181	.02182	45.829	.99976	**45**
16	211	211	45.226	976	44
17	240	240	44.639	975	43
18	269	269	44.066	974	42
19	298	298	43.508	974	41
20	.02327	.02328	42.964	.99973	**40**
21	356	357	42.433	972	39
22	385	386	41.916	972	38
23	414	415	41.411	971	37
24	443	444	40.917	970	36
25	.02472	.02473	40.436	.99969	**35**
26	501	502	39.965	969	34
27	530	531	39.506	968	33
28	560	560	39.057	967	32
29	589	589	38.618	966	31
30	.02618	.02619	38.188	.99966	**30**
31	647	648	37.769	965	29
32	676	677	37.358	964	28
33	705	706	36.956	963	27
34	734	735	36.563	963	26
35	.02763	.02764	36.178	.99962	**25**
36	792	793	35.801	961	24
37	821	822	35.431	960	23
38	850	851	35.070	959	22
39	879	881	34.715	959	21
40	.02908	.02910	34.368	.99958	**20**
41	938	939	34.027	957	19
42	967	968	33.694	956	18
43	.02996	.02997	33.366	955	17
44	.03025	.03026	33.045	954	16
45	.03054	.03055	32.730	.99953	**15**
46	083	084	32.421	952	14
47	112	114	32.118	952	13
48	141	143	31.821	951	12
49	170	172	31.528	950	11
50	.03199	.03201	31.242	.99949	**10**
51	228	230	30.960	948	9
52	257	259	30.683	947	8
53	286	288	30.412	946	7
54	316	317	30.145	945	6
55	.03345	.03346	29.882	.99944	**5**
56	374	376	29.624	943	4
57	403	405	29.371	942	3
58	432	434	29.122	941	2
59	461	463	28.877	940	1
60	.03490	.03492	28.636	.99939	**0**
	Cos	Ctn	Tan	Sin	′

88°

′	Sin	Tan	Ctn	Cos	
0	.03490	.03492	28.636	.99939	**60**
1	519	521	.399	938	59
2	548	550	28.166	937	58
3	577	579	27.937	936	57
4	606	609	.712	935	56
5	.03635	.03638	27.490	.99934	**55**
6	664	667	.271	933	54
7	693	696	27.057	932	53
8	723	725	26.845	931	52
9	752	754	.637	930	51
10	.03781	.03783	26.432	.99929	**50**
11	810	812	.230	927	49
12	839	842	26.031	926	48
13	868	871	25.835	925	47
14	897	900	.642	924	46
15	.03926	.03929	25.452	.99923	**45**
16	955	958	.264	922	44
17	.03984	.03987	25.080	921	43
18	.04013	.04016	24.898	919	42
19	042	046	.719	918	41
20	.04071	.04075	24.542	.99917	**40**
21	100	104	.368	916	39
22	129	133	.196	915	38
23	159	162	24.026	913	37
24	188	191	23.859	912	36
25	.04217	.04220	23.695	.99911	**35**
26	246	250	.532	910	34
27	275	279	.372	909	33
28	304	308	.214	907	32
29	333	337	23.058	906	31
30	.04362	.04366	22.904	.99905	**30**
31	391	395	.752	904	29
32	420	424	.602	902	28
33	449	454	.454	901	27
34	478	483	.308	900	26
35	.04507	.04512	22.164	.99898	**25**
36	536	541	22.022	897	24
37	565	570	21.881	896	23
38	594	599	.743	894	22
39	623	628	.606	893	21
40	.04653	.04658	21.470	.99892	**20**
41	682	687	.337	890	19
42	711	716	.205	889	18
43	740	745	21.075	888	17
44	769	774	20.946	886	16
45	.04798	.04803	20.819	.99885	**15**
46	827	833	.693	883	14
47	856	862	.569	882	13
48	885	891	.446	881	12
49	914	920	.325	879	11
50	.04943	.04949	20.206	.99878	**10**
51	.04972	.04978	20.087	876	9
52	.05001	.05007	19.970	875	8
53	030	037	.855	873	7
54	059	066	.740	872	6
55	.05088	.05095	19.627	.99870	**5**
56	117	124	.516	869	4
57	146	153	.405	867	3
58	175	182	.296	866	2
59	205	212	.188	864	1
60	.05234	.05241	19.081	.99863	**0**
	Cos	Ctn	Tan	Sin	′

87°

′	Sin	Tan	Ctn	Cos	
0	.05234	.05241	19.081	.99863	**60**
1	263	270	18.976	861	59
2	292	299	.871	860	58
3	321	328	.768	858	57
4	350	357	.666	857	56
5	.05379	.05387	18.564	.99855	**55**
6	408	416	.464	854	54
7	437	445	.366	852	53
8	466	474	.268	851	52
9	495	503	.171	849	51
10	.05524	.05533	18.075	.99847	**50**
11	553	562	17.980	846	49
12	582	591	.886	844	48
13	611	620	.793	842	47
14	640	649	.702	841	46
15	.05669	.05678	17.611	.99839	**45**
16	698	708	.521	838	44
17	727	737	.431	836	43
18	756	766	.343	834	42
19	785	795	.256	833	41
20	.05814	.05824	17.169	.99831	**40**
21	844	854	17.084	829	39
22	873	883	16.999	827	38
23	902	912	.915	826	37
24	931	941	.832	824	36
25	.05960	.05970	16.750	.99822	**35**
26	.05989	.05999	.668	821	34
27	.06018	.06029	.587	819	33
28	047	058	.507	817	32
29	076	087	.428	815	31
30	.06105	.06116	16.350	.99813	**30**
31	134	145	.272	812	29
32	163	175	.195	810	28
33	192	204	.119	808	27
34	221	233	16.043	806	26
35	.06250	.06262	15.969	.99804	**25**
36	279	291	.895	803	24
37	308	321	.821	801	23
38	337	350	.748	799	22
39	366	379	.676	797	21
40	.06395	.06408	15.605	.99795	**20**
41	424	438	.534	793	19
42	453	467	.464	792	18
43	482	496	.394	790	17
44	511	525	.325	788	16
45	.06540	.06554	15.257	.99786	**15**
46	569	584	.189	784	14
47	598	613	.122	782	13
48	627	642	15.056	780	12
49	656	671	14.990	778	11
50	.06685	.06700	14.924	.99776	**10**
51	714	730	.860	774	9
52	743	759	.795	772	8
53	773	788	.732	770	7
54	802	817	.669	768	6
55	.06831	.06847	14.606	.99766	**5**
56	860	876	.544	764	4
57	889	905	.482	762	3
58	918	934	.421	760	2
59	947	963	.361	758	1
60	.06976	.06993	14.301	.99756	**0**
	Cos	Ctn	Tan	Sin	′

86°

4° — Values of Trigonometric Functions — 5°

′	Sin	Tan	Ctn	Cos	
0	.06976	.06993	14.301	.99756	**60**
1	.07005	.07022	.241	754	59
2	034	051	.182	752	58
3	063	080	.124	750	57
4	092	110	.065	748	56
5	.07121	.07139	14.008	.99746	**55**
6	150	168	13.951	744	54
7	179	197	.894	742	53
8	208	227	.838	740	52
9	237	256	.782	738	51
10	.07266	.07285	13.727	.99736	**50**
11	295	314	.672	734	49
12	324	344	.617	731	48
13	353	373	.563	729	47
14	382	402	.510	727	46
15	.07411	.07431	13.457	.99725	**45**
16	440	461	.404	723	44
17	469	490	.352	721	43
18	498	519	.300	719	42
19	527	548	.248	716	41
20	.07556	.07578	13.197	.99714	**40**
21	585	607	.146	712	39
22	614	636	.096	710	38
23	643	665	13.046	708	37
24	672	695	12.996	705	36
25	.07701	.07724	12.947	.99703	**35**
26	730	753	.898	701	34
27	759	782	.850	699	33
28	788	812	.801	696	32
29	817	841	.754	694	31
30	.07846	.07870	12.706	.99692	**30**
31	875	899	.659	689	29
32	904	929	.612	687	28
33	933	958	.566	685	27
34	962	.07987	.520	683	26
35	.07991	.08017	12.474	.99680	**25**
36	.08020	046	.429	678	24
37	049	075	.384	676	23
38	078	104	.339	673	22
39	107	134	.295	671	21
40	.08136	.08163	12.251	.99668	**20**
41	165	192	.207	666	19
42	194	221	.163	664	18
43	223	251	.120	661	17
44	252	280	.077	659	16
45	.08281	.08309	12.035	.99657	**15**
46	310	339	11.992	654	14
47	339	368	.950	652	13
48	368	397	.909	649	12
49	397	427	.867	647	11
50	.08426	.08456	11.826	.99644	**10**
51	455	485	.785	642	9
52	484	514	.745	639	8
53	513	544	.705	637	7
54	542	573	.664	635	6
55	.08571	.08602	11.625	.99632	**5**
56	600	632	.585	630	4
57	629	661	.546	627	3
58	658	690	.507	625	2
59	687	720	.468	622	1
60	.08716	.08749	11.430	.99619	**0**
	Cos	Ctn	Tan	Sin	′

85°

′	Sin	Tan	Ctn	Cos	
0	.08716	.08749	11.430	.99619	**60**
1	745	778	.392	617	59
2	774	807	.354	614	58
3	803	837	.316	612	57
4	831	866	.279	609	56
5	.08860	.08895	11.242	.99607	**55**
6	889	925	.205	604	54
7	918	954	.168	602	53
8	947	.08983	.132	599	52
9	.08976	.09013	.095	596	51
10	.09005	.09042	11.059	.99594	**50**
11	034	071	11.024	591	49
12	063	101	10.988	588	48
13	092	130	.953	586	47
14	121	159	.918	583	46
15	.09150	.09189	10.883	.99580	**45**
16	179	218	.848	578	44
17	208	247	.814	575	43
18	237	277	.780	572	42
19	266	306	.746	570	41
20	.09295	.09335	10.712	.99567	**40**
21	324	365	.678	564	39
22	353	394	.645	562	38
23	382	423	.612	559	37
24	411	453	.579	556	36
25	.09440	.09482	10.546	.99553	**35**
26	469	511	.514	551	34
27	498	541	.481	548	33
28	527	570	.449	545	32
29	556	600	.417	542	31
30	.09585	.09629	10.385	.99540	**30**
31	614	658	.354	537	29
32	642	688	.322	534	28
33	671	717	.291	531	27
34	700	746	.260	528	26
35	.09729	.09776	10.229	.99526	**25**
36	758	805	.199	523	24
37	787	834	.168	520	23
38	816	864	.138	517	22
39	845	893	.108	514	21
40	.09874	.09923	10.078	.99511	**20**
41	903	952	.048	508	19
42	932	.09981	10.019	506	18
43	961	.10011	9.9893	503	17
44	.09990	040	.9601	500	16
45	.10019	.10069	9.9310	.99497	**15**
46	048	099	.9021	494	14
47	077	128	.8734	491	13
48	106	158	.8448	488	12
49	135	187	.8164	485	11
50	.10164	.10216	9.7882	.99482	**10**
51	192	246	.7601	479	9
52	221	275	.7322	476	8
53	250	305	.7044	473	7
54	279	334	.6768	470	6
55	.10308	.10363	9.6493	.99467	**5**
56	337	393	.6220	464	4
57	366	422	.5949	461	3
58	395	452	.5679	458	2
59	424	481	.5411	455	1
60	.10453	.10510	9.5144	.99452	**0**
	Cos	Ctn	Tan	Sin	′

84°

′	Sin	Tan	Ctn	Cos	
0	.10453	.10510	9.5144	.99452	**60**
1	482	540	.4878	449	59
2	511	569	.4614	446	58
3	540	599	.4352	443	57
4	569	628	.4090	440	56
5	.10597	.10657	9.3831	.99437	**55**
6	626	687	.3572	434	54
7	655	716	.3315	431	53
8	684	746	.3060	428	52
9	713	775	.2806	424	51
10	.10742	.10805	9.2553	.99421	**50**
11	771	834	.2302	418	49
12	800	863	.2052	415	48
13	829	893	.1803	412	47
14	858	922	.1555	409	46
15	.10887	.10952	9.1309	.99406	**45**
16	916	.10981	.1065	402	44
17	945	.11011	.0821	399	43
18	.10973	040	.0579	396	42
19	.11002	070	.0338	393	41
20	.11031	.11099	9.0098	.99390	**40**
21	060	128	8.9860	386	39
22	089	158	.9623	383	38
23	118	187	.9387	380	37
24	147	217	.9152	377	36
25	.11176	.11246	8.8919	.99374	**35**
26	205	276	.8686	370	34
27	234	305	.8455	367	33
28	263	335	.8225	364	32
29	291	364	.7996	360	31
30	.11320	.11394	8.7769	.99357	**30**
31	349	423	.7542	354	29
32	378	452	.7317	351	28
33	407	482	.7093	347	27
34	436	511	.6870	344	26
35	.11465	.11541	8.6648	.99341	**25**
36	494	570	.6427	337	24
37	523	600	.6208	334	23
38	552	629	.5989	331	22
39	580	659	.5772	327	21
40	.11609	.11688	8.5555	.99324	**20**
41	638	718	.5340	320	19
42	667	747	.5126	317	18
43	696	777	.4913	314	17
44	725	806	.4701	310	16
45	.11754	.11836	8.4490	.99307	**15**
46	783	865	.4280	303	14
47	812	895	.4071	300	13
48	840	924	.3863	297	12
49	869	954	.3656	293	11
50	.11898	.11983	8.3450	.99290	**10**
51	927	.12013	.3245	286	9
52	956	042	.3041	283	8
53	.11985	072	.2838	279	7
54	.12014	101	.2636	276	6
55	.12043	.12131	8.2434	.99272	**5**
56	071	160	.2234	269	4
57	100	190	.2035	265	3
58	129	219	.1837	262	2
59	158	249	.1640	258	1
60	.12187	.12278	8.1443	.99255	**0**
	Cos	Ctn	Tan	Sin	′

83°

′	Sin	Tan	Ctn	Cos	
0	.12187	.12278	8.1443	.99255	**60**
1	216	308	.1248	251	59
2	245	338	.1054	248	58
3	274	367	.0860	244	57
4	302	397	.0667	240	56
5	.12331	.12426	8.0476	.99237	**55**
6	360	456	.0285	233	54
7	389	485	8.0095	230	53
8	418	515	7.9906	226	52
9	447	544	.9718	222	51
10	.12476	.12574	7.9530	.99219	**50**
11	504	603	.9344	215	49
12	533	633	.9158	211	48
13	562	662	.8973	208	47
14	591	692	.8789	204	46
15	.12620	.12722	7.8606	.99200	**45**
16	649	751	.8424	197	44
17	678	781	.8243	193	43
18	706	810	.8062	189	42
19	735	840	.7882	186	41
20	.12764	.12869	7.7704	.99182	**40**
21	793	899	.7525	178	39
22	822	929	.7348	175	38
23	851	958	.7171	171	37
24	880	.12988	.6996	167	36
25	.12908	.13017	7.6821	.99163	**35**
26	937	047	.6647	160	34
27	966	076	.6473	156	33
28	.12995	106	.6301	152	32
29	.13024	136	.6129	148	31
30	.13053	.13165	7.5958	.99144	**30**
31	081	195	.5787	141	29
32	110	224	.5618	137	28
33	139	254	.5449	133	27
34	168	284	.5281	129	26
35	.13197	.13313	7.5113	.99125	**25**
36	226	343	.4947	122	24
37	254	372	.4781	118	23
38	283	402	.4615	114	22
39	312	432	.4451	110	21
40	.13341	.13461	7.4287	.99106	**20**
41	370	491	.4124	102	19
42	399	521	.3962	098	18
43	427	550	.3800	094	17
44	456	580	.3639	091	16
45	.13485	.13609	7.3479	.99087	**15**
46	514	639	.3319	083	14
47	543	669	.3160	079	13
48	572	698	.3002	075	12
49	600	728	.2844	071	11
50	.13629	.13758	7.2687	.99067	**10**
51	658	787	.2531	063	9
52	687	817	.2375	059	8
53	716	846	.2220	055	7
54	744	876	.2066	051	6
55	.13773	.13906	7.1912	.99047	**5**
56	802	935	.1759	043	4
57	831	965	.1607	039	3
58	860	.13995	.1455	035	2
59	889	.14024	.1304	031	1
60	.13917	.14054	7.1154	.99027	**0**
	Cos	Ctn	Tan	Sin	′

82°

8° — Values of Trigonometric Functions — 9°

′	Sin	Tan	Ctn	Cos	
0	.13917	.14054	7.1154	.99027	**60**
1	946	084	.1004	023	59
2	.13975	113	.0855	019	58
3	.14004	143	.0706	015	57
4	033	173	.0558	011	56
5	.14061	.14202	7.0410	.99006	**55**
6	090	232	.0264	.99002	54
7	119	262	7.0117	.98998	53
8	148	291	6.9972	994	52
9	177	321	.9827	990	51
10	.14205	.14351	6.9682	.98986	**50**
11	234	381	.9538	982	49
12	263	410	.9395	978	48
13	292	440	.9252	973	47
14	320	470	.9110	969	46
15	.14349	.14499	6.8969	.98965	**45**
16	378	529	.8828	961	44
17	407	559	.8687	957	43
18	436	588	.8548	953	42
19	464	618	.8408	948	41
20	.14493	.14648	6.8269	.98944	**40**
21	522	678	.8131	940	39
22	551	707	.7994	936	38
23	580	737	.7856	931	37
24	608	767	.7720	927	36
25	.14637	.14796	6.7584	.98923	**35**
26	666	826	.7448	919	34
27	695	856	.7313	914	33
28	723	886	.7179	910	32
29	752	915	.7045	906	31
30	.14781	.14945	6.6912	.98902	**30**
31	810	.14975	.6779	897	29
32	838	.15005	.6646	893	28
33	867	034	.6514	889	27
34	896	064	.6383	884	26
35	.14925	.15094	6.6252	.98880	**25**
36	954	124	.6122	876	24
37	.14982	153	.5992	871	23
38	.15011	183	.5863	867	22
39	040	213	.5734	863	21
40	.15069	.15243	6.5606	.98858	**20**
41	097	272	.5478	854	19
42	126	302	.5350	849	18
43	155	332	.5223	845	17
44	184	362	.5097	841	16
45	.15212	.15391	6.4971	.98836	**15**
46	241	421	.4846	832	14
47	270	451	.4721	827	13
48	299	481	.4596	823	12
49	327	511	.4472	818	11
50	.15356	.15540	6.4348	.98814	**10**
51	385	570	.4225	809	9
52	414	600	.4103	805	8
53	442	630	.3980	800	7
54	471	660	.3859	796	6
55	.15500	.15689	6.3737	.98791	**5**
56	529	719	.3617	787	4
57	557	749	.3496	782	3
58	586	779	.3376	778	2
59	615	809	.3257	773	1
60	.15643	.15838	6.3138	.98769	**0**
	Cos	Ctn	Tan	Sin	′

81°

′	Sin	Tan	Ctn	Cos	
0	.15643	.15838	6.3138	.98769	**60**
1	672	868	.3019	764	59
2	701	898	.2901	760	58
3	730	928	.2783	755	57
4	758	958	.2666	751	56
5	.15787	.15988	6.2549	.98746	**55**
6	816	.16017	.2432	741	54
7	845	047	.2316	737	53
8	873	077	.2200	732	52
9	902	107	.2085	728	51
10	.15931	.16137	6.1970	.98723	**50**
11	959	167	.1856	718	49
12	.15988	196	.1742	714	48
13	.16017	226	.1628	709	47
14	046	256	.1515	704	46
15	.16074	.16286	6.1402	.98700	**45**
16	103	316	.1290	695	44
17	132	346	.1178	690	43
18	160	376	.1066	686	42
19	189	405	.0955	681	41
20	.16218	.16435	6.0844	.98676	**40**
21	246	465	.0734	671	39
22	275	495	.0624	667	38
23	304	525	.0514	662	37
24	333	555	.0405	657	36
25	.16361	.16585	6.0296	.98652	**35**
26	390	615	.0188	648	34
27	419	645	6.0080	643	33
28	447	674	5.9972	638	32
29	476	704	.9865	633	31
30	.16505	.16734	5.9758	.98629	**30**
31	533	764	.9651	624	29
32	562	794	.9545	619	28
33	591	824	.9439	614	27
34	620	854	.9333	609	26
35	.16648	.16884	5.9228	.98604	**25**
36	677	914	.9124	600	24
37	706	944	.9019	595	23
38	734	.16974	.8915	590	22
39	763	.17004	.8811	585	21
40	.16792	.17033	5.8708	.98580	**20**
41	820	063	.8605	575	19
42	849	093	.8502	570	18
43	878	123	.8400	565	17
44	906	153	.8298	561	16
45	.16935	.17183	5.8197	.98556	**15**
46	964	213	.8095	551	14
47	.16992	243	.7994	546	13
48	.17021	273	.7894	541	12
49	050	303	.7794	536	11
50	.17078	.17333	5.7694	.98531	**10**
51	107	363	.7594	526	9
52	136	393	.7495	521	8
53	164	423	.7396	516	7
54	193	453	.7297	511	6
55	.17222	.17483	5.7199	.98506	**5**
56	250	513	.7101	501	4
57	279	543	.7004	496	3
58	308	573	.6906	491	2
59	336	603	.6809	486	1
60	.17365	.17633	5.6713	.98481	**0**
	Cos	Ctn	Tan	Sin	′

80°

′	Sin	Tan	Ctn	Cos	
0	.17365	.17633	5.6713	.98481	**60**
1	393	663	.6617	476	59
2	422	693	.6521	471	58
3	451	723	.6425	466	57
4	479	753	.6329	461	56
5	.17508	.17783	5.6234	.98455	**55**
6	537	813	.6140	450	54
7	565	843	.6045	445	53
8	594	873	.5951	440	52
9	623	903	.5857	435	51
10	.17651	.17933	5.5764	.98430	**50**
11	680	963	.5671	425	49
12	708	.17993	.5578	420	48
13	737	.18023	.5485	414	47
14	766	053	.5393	409	46
15	.17794	.18083	5.5301	.98404	**45**
16	823	113	.5209	399	44
17	852	143	.5118	394	43
18	880	173	.5026	389	42
19	909	203	.4936	383	41
20	.17937	.18233	5.4845	.98378	**40**
21	966	263	.4755	373	39
22	.17995	293	.4665	368	38
23	.18023	323	.4575	362	37
24	052	353	.4486	357	36
25	.18081	.18384	5.4397	.98352	**35**
26	109	414	.4308	347	34
27	138	444	.4219	341	33
28	166	474	.4131	336	32
29	195	504	.4043	331	31
30	.18224	.18534	5.3955	.98325	**30**
31	252	564	.3868	320	29
32	281	594	.3781	315	28
33	309	624	.3694	310	27
34	338	654	.3607	304	26
35	.18367	.18684	5.3521	.98299	**25**
36	395	714	.3435	294	24
37	424	745	.3349	288	23
38	452	775	.3263	283	22
39	481	805	.3178	277	21
40	.18509	.18835	5.3093	.98272	**20**
41	538	865	.3008	267	19
42	567	895	.2924	261	18
43	595	925	.2839	256	17
44	624	955	.2755	250	16
45	.18652	.18986	5.2672	.98245	**15**
46	681	.19016	.2588	240	14
47	710	046	.2505	234	13
48	738	076	.2422	229	12
49	767	106	.2339	223	11
50	.18795	.19136	5.2257	.98218	**10**
51	824	166	.2174	212	9
52	852	197	.2092	207	8
53	881	227	.2011	201	7
54	910	257	.1929	196	6
55	.18938	.19287	5.1848	.98190	**5**
56	967	317	.1767	185	4
57	.18995	347	.1686	179	3
58	.19024	378	.1606	174	2
59	052	408	.1526	168	1
60	.19081	.19438	5.1446	.98163	**0**
	Cos	Ctn	Tan	Sin	′

79°

′	Sin	Tan	Ctn	Cos	
0	.19081	.19438	5.1446	.98163	**60**
1	109	468	.1366	157	59
2	138	498	.1286	152	58
3	167	529	.1207	146	57
4	195	559	.1128	140	56
5	.19224	.19589	5.1049	.98135	**55**
6	252	619	.0970	129	54
7	281	649	.0892	124	53
8	309	680	.0814	118	52
9	338	710	.0736	112	51
10	.19366	.19740	5.0658	.98107	**50**
11	395	770	.0581	101	49
12	423	801	.0504	096	48
13	452	831	.0427	090	47
14	481	861	.0350	084	46
15	.19509	.19891	5.0273	.98079	**45**
16	538	921	.0197	073	44
17	566	952	.0121	067	43
18	595	.19982	5.0045	061	42
19	623	.20012	4.9969	056	41
20	.19652	.20042	4.9894	.98050	**40**
21	680	073	.9819	044	39
22	709	103	.9744	039	38
23	737	133	.9669	033	37
24	766	164	.9594	027	36
25	.19794	.20194	4.9520	.98021	**35**
26	823	224	.9446	016	34
27	851	254	.9372	010	33
28	880	285	.9298	.98004	32
29	908	315	.9225	.97998	31
30	.19937	.20345	4.9152	.97992	**30**
31	965	376	.9078	987	29
32	.19994	406	.9006	981	28
33	.20022	436	.8933	975	27
34	051	466	.8860	969	26
35	.20079	.20497	4.8788	.97963	**25**
36	108	527	.8716	958	24
37	136	557	.8644	952	23
38	165	588	.8573	946	22
39	193	618	.8501	940	21
40	.20222	.20648	4.8430	.97934	**20**
41	250	679	.8359	928	19
42	279	709	.8288	922	18
43	307	739	.8218	916	17
44	336	770	.8147	910	16
45	.20364	.20800	4.8077	.97905	**15**
46	393	830	.8007	899	14
47	421	861	.7937	893	13
48	450	891	.7867	887	12
49	478	921	.7798	881	11
50	.20507	.20952	4.7729	.97875	**10**
51	535	.20982	.7659	869	9
52	563	.21013	.7591	863	8
53	592	043	.7522	857	7
54	620	073	.7453	851	6
55	.20649	.21104	4.7385	.97845	**5**
56	677	134	.7317	839	4
57	706	164	.7249	833	3
58	734	195	.7181	827	2
59	763	225	.7114	821	1
60	.20791	.21256	4.7046	.97815	**0**
	Cos	Ctn	Tan	Sin	′

78°

′	Sin	Tan	Ctn	Cos	
0	.20791	.21256	4.7046	.97815	**60**
1	820	286	.6979	809	59
2	848	316	.6912	803	58
3	877	347	.6845	797	57
4	905	377	.6779	791	56
5	.20933	.21408	4.6712	.97784	**55**
6	962	438	.6646	778	54
7	.20990	469	.6580	772	53
8	.21019	499	.6514	766	52
9	047	529	.6448	760	51
10	.21076	.21560	4.6382	.97754	**50**
11	104	590	.6317	748	49
12	132	621	.6252	742	48
13	161	651	.6187	735	47
14	189	682	.6122	729	46
15	.21218	.21712	4.6057	.97723	**45**
16	246	743	.5993	717	44
17	275	773	.5928	711	43
18	303	804	.5864	705	42
19	331	834	.5800	698	41
20	.21360	.21864	4.5736	.97692	**40**
21	388	895	.5673	686	39
22	417	925	.5609	680	38
23	445	956	.5546	673	37
24	474	.21986	.5483	667	36
25	.21502	.22017	4.5420	.97661	**35**
26	530	047	.5357	655	34
27	559	078	.5294	648	33
28	587	108	.5232	642	32
29	616	139	.5169	636	31
30	.21644	.22169	4.5107	.97630	**30**
31	672	200	.5045	623	29
32	701	231	.4983	617	28
33	729	261	.4922	611	27
34	758	292	.4860	604	26
35	.21786	.22322	4.4799	.97598	**25**
36	814	353	.4737	592	24
37	843	383	.4676	585	23
38	871	414	.4615	579	22
39	899	444	.4555	573	21
40	.21928	.22475	4.4494	.97566	**20**
41	956	505	.4434	560	19
42	.21985	536	.4373	553	18
43	.22013	567	.4313	547	17
44	041	597	.4253	541	16
45	.22070	.22628	4.4194	.97534	**15**
46	098	658	.4134	528	14
47	126	689	.4075	521	13
48	155	719	.4015	515	12
49	183	750	.3956	508	11
50	.22212	.22781	4.3897	.97502	**10**
51	240	811	.3838	496	9
52	268	842	.3779	489	8
53	297	872	.3721	483	7
54	325	903	.3662	476	6
55	.22353	.22934	4.3604	.97470	**5**
56	382	964	.3546	463	4
57	410	.22995	.3488	457	3
58	438	.23026	.3430	450	2
59	467	056	.3372	444	1
60	.22495	.23087	4.3315	.97437	**0**
	Cos	Ctn	Tan	Sin	′

77°

′	Sin	Tan	Ctn	Cos	
0	.22495	.23087	4.3315	.97437	**60**
1	523	117	.3257	430	59
2	552	148	.3200	424	58
3	580	179	.3143	417	57
4	608	209	.3086	411	56
5	.22637	.23240	4.3029	.97404	**55**
6	665	271	.2972	398	54
7	693	301	.2916	391	53
8	722	332	.2859	384	52
9	750	363	.2803	378	51
10	.22778	.23393	4.2747	.97371	**50**
11	807	424	.2691	365	49
12	835	455	.2635	358	48
13	863	485	.2580	351	47
14	892	516	.2524	345	46
15	.22920	.23547	4.2468	.97338	**45**
16	948	578	2413	331	44
17	.22977	608	.2358	325	43
18	.23005	639	.2303	318	42
19	033	670	.2248	311	41
20	.23062	.23700	4.2193	.97304	**40**
21	090	731	.2139	298	39
22	118	762	.2084	291	38
23	146	793	.2030	284	37
24	175	823	.1976	278	36
25	.23203	.23854	4.1922	.97271	**35**
26	231	885	.1868	264	34
27	260	916	.1814	257	33
28	288	946	.1760	251	32
29	316	.23977	.1706	244	31
30	.23345	.24008	4.1653	.97237	**30**
31	373	039	.1600	230	29
32	401	069	.1547	223	28
33	429	100	.1493	217	27
34	458	131	.1441	210	26
35	.23486	.24162	4.1388	.97203	**25**
36	514	193	.1335	196	24
37	542	223	.1282	189	23
38	571	254	.1230	182	22
39	599	285	.1178	176	21
40	.23627	.24316	4.1126	.97169	**20**
41	656	347	.1074	162	19
42	684	377	.1022	155	18
43	712	408	.0970	148	17
44	740	439	.0918	141	16
45	.23769	.24470	4.0867	.97134	**15**
46	797	501	.0815	127	14
47	825	532	.0764	120	13
48	853	562	.0713	113	12
49	882	593	.0662	106	11
50	.23910	.24624	4.0611	.97100	**10**
51	938	655	.0560	093	9
52	966	686	.0509	086	8
53	.23995	717	.0459	079	7
54	.24023	747	.0408	072	6
55	.24051	.24778	4.0358	.97065	**5**
56	079	809	.0308	058	4
57	108	840	.0257	051	3
58	136	871	.0207	044	2
59	164	902	.0158	037	1
60	.24192	.24933	4.0108	.97030	**0**
	Cos	Ctn	Tan	Sin	

76°

′	Sin	Tan	Ctn	Cos	
0	.24192	.24933	4.0108	.97030	**60**
1	220	964	.0058	023	59
2	249	.24995	4.0009	015	58
3	277	.25026	3.9959	008	57
4	305	056	.9910	.97001	56
5	.24333	.25087	3.9861	.96994	**55**
6	362	118	.9812	987	54
7	390	149	.9763	980	53
8	418	180	.9714	973	52
9	446	211	.9665	966	51
10	.24474	.25242	3.9617	.96959	**50**
11	503	273	.9568	952	49
12	531	304	.9520	945	48
13	559	335	.9471	937	47
14	587	366	.9423	930	46
15	.24615	.25397	3.9375	.96923	**45**
16	644	428	.9327	916	44
17	672	459	.9279	909	43
18	700	490	.9232	902	42
19	728	521	.9184	894	41
20	.24756	.25552	3.9136	.96887	**40**
21	784	583	.9089	880	39
22	813	614	.9042	873	38
23	841	645	.8995	866	37
24	869	676	.8947	858	36
25	.24897	.25707	3.8900	.96851	**35**
26	925	738	.8854	844	34
27	954	769	.8807	837	33
28	.24982	800	.8760	829	32
29	.25010	831	.8714	822	31
30	.25038	.25862	3.8667	.96815	**30**
31	066	893	.8621	807	29
32	094	924	.8575	800	28
33	122	955	.8528	793	27
34	151	.25986	.8482	786	26
35	.25179	.26017	3.8436	.96778	**25**
36	207	048	.8391	771	24
37	235	079	.8345	764	23
38	263	110	.8299	756	22
39	291	141	.8254	749	21
40	.25320	.26172	3.8208	.96742	**20**
41	348	203	.8163	734	19
42	376	235	.8118	727	18
43	404	266	.8073	719	17
44	432	297	.8028	712	16
45	.25460	.26328	3.7983	.96705	**15**
46	488	359	.7938	697	14
47	516	390	.7893	690	13
48	545	421	.7848	682	12
49	573	452	.7804	675	11
50	.25601	.26483	3.7760	.96667	**10**
51	629	515	.7715	660	9
52	657	546	.7671	653	8
53	685	577	.7627	645	7
54	713	608	.7583	638	6
55	.25741	.26639	3.7539	.96630	**5**
56	769	670	.7495	623	4
57	798	701	.7451	615	3
58	826	733	.7408	608	2
59	854	764	.7364	600	1
60	.25882	.26795	3.7321	.96593	**0**
	Cos	Ctn	Tan	Sin	′

75°

′	Sin	Tan	Ctn	Cos	
0	.25882	.26795	3.7321	.96593	**60**
1	910	826	.7277	585	59
2	938	857	.7234	578	58
3	966	888	.7191	570	57
4	.25994	920	.7148	562	56
5	.26022	.26951	3.7105	.96555	**55**
6	050	.26982	.7062	547	54
7	079	27013	.7019	540	53
8	107	044	.6976	532	52
9	135	076	.6933	524	51
10	.26163	.27107	3.6891	.96517	**50**
11	191	138	.6848	509	49
12	219	169	.6806	502	48
13	247	201	.6764	494	47
14	275	232	.6722	486	46
15	.26303	.27263	3.6680	.96479	**45**
16	331	294	.6638	471	44
17	359	326	.6596	463	43
18	387	357	.6554	456	42
19	415	388	.6512	448	41
20	.26443	.27419	3.6470	.96440	**40**
21	471	451	.6429	433	39
22	500	482	.6387	425	38
23	528	513	.6346	417	37
24	556	545	.6305	410	36
25	.26584	.27576	3.6264	.96402	**35**
26	612	607	.6222	394	34
27	640	638	.6181	386	33
28	668	670	.6140	379	32
29	696	701	.6100	371	31
30	.26724	.27732	3.6059	.96363	**30**
31	752	764	.6018	355	29
32	780	795	.5978	347	28
33	808	826	.5937	340	27
34	836	858	.5897	332	26
35	.26864	.27889	3.5856	.96324	**25**
36	892	921	.5816	316	24
37	920	952	.5776	308	23
38	948	.27983	.5736	301	22
39	.26976	.28015	.5696	293	21
40	.27004	.28046	3.5656	.96285	**20**
41	032	077	.5616	277	19
42	060	109	.5576	269	18
43	088	140	.5536	261	17
44	116	172	.5497	253	16
45	.27144	.28203	3.5457	.96246	**15**
46	172	234	.5418	238	14
47	200	266	.5379	230	13
48	228	297	.5339	222	12
49	256	329	.5300	214	11
50	.27284	.28360	3.5261	.96206	**10**
51	312	391	.5222	198	9
52	340	423	.5183	190	8
53	368	454	.5144	182	7
54	396	486	.5105	174	6
55	.27424	.28517	3.5067	.96166	**5**
56	452	549	.5028	158	4
57	480	580	.4989	150	3
58	508	612	.4951	142	2
59	536	643	.4912	134	1
60	.27564	.28675	3.4874	.96126	**0**
	Cos	Ctn	Tan	Sin	′

74°

′	Sin	Tan	Ctn	Cos	
0	.27564	.28675	3.4874	.96126	**60**
1	592	706	.4836	118	59
2	620	738	.4798	110	58
3	648	769	.4760	102	57
4	676	801	.4722	094	56
5	.27704	.28832	3.4684	.96086	**55**
6	731	864	.4646	078	54
7	759	895	.4608	070	53
8	787	927	.4570	062	52
9	815	958	.4533	054	51
10	.27843	.28990	3.4495	.96046	**50**
11	871	.29021	.4458	037	49
12	899	053	.4420	029	48
13	927	084	.4383	021	47
14	955	116	.4346	013	46
15	.27983	.29147	3.4308	.96005	**45**
16	.28011	179	.4271	.95997	44
17	039	210	.4234	989	43
18	067	242	.4197	981	42
19	095	274	.4160	972	41
20	.28123	.29305	3.4124	.95964	**40**
21	150	337	.4087	956	39
22	178	368	.4050	948	38
23	206	400	.4014	940	37
24	234	432	.3977	931	36
25	.28262	.29463	3.3941	.95923	**35**
26	290	495	.3904	915	34
27	318	526	.3868	907	33
28	346	558	.3832	898	32
29	374	590	.3796	890	31
30	.28402	.29621	3.3759	.95882	**30**
31	429	653	.3723	874	29
32	457	685	.3687	865	28
33	485	716	.3652	857	27
34	513	748	.3616	849	26
35	.28541	.29780	3.3580	.95841	**25**
36	569	811	.3544	832	24
37	597	843	.3509	824	23
38	625	875	.3473	816	22
39	652	906	.3438	807	21
40	.28680	.29938	3.3402	.95799	**20**
41	708	.29970	.3367	791	19
42	736	.30001	.3332	782	18
43	764	033	.3297	774	17
44	792	065	.3261	766	16
45	.28820	.30097	3.3226	.95757	**15**
46	847	128	.3191	749	14
47	875	160	.3156	740	13
48	903	192	.3122	732	12
49	931	224	.3087	724	11
50	.28959	.30255	3.3052	.95715	**10**
51	.28987	287	.3017	707	9
52	.29015	319	.2983	698	8
53	042	351	.2948	690	7
54	070	382	.2914	681	6
55	.29098	.30414	3.2879	.95673	**5**
56	126	446	.2845	664	4
57	154	478	.2811	656	3
58	182	509	.2777	647	2
59	209	541	.2743	639	1
60	.29237	.30573	3.2709	.95630	**0**
	Cos	Ctn	Tan	Sin	′

73°

′	Sin	Tan	Ctn	Cos	
0	.29237	.30573	3.2709	.95630	**60**
1	265	605	.2675	622	59
2	293	637	.2641	613	58
3	321	669	.2607	605	57
4	348	700	.2573	596	56
5	.29376	.30732	3.2539	.95588	**55**
6	404	764	.2506	579	54
7	432	796	.2472	571	53
8	460	828	.2438	562	52
9	487	860	.2405	554	51
10	.29515	.30891	3.2371	.95545	**50**
11	543	923	.2338	536	49
12	571	955	.2305	528	48
13	599	.30987	.2272	519	47
14	626	.31019	.2238	511	46
15	.29654	.31051	3.2205	.95502	**45**
16	682	083	.2172	493	44
17	710	115	.2139	485	43
18	737	147	.2106	476	42
19	765	178	.2073	467	41
20	.29793	.31210	3.2041	.95459	**40**
21	821	242	.2008	450	39
22	849	274	.1975	441	38
23	876	306	.1943	433	37
24	904	338	.1910	424	36
25	.29932	.31370	3.1878	.95415	**35**
26	960	402	.1845	407	34
27	.29987	434	.1813	398	33
28	.30015	466	.1780	389	32
29	043	498	.1748	380	31
30	.30071	.31530	3.1716	.95372	**30**
31	098	562	.1684	363	29
32	126	594	.1652	354	28
33	154	626	.1620	345	27
34	182	658	.1588	337	26
35	.30209	.31690	3.1556	.95328	**25**
36	237	722	.1524	319	24
37	265	754	.1492	310	23
38	292	786	.1460	301	22
39	320	818	.1429	293	21
40	.30348	.31850	3.1397	.95284	**20**
41	376	882	.1366	275	19
42	403	914	.1334	266	18
43	431	946	.1303	257	17
44	459	.31978	.1271	248	16
45	.30486	.32010	3.1240	.95240	**15**
46	514	042	.1209	231	14
47	542	074	.1178	222	13
48	570	106	.1146	213	12
49	597	139	.1115	204	11
50	.30625	.32171	3.1084	.95195	**10**
51	653	203	.1053	186	9
52	680	235	.1022	177	8
53	708	267	.0991	168	7
54	736	299	.0961	159	6
55	.30763	.32331	3.0930	.95150	**5**
56	791	363	.0899	142	4
57	819	396	.0868	133	3
58	846	428	.0838	124	2
59	874	460	.0807	115	1
60	.30902	.32492	3.0777	.95106	**0**
	Cos	Ctn	Tan	Sin	′

72°

′	Sin	Tan	Ctn	Cos	
0	.30902	.32492	3.0777	.95106	**60**
1	929	524	.0746	097	59
2	957	556	.0716	088	58
3	.30985	588	.0686	079	57
4	.31012	621	.0655	070	56
5	.31040	.32653	3.0625	.95061	**55**
6	068	685	.0595	052	54
7	095	717	.0565	043	53
8	123	749	.0535	033	52
9	151	782	.0505	024	51
10	.31178	.32814	3.0475	.95015	**50**
11	206	846	.0445	.95006	49
12	233	878	.0415	.94997	48
13	261	911	.0385	988	47
14	289	943	.0356	979	46
15	.31316	.32975	3.0326	.94970	**45**
16	344	.33007	.0296	961	44
17	372	040	.0267	952	43
18	399	072	.0237	943	42
19	427	104	.0208	933	41
20	.31454	.33136	3.0178	.94924	**40**
21	482	169	.0149	915	39
22	510	201	.0120	906	38
23	537	233	.0090	897	37
24	565	266	.0061	888	36
25	.31593	.33298	3.0032	.94878	**35**
26	620	330	3.0003	869	34
27	648	363	2.9974	860	33
28	675	395	.9945	851	32
29	703	427	.9916	842	31
30	.31730	.33460	2.9887	.94832	**30**
31	758	492	.9858	823	29
32	786	524	.9829	814	28
33	813	557	.9800	805	27
34	841	589	.9772	795	26
35	.31868	.33621	2.9743	.94786	**25**
36	896	654	.9714	777	24
37	923	686	.9686	768	23
38	951	718	.9657	758	22
39	.31979	751	.9629	749	21
40	.32006	.33783	2.9600	.94740	**20**
41	034	816	.9572	730	19
42	061	848	.9544	721	18
43	089	881	.9515	712	17
44	116	913	.9487	702	16
45	.32144	.33945	2.9459	.94693	**15**
46	171	.33978	.9431	684	14
47	199	.34010	.9403	674	13
48	227	043	.9375	665	12
49	254	075	.9347	656	11
50	.32282	.34108	2.9319	.94646	**10**
51	309	140	.9291	637	9
52	337	173	.9263	627	8
53	364	205	.9235	618	7
54	392	238	.9208	609	6
55	.32419	.34270	2.9180	.94599	**5**
56	447	303	.9152	590	4
57	474	335	.9125	580	3
58	502	368	.9097	571	2
59	529	400	.9070	561	1
60	.32557	.34433	2.9042	.94552	**0**
	Cos	Ctn	Tan	Sin	′

71°

′	Sin	Tan	Ctn	Cos	
0	.32557	.34433	2.9042	.94552	**60**
1	584	465	.9015	542	59
2	612	498	.8987	533	58
3	639	530	.8960	523	57
4	667	563	.8933	514	56
5	.32694	.34596	2.8905	.94504	**55**
6	722	628	.8878	495	54
7	749	661	.8851	485	53
8	777	693	.8824	476	52
9	804	726	.8797	466	51
10	.32832	.34758	2.8770	.94457	**50**
11	859	791	.8743	447	49
12	887	824	.8716	438	48
13	914	856	.8689	428	47
14	942	889	.8662	418	46
15	.32969	.34922	2.8636	.94409	**45**
16	.32997	954	.8609	399	44
17	.33024	.34987	.8582	390	43
18	051	.35020	.8556	380	42
19	079	052	.8529	370	41
20	.33106	.35085	2.8502	.94361	**40**
21	134	118	.8476	351	39
22	161	150	.8449	342	38
23	189	183	.8423	332	37
24	216	216	.8397	322	36
25	.33244	.35248	2.8370	.94313	**35**
26	271	281	.8344	303	34
27	298	314	.8318	293	33
28	326	346	.8291	284	32
29	353	379	.8265	274	31
30	.33381	.35412	2.8239	.94264	**30**
31	408	445	.8213	254	29
32	436	477	.8187	245	28
33	463	510	.8161	235	27
34	490	543	.8135	225	26
35	.33518	.35576	2.8109	.94215	**25**
36	545	608	.8083	206	24
37	573	641	.8057	196	23
38	600	674	.8032	186	22
39	627	707	.8006	176	21
40	.33655	.35740	2.7980	.94167	**20**
41	682	772	.7955	157	19
42	710	805	.7929	147	18
43	737	838	.7903	137	17
44	764	871	.7878	127	16
45	.33792	.35904	2.7852	.94118	**15**
46	819	937	.7827	108	14
47	846	.35969	.7801	098	13
48	874	.36002	.7776	088	12
49	901	035	.7751	078	11
50	.33929	.36068	2.7725	.94068	**10**
51	956	101	.7700	058	9
52	.33983	134	.7675	049	8
53	.34011	167	.7650	039	7
54	038	199	.7625	029	6
55	.34065	.36232	2.7600	.94019	**5**
56	093	265	.7575	.94009	4
57	120	298	.7550	.93999	3
58	147	331	.7525	989	2
59	175	364	.7500	979	1
60	.34202	.36397	2.7475	.93969	**0**
	Cos	Ctn	Tan	Sin	′

70°

′	Sin	Tan	Ctn	Cos	
0	.34202	.36397	2.7475	.93969	**60**
1	229	430	.7450	959	59
2	257	463	.7425	949	58
3	284	496	.7400	939	57
4	311	529	.7376	929	56
5	.34339	.36562	2.7351	.93919	**55**
6	366	595	.7326	909	54
7	393	628	.7302	899	53
8	421	661	.7277	889	52
9	448	694	.7253	879	51
10	.34475	.36727	2.7228	.93869	**50**
11	503	760	.7204	859	49
12	530	793	.7179	849	48
13	557	826	.7155	839	47
14	584	859	.7130	829	46
15	.34612	.36892	2.7106	.93819	**45**
16	639	925	.7082	809	44
17	666	958	.7058	799	43
18	694	.36991	.7034	789	42
19	721	.37024	.7009	779	41
20	.34748	.37057	2.6985	.93769	**40**
21	775	090	.6961	759	39
22	803	123	.6937	748	38
23	830	157	.6913	738	37
24	857	190	.6889	728	36
25	.34884	.37223	2.6865	.93718	**35**
26	912	256	.6841	708	34
27	939	289	.6818	698	33
28	966	322	.6794	688	32
29	.34993	355	.6770	677	31
30	.35021	.37388	2.6746	.93667	**30**
31	048	422	.6723	657	29
32	075	455	.6699	647	28
33	102	488	.6675	637	27
34	130	521	.6652	626	26
35	.35157	.37554	2.6628	.93616	**25**
36	184	588	.6605	606	24
37	211	621	.6581	596	23
38	239	654	.6558	585	22
39	266	687	.6534	575	21
40	.35293	.37720	2.6511	.93565	**20**
41	320	754	.6488	555	19
42	347	787	.6464	544	18
43	375	820	.6441	534	17
44	402	853	.6418	524	16
45	.35429	.37887	2.6395	.93514	**15**
46	456	920	.6371	503	14
47	484	953	.6348	493	13
48	511	.37986	.6325	483	12
49	538	.38020	.6302	472	11
50	.35565	.38053	2.6279	.93462	**10**
51	592	086	.6256	452	9
52	619	120	.6233	441	8
53	647	153	.6210	431	7
54	674	186	.6187	420	6
55	.35701	.38220	2.6165	.93410	**5**
56	728	253	.6142	400	4
57	755	286	.6119	389	3
58	782	320	.6096	379	2
59	810	353	.6074	368	1
60	.35837	.38386	2 6051	.93358	**0**
	Cos	Ctn	Tan	Sin	′

69°

′	Sin	Tan	Ctn	Cos	
0	.35837	.38386	2.6051	.93358	**60**
1	864	420	.6028	348	59
2	891	453	.6006	337	58
3	918	487	.5983	327	57
4	945	520	.5961	316	56
5	.35973	.38553	2.5938	.93306	**55**
6	.36000	587	.5916	295	54
7	027	620	.5893	285	53
8	054	654	.5871	274	52
9	081	687	.5848	264	51
10	.36108	.38721	2.5826	.93253	**50**
11	135	754	.5804	243	49
12	162	787	.5782	232	48
13	190	821	.5759	222	47
14	217	854	.5737	211	46
15	.36244	.38888	2.5715	.93201	**45**
16	271	921	.5693	190	44
17	298	955	.5671	180	43
18	325	.38988	.5649	169	42
19	352	.39022	.5627	159	41
20	.36379	.39055	2.5605	.93148	**40**
21	406	089	.5583	137	39
22	434	122	.5561	127	38
23	461	156	.5539	116	37
24	488	190	.5517	106	36
25	.36515	.39223	2.5495	.93095	**35**
26	542	257	.5473	084	34
27	569	290	.5452	074	33
28	596	324	.5430	063	32
29	623	357	.5408	052	31
30	.36650	.39391	2.5386	.93042	**30**
31	677	425	.5365	031	29
32	704	458	.5343	020	28
33	731	492	.5322	.93010	27
34	758	526	.5300	.92999	26
35	.36785	.39559	2.5279	.92988	**25**
36	812	593	.5257	978	24
37	839	626	.5236	967	23
38	867	660	.5214	956	22
39	894	694	.5193	945	21
40	.36921	.39727	2.5172	.92935	**20**
41	948	761	.5150	924	19
42	.36975	795	.5129	913	18
43	.37002	829	.5108	902	17
44	029	862	.5086	892	16
45	.37056	.39896	2.5065	.92881	**15**
46	083	930	.5044	870	14
47	110	963	.5023	859	13
48	137	.39997	.5002	849	12
49	164	.40031	.4981	838	11
50	.37191	.40065	2.4960	.92827	**10**
51	218	098	.4939	816	9
52	245	132	.4918	805	8
53	272	166	.4897	794	7
54	299	200	.4876	784	6
55	.37326	.40234	2.4855	.92773	**5**
56	353	267	.4834	762	4
57	380	301	.4813	751	3
58	407	335	.4792	740	2
59	434	369	.4772	729	1
60	.37461	.40403	2.4751	.92718	**0**
	Cos	Ctn	Tan	Sin	′

68°

22° — Values of Trigonometric Functions — 23°

′	Sin	Tan	Ctn	Cos	
0	.37461	.40403	2.4751	.92718	**60**
1	488	436	.4730	707	59
2	515	470	.4709	697	58
3	542	504	.4689	686	57
4	569	538	.4668	675	56
5	.37595	.40572	2.4648	.92664	**55**
6	622	606	.4627	653	54
7	649	640	.4606	642	53
8	676	674	.4586	631	52
9	703	707	.4566	620	51
10	.37730	.40741	2.4545	.92609	**50**
11	757	775	.4525	598	49
12	784	809	.4504	587	48
13	811	843	.4484	576	47
14	838	877	.4464	565	46
15	.37865	.40911	2.4443	.92554	**45**
16	892	945	.4423	543	44
17	919	.40979	.4403	532	43
18	946	.41013	.4383	521	42
19	973	047	.4362	510	41
20	.37999	.41081	2.4342	.92499	**40**
21	.38026	115	.4322	488	39
22	053	149	.4302	477	38
23	080	183	.4282	466	37
24	107	217	.4262	455	36
25	.38134	.41251	2.4242	.92444	**35**
26	161	285	.4222	432	34
27	188	319	.4202	421	33
28	215	353	.4182	410	32
29	241	387	.4162	399	31
30	.38268	.41421	2.4142	.92388	**30**
31	295	455	.4122	377	29
32	322	490	.4102	366	28
33	349	524	.4083	355	27
34	376	558	.4063	343	26
35	.38403	.41592	2.4043	.92332	**25**
36	430	626	.4023	321	24
37	456	660	.4004	310	23
38	483	694	.3984	299	22
39	510	728	.3964	287	21
40	.38537	.41763	2.3945	.92276	**20**
41	564	797	.3925	265	19
42	591	831	.3906	254	18
43	617	865	.3886	243	17
44	644	899	.3867	231	16
45	.38671	.41933	2.3847	.92220	**15**
46	698	.41968	.3828	209	14
47	725	.42002	.3808	198	13
48	752	036	.3789	186	12
49	778	070	.3770	175	11
50	.38805	.42105	2.3750	.92164	**10**
51	832	139	.3731	152	9
52	859	173	.3712	141	8
53	886	207	.3693	130	7
54	912	242	.3673	119	6
55	.38939	.42276	2.3654	.92107	**5**
56	966	310	.3635	096	4
57	.38993	345	.3616	085	3
58	.39020	379	.3597	073	2
59	046	413	.3578	062	1
60	.39073	.42447	2.3559	.92050	**0**
	Cos	Ctn	Tan	Sin	′

67°

′	Sin	Tan	Ctn	Cos	
0	.39073	.42447	2.3559	.92050	**60**
1	100	482	.3539	039	59
2	127	516	.3520	028	58
3	153	551	.3501	016	57
4	180	585	.3483	.92005	56
5	.39207	.42619	2.3464	.91994	**55**
6	234	654	.3445	982	54
7	260	688	.3426	971	53
8	287	722	.3407	959	52
9	314	757	.3388	948	51
10	.39341	.42791	2.3369	.91936	**50**
11	367	826	.3351	925	49
12	394	860	.3332	914	48
13	421	894	.3313	902	47
14	448	929	.3294	891	46
15	.39474	.42963	2.3276	.91879	**45**
16	501	.42998	.3257	868	44
17	528	.43032	.3238	856	43
18	555	067	.3220	845	42
19	581	101	.3201	833	41
20	.39608	.43136	2.3183	.91822	**40**
21	635	170	.3164	810	39
22	661	205	.3146	799	38
23	688	239	.3127	787	37
24	715	274	.3109	775	36
25	.39741	.43308	2.3090	.91764	**35**
26	768	343	.3072	752	34
27	795	378	.3053	741	33
28	822	412	.3035	729	32
29	848	447	.3017	718	31
30	.39875	.43481	2.2998	.91706	**30**
31	902	516	.2980	694	29
32	928	550	.2962	683	28
33	955	585	.2944	671	27
34	.39982	620	.2925	660	26
35	.40008	.43654	2.2907	.91648	**25**
36	035	689	.2889	636	24
37	062	724	.2871	625	23
38	088	758	.2853	613	22
39	115	793	.2835	601	21
40	.40141	.43828	2.2817	.91590	**20**
41	168	862	.2799	578	19
42	195	897	.2781	566	18
43	221	932	.2763	555	17
44	248	.43966	.2745	543	16
45	.40275	.44001	2.2727	.91531	**15**
46	301	036	.2709	519	14
47	328	071	.2691	508	13
48	355	105	.2673	496	12
49	381	140	.2655	484	11
50	.40408	.44175	2.2637	.91472	**10**
51	434	210	.2620	461	9
52	461	244	.2602	449	8
53	488	279	.2584	437	7
54	514	314	.2566	425	6
55	.40541	.44349	2.2549	.91414	**5**
56	567	384	.2531	402	4
57	594	418	.2513	390	3
58	621	453	.2496	378	2
59	647	488	.2478	366	1
60	.40674	.44523	2.2460	.91355	**0**
	Cos	Ctn	Tan	Sin	′

66°

′	Sin	Tan	Ctn	Cos	
0	.40674	.44523	2.2460	.91355	**60**
1	700	558	.2443	343	59
2	727	593	.2425	331	58
3	753	627	.2408	319	57
4	780	662	.2390	307	56
5	.40806	.44697	2.2373	.91295	**55**
6	833	732	.2355	283	54
7	860	767	.2338	272	53
8	886	802	.2320	260	52
9	913	837	.2303	248	51
10	.40939	.44872	2.2286	.91236	**50**
11	966	907	.2268	224	49
12	.40992	942	.2251	212	48
13	.41019	.44977	.2234	200	47
14	045	.45012	.2216	188	46
15	.41072	.45047	2.2199	.91176	**45**
16	098	082	.2182	164	44
17	125	117	.2165	152	43
18	151	152	.2148	140	42
19	178	187	.2130	128	41
20	.41204	.45222	2.2113	.91116	**40**
21	231	257	.2096	104	39
22	257	292	.2079	092	38
23	284	327	.2062	080	37
24	310	362	.2045	068	36
25	.41337	.45397	2.2028	.91056	**35**
26	363	432	.2011	044	34
27	390	467	.1994	032	33
28	416	502	.1977	020	32
29	443	538	.1960	.91008	31
30	.41469	.45573	2.1943	.90996	**30**
31	496	608	.1926	984	29
32	522	643	.1909	972	28
33	549	678	.1892	960	27
34	575	713	.1876	948	26
35	.41602	.45748	2.1859	.90936	**25**
36	628	784	.1842	924	24
37	655	819	.1825	911	23
38	681	854	.1808	899	22
39	707	889	.1792	887	21
40	.41734	.45924	2.1775	.90875	**20**
41	760	960	.1758	863	19
42	787	.45995	.1742	851	18
43	813	.46030	.1725	839	17
44	840	065	.1708	826	16
45	.41866	.46101	2.1692	.90814	**15**
46	892	136	.1675	802	14
47	919	171	.1659	790	13
48	945	206	.1642	778	12
49	972	242	.1625	766	11
50	.41998	.46277	2.1609	.90753	**10**
51	.42024	312	.1592	741	9
52	051	348	.1576	729	8
53	077	383	.1560	717	7
54	104	418	.1543	704	6
55	.42130	.46454	2.1527	.90692	**5**
56	156	489	.1510	680	4
57	183	525	.1494	668	3
58	209	560	.1478	655	2
59	235	595	.1461	643	1
60	.42262	.46631	2.1445	.90631	**0**
	Cos	Ctn	Tan	Sin	′

65°

′	Sin	Tan	Ctn	Cos	
0	.42262	.46631	2.1445	.90631	**60**
1	288	666	.1429	618	59
2	315	702	.1413	606	58
3	341	737	.1396	594	57
4	367	772	.1380	582	56
5	.42394	.46808	2.1364	.90569	**55**
6	420	843	.1348	557	54
7	446	879	.1332	545	53
8	473	914	.1315	532	52
9	499	950	.1299	520	51
10	.42525	.46985	2.1283	.90507	**50**
11	552	.47021	.1267	495	49
12	578	056	.1251	483	48
13	604	092	.1235	470	47
14	631	128	.1219	458	46
15	.42657	.47163	2.1203	.90446	**45**
16	683	199	.1187	433	44
17	709	234	.1171	421	43
18	736	270	.1155	408	42
19	762	305	.1139	396	41
20	.42788	.47341	2.1123	.90383	**40**
21	815	377	.1107	371	39
22	841	412	.1092	358	38
23	867	448	.1076	346	37
24	894	483	.1060	334	36
25	.42920	.47519	2.1044	.90321	**35**
26	946	555	.1028	309	34
27	972	590	.1013	296	33
28	.42999	626	.0997	284	32
29	.43025	662	.0981	271	31
30	.43051	.47698	2.0965	.90259	**30**
31	077	733	.0950	246	29
32	104	769	.0934	233	28
33	130	805	.0918	221	27
34	156	840	.0903	208	26
35	.43182	.47876	2.0887	.90196	**25**
36	209	912	.0872	183	24
37	235	948	.0856	171	23
38	261	.47984	.0840	158	22
39	287	.48019	.0825	146	21
40	.43313	.48055	2.0809	.90133	**20**
41	340	091	.0794	120	19
42	366	127	.0778	108	18
43	392	163	.0763	095	17
44	418	198	.0748	082	16
45	.43445	.48234	2.0732	.90070	**15**
46	471	270	.0717	057	14
47	497	306	.0701	045	13
48	523	342	.0686	032	12
49	549	378	.0671	019	11
50	.43575	.48414	2.0655	.90007	**10**
51	602	450	.0640	.89994	9
52	628	486	0625	981	8
53	654	521	.0609	968	7
54	680	557	.0594	956	6
55	.43706	.48593	2.0579	.89943	**5**
56	733	629	.0564	930	4
57	759	665	.0549	918	3
58	785	701	.0533	905	2
59	811	737	.0518	892	1
60	.43837	.48773	2 0503	.89879	**0**
	Cos	Ctn	Tan	Sin	

64°

26° — Values of Trigonometric Functions — 27°

′	Sin	Tan	Ctn	Cos	
0	.43837	.48773	2.0503	.89879	**60**
1	863	809	.0488	867	59
2	889	845	.0473	854	58
3	916	881	.0458	841	57
4	942	917	.0443	828	56
5	.43968	.48953	2.0428	.89816	**55**
6	.43994	.48989	.0413	803	54
7	.44020	.49026	.0398	790	53
8	046	062	.0383	777	52
9	072	098	.0368	764	51
10	.44098	.49134	2.0353	.89752	**50**
11	124	170	.0338	739	49
12	151	206	.0323	726	48
13	177	242	.0308	713	47
14	203	278	.0293	700	46
15	.44229	.49315	2.0278	.89687	**45**
16	255	351	.0263	674	44
17	281	387	.0248	662	43
18	307	423	.0233	649	42
19	333	459	.0219	636	41
20	.44359	.49495	2.0204	.89623	**40**
21	385	532	.0189	610	39
22	411	568	.0174	597	38
23	437	604	.0160	584	37
24	464	640	.0145	571	36
25	.44490	.49677	2.0130	.89558	**35**
26	516	713	.0115	545	34
27	542	749	.0101	532	33
28	568	786	.0086	519	32
29	594	822	.0072	506	31
30	.44620	.49858	2.0057	.89493	**30**
31	646	894	.0042	480	29
32	672	931	.0028	467	28
33	698	.49967	2.0013	454	27
34	724	.50004	1.9999	441	26
35	.44750	.50040	1.9984	.89428	**25**
36	776	076	.9970	415	24
37	802	113	.9955	402	23
38	828	149	.9941	389	22
39	854	185	.9926	376	21
40	.44880	.50222	1.9912	.89363	**20**
41	906	258	.9897	350	19
42	932	295	.9883	337	18
43	958	331	.9868	324	17
44	.44984	368	.9854	311	16
45	.45010	.50404	1.9840	.89298	**15**
46	036	441	.9825	285	14
47	062	477	.9811	272	13
48	088	514	.9797	259	12
49	114	550	.9782	245	11
50	.45140	.50587	1.9768	.89232	**10**
51	166	623	.9754	219	9
52	192	660	.9740	206	8
53	218	696	.9725	193	7
54	243	733	.9711	180	6
55	.45269	.50769	1.9697	.89167	**5**
56	295	806	.9683	153	4
57	321	843	.9669	140	3
58	347	879	.9654	127	2
59	373	916	.9640	114	1
60	.45399	.50953	1.9626	.89101	**0**
	Cos	Ctn	Tan	Sin	′

63°

′	Sin	Tan	Ctn	Cos	
0	.45399	.50953	1.9626	.89101	**60**
1	425	.50989	.9612	087	59
2	451	.51026	.9598	074	58
3	477	063	.9584	061	57
4	503	099	.9570	048	56
5	.45529	.51136	1.9556	.89035	**55**
6	554	173	.9542	021	54
7	580	209	.9528	.89008	53
8	606	246	.9514	.88995	52
9	632	283	.9500	981	51
10	.45658	.51319	1.9486	.88968	**50**
11	684	356	.9472	955	49
12	710	393	.9458	942	48
13	736	430	.9444	928	47
14	762	467	.9430	915	46
15	.45787	.51503	1.9416	.88902	**45**
16	813	540	.9402	888	44
17	839	577	.9388	875	43
18	865	614	.9375	862	42
19	891	651	.9361	848	41
20	.45917	.51688	1.9347	.88835	**40**
21	942	724	.9333	822	39
22	968	761	.9319	808	38
23	.45994	798	.9306	795	37
24	.46020	835	.9292	782	36
25	.46046	.51872	1.9278	.88768	**35**
26	072	909	.9265	755	34
27	097	946	.9251	741	33
28	123	.51983	.9237	728	32
29	149	.52020	.9223	715	31
30	.46175	.52057	1.9210	.88701	**30**
31	201	094	.9196	688	29
32	226	131	.9183	674	28
33	252	168	.9169	661	27
34	278	205	.9155	647	26
35	.46304	.52242	1.9142	.88634	**25**
36	330	279	.9128	620	24
37	355	316	.9115	607	23
38	381	353	.9101	593	22
39	407	390	.9088	580	21
40	.46433	.52427	1.9074	.88566	**20**
41	458	464	.9061	553	19
42	484	501	.9047	539	18
43	510	538	.9034	526	17
44	536	575	.9020	512	16
45	.46561	.52613	1.9007	.88499	**15**
46	587	650	.8993	485	14
47	613	687	.8980	472	13
48	639	724	.8967	458	12
49	664	761	.8953	445	11
50	.46690	.52798	1.8940	.88431	**10**
51	716	836	.8927	417	9
52	742	873	.8913	404	8
53	767	910	.8900	390	7
54	793	947	.8887	377	6
55	.46819	.52985	1.8873	.88363	**5**
56	844	.53022	.8860	349	4
57	870	059	.8847	336	3
58	896	096	.8834	322	2
59	921	134	.8820	308	1
60	.46947	.53171	1.8807	.88295	**0**
	Cos	Ctn	Tan	Sin	′

62°

28° — Values of Trigonometric Functions — 29°

'	Sin	Tan	Ctn	Cos	
0	.46947	.53171	1.8807	.88295	60
1	973	208	.8794	281	59
2	.46999	246	.8781	267	58
3	.47024	283	.8768	254	57
4	050	320	.8755	240	56
5	.47076	.53358	1.8741	.88226	55
6	101	395	.8728	213	54
7	127	432	.8715	199	53
8	153	470	.8702	185	52
9	178	507	.8689	172	51
10	.47204	.53545	1.8676	.88158	50
11	229	582	.8663	144	49
12	255	620	.8650	130	48
13	281	657	.8637	117	47
14	306	694	.8624	103	46
15	.47332	.53732	1.8611	.88089	45
16	358	769	.8598	075	44
17	383	807	.8585	062	43
18	409	844	.8572	048	42
19	434	882	.8559	034	41
20	.47460	.53920	1.8546	.88020	40
21	486	957	.8533	.88006	39
22	511	.53995	.8520	.87993	38
23	537	.54032	.8507	979	37
24	562	070	.8495	965	36
25	.47588	.54107	1.8482	.87951	35
26	614	145	.8469	937	34
27	639	183	.8456	923	33
28	665	220	.8443	909	32
29	690	258	.8430	896	31
30	.47716	.54296	1.8418	.87882	30
31	741	333	.8405	868	29
32	767	371	.8392	854	28
33	793	409	.8379	840	27
34	818	446	.8367	826	26
35	.47844	.54484	1.8354	.87812	25
36	869	522	.8341	798	24
37	895	560	.8329	784	23
38	920	597	.8316	770	22
39	946	635	.8303	756	21
40	.47971	.54673	1.8291	.87743	20
41	.47997	711	.8278	729	19
42	.48022	748	.8265	715	18
43	048	786	.8253	701	17
44	073	824	.8240	687	16
45	.48099	.54862	1.8228	.87673	15
46	124	900	.8215	659	14
47	150	938	.8202	645	13
48	175	.54975	.8190	631	12
49	201	.55013	.8177	617	11
50	.48226	.55051	1.8165	.87603	10
51	252	089	.8152	589	9
52	277	127	.8140	575	8
53	303	165	.8127	561	7
54	328	203	.8115	546	6
55	.48354	.55241	1.8103	.87532	5
56	379	279	.8090	518	4
57	405	317	.8078	504	3
58	430	355	.8065	490	2
59	456	393	.8053	476	1
60	.48481	.55431	1.8040	.87462	0
	Cos	Ctn	Tan	Sin	'

61°

'	Sin	Tan	Ctn	Cos	
0	.48481	.55431	1.8040	.87462	60
1	506	469	.8028	448	59
2	532	507	.8016	434	58
3	557	545	.8003	420	57
4	583	583	.7991	406	56
5	.48608	.55621	1.7979	.87391	55
6	634	659	.7966	377	54
7	659	697	.7954	363	53
8	684	736	.7942	349	52
9	710	774	.7930	335	51
10	.48735	.55812	1.7917	.87321	50
11	761	850	.7905	306	49
12	786	888	.7893	292	48
13	811	926	.7881	278	47
14	837	.55964	.7868	264	46
15	.48862	.56003	1.7856	.87250	45
16	888	041	.7844	235	44
17	913	079	.7832	221	43
18	938	117	.7820	207	42
19	964	156	.7808	193	41
20	.48989	.56194	1.7796	.87178	40
21	.49014	232	.7783	164	39
22	040	270	.7771	150	38
23	065	309	.7759	136	37
24	090	347	.7747	121	36
25	.49116	.56385	1.7735	.87107	35
26	141	424	.7723	093	34
27	166	462	.7711	079	33
28	192	501	.7699	064	32
29	217	539	.7687	050	31
30	.49242	.56577	1.7675	.87036	30
31	268	616	.7663	021	29
32	293	654	.7651	.87007	28
33	318	693	.7639	.86993	27
34	344	731	.7627	978	26
35	.49369	.56769	1.7615	.86964	25
36	394	808	.7603	949	24
37	419	846	.7591	935	23
38	445	885	.7579	921	22
39	470	923	.7567	906	21
40	.49495	.56962	1.7556	.86892	20
41	521	.57000	.7544	878	19
42	546	039	.7532	863	18
43	571	078	.7520	849	17
44	596	116	.7508	834	16
45	.49622	.57155	1.7496	.86820	15
46	647	193	.7485	805	14
47	672	232	.7473	791	13
48	697	271	.7461	777	12
49	723	309	.7449	762	11
50	.49748	.57348	1.7437	.86748	10
51	773	386	.7426	733	9
52	798	425	.7414	719	8
53	824	464	.7402	704	7
54	849	503	.7391	690	6
55	.49874	.57541	1.7379	.86675	5
56	899	580	.7367	661	4
57	924	619	.7355	646	3
58	950	657	.7344	632	2
59	.49975	696	.7332	617	1
60	.50000	.57735	1.7321	.86603	0
	Cos	Ctn	Tan	Sin	'

60°

30°—Values of Trigonometric Functions—31°

′	Sin	Tan	Ctn	Cos	
0	.50000	.57735	1.7321	.86603	**60**
1	025	774	.7309	588	59
2	050	813	.7297	573	58
3	076	851	.7286	559	57
4	101	890	.7274	544	56
5	.50126	.57929	1.7262	.86530	**55**
6	151	.57968	.7251	515	54
7	176	.58007	.7239	501	53
8	201	046	.7228	486	52
9	227	085	.7216	471	51
10	.50252	.58124	1.7205	.86457	**50**
11	277	162	.7193	442	49
12	302	201	.7182	427	48
13	327	240	.7170	413	47
14	352	279	.7159	398	46
15	.50377	.58318	1.7147	.86384	**45**
16	403	357	.7136	369	44
17	428	396	.7124	354	43
18	453	435	.7113	340	42
19	478	474	.7102	325	41
20	.50503	.58513	1.7090	.86310	**40**
21	528	552	.7079	295	39
22	553	591	.7067	281	38
23	578	631	.7056	266	37
24	603	670	.7045	251	36
25	.50628	.58709	1.7033	.86237	**35**
26	654	748	.7022	222	34
27	679	787	.7011	207	33
28	704	826	.6999	192	32
29	729	865	.6988	178	31
30	.50754	.58905	1.6977	.86163	**30**
31	779	944	.6965	148	29
32	804	.58983	.6954	133	28
33	829	.59022	.6943	119	27
34	854	061	.6932	104	26
35	.50879	.59101	1.6920	.86089	**25**
36	904	140	.6909	074	24
37	929	179	.6898	059	23
38	954	218	.6887	045	22
39	.50979	258	.6875	030	21
40	.51004	.59297	1.6864	.86015	**20**
41	029	336	.6853	.86000	19
42	054	376	.6842	.85985	18
43	079	415	.6831	970	17
44	104	454	.6820	956	16
45	.51129	.59494	1.6808	.85941	**15**
46	154	533	.6797	926	14
47	179	573	.6786	911	13
48	204	612	.6775	896	12
49	229	651	.6764	881	11
50	.51254	.59691	1.6753	.85866	**10**
51	279	730	.6742	851	9
52	304	770	.6731	836	8
53	329	809	.6720	821	7
54	354	849	.6709	806	6
55	.51379	.59888	1.6698	.85792	**5**
56	404	928	.6687	777	4
57	429	.59967	.6676	762	3
58	454	.60007	.6665	747	2
59	479	046	.6654	732	1
60	.51504	.60086	1.6643	.85717	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

59°

′	Sin	Tan	Ctn	Cos	
0	.51504	.60086	1.6643	.85717	**60**
1	529	126	.6632	702	59
2	554	165	.6621	687	58
3	579	205	.6610	672	57
4	604	245	.6599	657	56
5	.51628	.60284	1.6588	.85642	**55**
6	653	324	.6577	627	54
7	678	364	.6566	612	53
8	703	403	.6555	597	52
9	728	443	.6545	582	51
10	.51753	.60483	1.6534	.85567	**50**
11	778	522	.6523	551	49
12	803	562	.6512	536	48
13	828	602	.6501	521	47
14	852	642	.6490	506	46
15	.51877	.60681	1.6479	.85491	**45**
16	902	721	.6469	476	44
17	927	761	.6458	461	43
18	952	801	.6447	446	42
19	.51977	841	.6436	431	41
20	.52002	.60881	1.6426	.85416	**40**
21	026	921	.6415	401	39
22	051	.60960	.6404	385	38
23	076	.61000	.6393	370	37
24	101	040	.6383	355	36
25	.52126	.61080	1.6372	.85340	**35**
26	151	120	.6361	325	34
27	175	160	.6351	310	33
28	200	200	.6340	294	32
29	225	240	.6329	279	31
30	.52250	.61280	1.6319	.85264	**30**
31	275	320	.6308	249	29
32	299	360	.6297	234	28
33	324	400	.6287	218	27
34	349	440	.6276	203	26
35	.52374	.61480	1.6265	.85188	**25**
36	399	520	.6255	173	24
37	423	561	.6244	157	23
38	448	601	.6234	142	22
39	473	641	.6223	127	21
40	.52498	.61681	1.6212	.85112	**20**
41	522	721	.6202	096	19
42	547	761	.6191	081	18
43	572	801	.6181	066	17
44	597	842	.6170	051	16
45	.52621	.61882	1.6160	.85035	**15**
46	646	922	.6149	020	14
47	671	.61962	.6139	.85005	13
48	696	.62003	.6128	.84989	12
49	720	043	.6118	974	11
50	.52745	.62083	1.6107	.84959	**10**
51	770	124	.6097	943	9
52	794	164	.6087	928	8
53	819	204	.6076	913	7
54	844	245	.6066	897	6
55	.52869	.62285	1.6055	.84882	**5**
56	893	325	.6045	866	4
57	918	366	.6034	851	3
58	943	406	.6024	836	2
59	967	446	.6014	820	1
60	.52992	.62487	1.6003	.84805	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

58°

′	Sin	Tan	Ctn	Cos	
0	.52992	.62487	1.6003	.84805	**60**
1	.53017	527	.5993	789	59
2	041	568	.5983	774	58
3	066	608	.5972	759	57
4	091	649	.5962	743	56
5	.53115	.62689	1.5952	.84728	**55**
6	140	730	.5941	712	54
7	164	770	.5931	697	53
8	189	811	.5921	681	52
9	214	852	.5911	666	51
10	.53238	.62892	1.5900	.84650	**50**
11	263	933	.5890	635	49
12	288	.62973	.5880	619	48
13	312	.63014	.5869	604	47
14	337	055	.5859	588	46
15	.53361	.63095	1.5849	.84573	**45**
16	386	136	.5839	557	44
17	411	177	.5829	542	43
18	435	217	.5818	526	42
19	460	258	.5808	511	41
20	.53484	.63299	1.5798	.84495	**40**
21	509	340	.5788	480	39
22	534	380	.5778	464	38
23	558	421	.5768	448	37
24	583	462	.5757	433	36
25	.53607	.63503	1.5747	.84417	**35**
26	632	544	.5737	402	34
27	656	584	.5727	386	33
28	681	625	.5717	370	32
29	705	666	.5707	355	31
30	.53730	.63707	1.5697	.84339	**30**
31	754	748	.5687	324	29
32	779	789	.5677	308	28
33	804	830	.5667	292	27
34	828	871	.5657	277	26
35	.53853	.63912	1.5647	.84261	**25**
36	877	953	.5637	245	24
37	902	.63994	.5627	230	23
38	926	.64035	.5617	214	22
39	951	076	.5607	198	21
40	.53975	.64117	1.5597	.84182	**20**
41	.54000	158	.5587	167	19
42	024	199	.5577	151	18
43	049	240	.5567	135	17
44	073	281	.5557	120	16
45	.54097	.64322	1.5547	.84104	**15**
46	122	363	.5537	088	14
47	146	404	.5527	072	13
48	171	446	.5517	057	12
49	195	487	.5507	041	11
50	.54220	.64528	1.5497	.84025	**10**
51	244	569	.5487	.84009	9
52	269	610	.5477	.83994	8
53	293	652	.5468	978	7
54	317	693	.5458	962	6
55	.54342	.64734	1.5448	.83946	**5**
56	366	775	.5438	930	4
57	391	817	.5428	915	3
58	415	858	.5418	899	2
59	440	899	.5408	883	1
60	.54464	.64941	1.5399	.83867	**0**
	Cos	Ctn	Tan	Sin	′

57°

′	Sin	Tan	Ctn	Cos	
0	.54464	.64941	1.5399	.83867	**60**
1	488	.64982	.5389	851	59
2	513	.65024	.5379	835	58
3	537	065	.5369	819	57
4	561	106	.5359	804	56
5	.54586	.65148	1.5350	.83788	**55**
6	610	189	.5340	772	54
7	635	231	.5330	756	53
8	659	272	.5320	740	52
9	683	314	.5311	724	51
10	.54708	.65355	1.5301	.83708	**50**
11	732	397	.5291	692	49
12	756	438	.5282	676	48
13	781	480	.5272	660	47
14	805	521	.5262	645	46
15	.54829	.65563	1.5253	.83629	**45**
16	854	604	.5243	613	44
17	878	646	.5233	597	43
18	902	688	.5224	581	42
19	927	729	.5214	565	41
20	.54951	.65771	1.5204	.83549	**40**
21	975	813	.5195	533	39
22	.54999	854	.5185	517	38
23	.55024	896	.5175	501	37
24	048	938	.5166	485	36
25	.55072	.65980	1.5156	.83469	**35**
26	097	.66021	.5147	453	34
27	121	063	.5137	437	33
28	145	105	.5127	421	32
29	169	147	.5118	405	31
30	.55194	.66189	1.5108	.83389	**30**
31	218	230	.5099	373	29
32	242	272	.5089	356	28
33	266	314	.5080	340	27
34	291	356	.5070	324	26
35	.55315	.66398	1.5061	.83308	**25**
36	339	440	.5051	292	24
37	363	482	.5042	276	23
38	388	524	.5032	260	22
39	412	566	.5023	244	21
40	.55436	.66608	1.5013	.83228	**20**
41	460	650	.5004	212	19
42	484	692	.4994	195	18
43	509	734	.4985	179	17
44	533	776	.4975	163	16
45	.55557	.66818	1.4966	.83147	**15**
46	581	860	.4957	131	14
47	605	902	.4947	115	13
48	630	944	.4938	098	12
49	654	.66986	.4928	082	11
50	.55678	.67028	1.4919	.83066	**10**
51	702	071	.4910	050	9
52	726	113	.4900	034	8
53	750	155	.4891	017	7
54	775	197	.4882	.83001	6
55	.55799	.67239	1.4872	.82985	**5**
56	823	282	.4863	969	4
57	847	324	.4854	953	3
58	871	366	.4844	936	2
59	895	409	.4835	920	1
60	.55919	.67451	1.4826	.82904	**0**
	Cos	Ctn	Tan	Sin	′

56°

′	Sin	Tan	Ctn	Cos	
0	.55919	.67451	1.4826	.82904	60
1	943	493	.4816	887	59
2	968	536	.4807	871	58
3	.55992	578	.4798	855	57
4	.56016	620	.4788	839	56
5	.56040	.67663	1.4779	.82822	55
6	064	705	.4770	806	54
7	088	748	.4761	790	53
8	112	790	.4751	773	52
9	136	832	.4742	757	51
10	.56160	.67875	1.4733	.82741	50
11	184	917	.4724	724	49
12	208	.67960	.4715	708	48
13	232	.68002	.4705	692	47
14	256	045	.4696	675	46
15	.56280	.68088	1.4687	.82659	45
16	305	130	.4678	643	44
17	329	173	.4669	626	43
18	353	215	.4659	610	42
19	377	258	.4650	593	41
20	.56401	.68301	1.4641	.82577	40
21	425	343	.4632	561	39
22	449	386	.4623	544	38
23	473	429	.4614	528	37
24	497	471	.4605	511	36
25	.56521	.68514	1.4596	.82495	35
26	545	557	.4586	478	34
27	569	600	.4577	462	33
28	593	642	.4568	446	32
29	617	685	.4559	429	31
30	.56641	.68728	1.4550	.82413	30
31	665	771	.4541	396	29
32	689	814	.4532	380	28
33	713	857	.4523	363	27
34	736	900	.4514	347	26
35	.56760	.68942	1.4505	.82330	25
36	784	.68985	.4496	314	24
37	808	.69028	.4487	297	23
38	832	071	.4478	281	22
39	856	114	.4469	264	21
40	.56880	.69157	1.4460	.82248	20
41	904	200	.4451	231	19
42	928	243	.4442	214	18
43	952	286	.4433	198	17
44	.56976	329	.4424	181	16
45	.57000	.69372	1.4415	.82165	15
46	024	416	.4406	148	14
47	047	459	.4397	132	13
48	071	502	.4388	115	12
49	095	545	.4379	098	11
50	.57119	.69588	1.4370	.82082	10
51	143	631	.4361	065	9
52	167	675	.4352	048	8
53	191	718	.4344	032	7
54	215	761	.4335	.82015	6
55	.57238	.69804	1.4326	.81999	5
56	262	847	.4317	982	4
57	286	891	.4308	965	3
58	310	934	.4299	949	2
59	334	.69977	.4290	932	1
60	.57358	.70021	1.4281	.81915	0
	Cos	Ctn	Tan	Sin	′

55°

′	Sin	Tan	Ctn	Cos	
0	.57358	.70021	1.4281	.81915	60
1	381	064	.4273	899	59
2	405	107	.4264	882	58
3	429	151	.4255	865	57
4	453	194	.4246	848	56
5	.57477	.70238	1.4237	.81832	55
6	501	281	.4229	815	54
7	524	325	.4220	798	53
8	548	368	.4211	782	52
9	572	412	.4202	765	51
10	.57596	.70455	1.4193	.81748	50
11	619	499	.4185	731	49
12	643	542	.4176	714	48
13	667	586	.4167	698	47
14	691	629	.4158	681	46
15	.57715	.70673	1.4150	.81664	45
16	738	717	.4141	647	44
17	762	760	.4132	631	43
18	786	804	.4124	614	42
19	810	848	.4115	597	41
20	.57833	.70891	1.4106	.81580	40
21	857	935	.4097	563	39
22	881	.70979	.4089	546	38
23	904	.71023	.4080	530	37
24	928	066	.4071	513	36
25	.57952	.71110	1.4063	.81496	35
26	976	154	.4054	479	34
27	.57999	198	.4045	462	33
28	.58023	242	.4037	445	32
29	047	285	.4028	428	31
30	.58070	.71329	1.4019	.81412	30
31	094	373	.4011	395	29
32	118	417	.4002	378	28
33	141	461	.3994	361	27
34	165	505	.3985	344	26
35	.58189	.71549	1.3976	.81327	25
36	212	593	.3968	310	24
37	236	637	.3959	293	23
38	260	681	.3951	276	22
39	283	725	.3942	259	21
40	.58307	.71769	1.3934	.81242	20
41	330	813	.3925	225	19
42	354	857	.3916	208	18
43	378	901	.3908	191	17
44	401	946	.3899	174	16
45	.58425	.71990	1.3891	.81157	15
46	449	.72034	.3882	140	14
47	472	078	.3874	123	13
48	496	122	.3865	106	12
49	519	167	.3857	089	11
50	.58543	.72211	1.3848	.81072	10
51	567	255	.3840	055	9
52	590	299	.3831	038	8
53	614	344	.3823	021	7
54	637	388	.3814	.81004	6
55	.58661	.72432	1.3806	.80987	5
56	684	477	.3798	970	4
57	708	521	.3789	953	3
58	731	565	.3781	936	2
59	755	610	.3772	919	1
60	.58779	.72654	1.3764	.80902	0
	Cos	Ctn	Tan	Sin	′

54°

′	Sin	Tan	Ctn	Cos	
0	.58779	.72654	1.3764	.80902	**60**
1	802	699	.3755	885	59
2	826	743	.3747	867	58
3	849	788	.3739	850	57
4	873	832	.3730	833	56
5	.58896	.72877	1.3722	.80816	**55**
6	920	921	.3713	799	54
7	943	.72966	.3705	782	53
8	967	.73010	.3697	765	52
9	.58990	055	.3688	748	51
10	.59014	.73100	1.3680	.80730	**50**
11	037	144	.3672	713	49
12	061	189	.3663	696	48
13	084	234	.3655	679	47
14	108	278	.3647	662	46
15	.59131	.73323	1.3638	.80644	**45**
16	154	368	.3630	627	44
17	178	413	.3622	610	43
18	201	457	.3613	593	42
19	225	502	.3605	576	41
20	.59248	.73547	1.3597	.80558	**40**
21	272	592	.3588	541	39
22	295	637	.3580	524	38
23	318	681	.3572	507	37
24	342	726	.3564	489	36
25	.59365	.73771	1.3555	.80472	**35**
26	389	816	.3547	455	34
27	412	861	.3539	438	33
28	436	906	.3531	420	32
29	459	951	.3522	403	31
30	.59482	.73996	1.3514	.80386	**30**
31	506	.74041	.3506	368	29
32	529	086	.3498	351	28
33	552	131	.3490	334	27
34	576	176	.3481	316	26
35	.59599	.74221	1.3473	.80299	**25**
36	622	267	.3465	282	24
37	646	312	.3457	264	23
38	669	357	.3449	247	22
39	693	402	.3440	230	21
40	.59716	.74447	1.3432	.80212	**20**
41	739	492	.3424	195	19
42	763	538	.3416	178	18
43	786	583	.3408	160	17
44	809	628	.3400	143	16
45	.59832	.74674	1.3392	.80125	**15**
46	856	719	.3384	108	14
47	879	764	.3375	091	13
48	902	810	.3367	073	12
49	926	855	.3359	056	11
50	.59949	.74900	1.3351	.80038	**10**
51	972	946	.3343	021	9
52	.59995	.74991	.3335	.80003	8
53	.60019	.75037	.3327	.79986	7
54	042	082	.3319	968	6
55	.60065	.75128	1.3311	.79951	**5**
56	089	173	.3303	934	4
57	112	219	.3295	916	3
58	135	264	.3287	899	2
59	158	310	.3278	881	1
60	.60182	.75355	1.3270	.79864	**0**
	Cos	Ctn	Tan	Sin	′

53°

′	Sin	Tan	Ctn	Cos	
0	.60182	.75355	1.3270	.79864	**60**
1	205	401	.3262	846	59
2	228	447	.3254	829	58
3	251	492	.3246	811	57
4	274	538	.3238	793	56
5	.60298	.75584	1.3230	.79776	**55**
6	321	629	.3222	758	54
7	344	675	.3214	741	53
8	367	721	.3206	723	52
9	390	767	.3198	706	51
10	.60414	.75812	1.3190	.79688	**50**
11	437	858	.3182	671	49
12	460	904	.3175	653	48
13	483	950	.3167	635	47
14	506	.75996	.3159	618	46
15	.60529	.76042	1.3151	.79600	**45**
16	553	088	.3143	583	44
17	576	134	.3135	565	43
18	599	180	.3127	547	42
19	622	226	.3119	530	41
20	.60645	.76272	1.3111	.79512	**40**
21	668	318	.3103	494	39
22	691	364	.3095	477	38
23	714	410	.3087	459	37
24	738	456	.3079	441	36
25	.60761	.76502	1.3072	.79424	**35**
26	784	548	.3064	406	34
27	807	594	.3056	388	33
28	830	640	.3048	371	32
29	853	686	.3040	353	31
30	.60876	.76733	1.3032	.79335	**30**
31	899	779	.3024	318	29
32	922	825	.3017	300	28
33	945	871	.3009	282	27
34	968	918	.3001	264	26
35	.60991	.76964	1.2993	.79247	**25**
36	.61015	.77010	.2985	229	24
37	038	057	.2977	211	23
38	061	103	.2970	193	22
39	084	149	.2962	176	21
40	.61107	.77196	1.2954	.79158	**20**
41	130	242	.2946	140	19
42	153	289	.2938	122	18
43	176	335	.2931	105	17
44	199	382	.2923	087	16
45	.61222	.77428	1.2915	.79069	**15**
46	245	475	.2907	051	14
47	268	521	.2900	033	13
48	291	568	.2892	.79016	12
49	314	615	.2884	.78998	11
50	.61337	.77661	1.2876	.78980	**10**
51	360	708	.2869	962	9
52	383	754	.2861	944	8
53	406	801	.2853	926	7
54	429	848	.2846	908	6
55	.61451	.77895	1.2838	.78891	**5**
56	474	941	.2830	873	4
57	497	.77988	.2822	855	3
58	520	.78035	.2815	837	2
59	543	082	.2807	819	1
60	.61566	.78129	1.2799	.78801	**0**
	Cos	Ctn	Tan	Sin	′

52°

'	Sin	Tan	Ctn	Cos	
0	.61566	.78129	1.2799	.78801	**60**
1	589	175	.2792	783	59
2	612	222	.2784	765	58
3	635	269	.2776	747	57
4	658	316	.2769	729	56
5	.61681	.78363	1.2761	.78711	**55**
6	704	410	.2753	694	54
7	726	457	.2746	676	53
8	749	504	.2738	658	52
9	772	551	.2731	640	51
10	.61795	.78598	1.2723	.78622	**50**
11	818	645	.2715	604	49
12	841	692	.2708	586	48
13	864	739	.2700	568	47
14	887	786	.2693	550	46
15	.61909	.78834	1.2685	.78532	**45**
16	932	881	.2677	514	44
17	955	928	.2670	496	43
18	.61978	.78975	.2662	478	42
19	.62001	.79022	.2655	460	41
20	.62024	.79070	1.2647	.78442	**40**
21	046	117	.2640	424	39
22	069	164	.2632	405	38
23	092	212	.2624	387	37
24	115	259	.2617	369	36
25	.62138	.79306	1.2609	.78351	**35**
26	160	354	.2602	333	34
27	183	401	.2594	315	33
28	206	449	.2587	297	32
29	229	496	.2579	279	31
30	.62251	.79544	1.2572	.78261	**30**
31	274	591	.2564	243	29
32	297	639	.2557	225	28
33	320	686	.2549	206	27
34	342	734	.2542	188	26
35	.62365	.79781	1.2534	.78170	**25**
36	388	829	.2527	152	24
37	411	877	.2519	134	23
38	433	924	.2512	116	22
39	456	.79972	.2504	098	21
40	.62479	.80020	1.2497	.78079	**20**
41	502	067	.2489	061	19
42	524	115	.2432	043	18
43	547	163	.2475	025	17
44	570	211	.2467	.78007	16
45	.62592	.80258	1.2460	.77988	**15**
46	615	306	.2452	970	14
47	638	354	.2445	952	13
48	660	402	.2437	934	12
49	683	450	.2430	916	11
50	.62706	.80498	1.2423	.77897	**10**
51	728	546	.2415	879	9
52	751	594	.2408	861	8
53	774	642	.2401	843	7
54	796	690	.2393	824	6
55	.62819	.80738	1.2386	.77806	**5**
56	842	786	.2378	788	4
57	864	834	.2371	769	3
58	887	882	.2364	751	2
59	909	930	.2356	733	1
60	.62932	.80978	1.2349	.77715	**0**
	Cos	Ctn	Tan	Sin	'

51°

'	Sin	Tan	Ctn	Cos	
0	.62932	.80978	1.2349	.77715	**60**
1	955	.81027	.2342	696	59
2	.62977	075	.2334	678	58
3	.63000	123	.2327	660	57
4	022	171	.2320	641	56
5	.63045	.81220	1.2312	.77623	**55**
6	068	268	.2305	605	54
7	090	316	.2298	586	53
8	113	364	.2290	568	52
9	135	413	.2283	550	51
10	.63158	.81461	1.2276	.77531	**50**
11	180	510	.2268	513	49
12	203	558	.2261	494	48
13	225	606	.2254	476	47
14	248	655	.2247	458	46
15	.63271	.81703	1.2239	.77439	**45**
16	293	752	.2232	421	44
17	316	800	.2225	402	43
18	338	849	.2218	384	42
19	361	898	.2210	366	41
20	.63383	.81946	1.2203	.77347	**40**
21	406	.81995	.2196	329	39
22	428	.82044	.2189	310	38
23	451	092	.2181	292	37
24	473	141	.2174	273	36
25	.63496	.82190	1.2167	.77255	**35**
26	518	238	.2160	236	34
27	540	287	.2153	218	33
28	563	336	.2145	199	32
29	585	385	.2138	181	31
30	.63608	.82434	1.2131	.77162	**30**
31	630	483	.2124	144	29
32	653	531	.2117	125	28
33	675	580	.2109	107	27
34	698	629	.2102	088	26
35	.63720	.82678	1.2095	.77070	**25**
36	742	727	.2088	051	24
37	765	776	.2081	033	23
38	787	825	.2074	.77014	22
39	810	874	.2066	.76996	21
40	.63832	.82923	1.2059	.76977	**20**
41	854	.82972	.2052	959	19
42	877	.83022	.2045	940	18
43	899	071	.2038	921	17
44	922	120	.2031	903	16
45	.63944	.83169	1.2024	.76884	**15**
46	966	218	.2017	866	14
47	.63989	268	.2009	847	13
48	.64011	317	.2002	828	12
49	033	366	.1995	810	11
50	.64056	.83415	1.1988	.76791	**10**
51	078	465	.1981	772	9
52	100	514	.1974	754	8
53	123	564	.1967	735	7
54	145	613	.1960	717	6
55	.64167	.83662	1.1953	.76698	**5**
56	190	712	.1946	679	4
57	212	761	.1939	661	3
58	234	811	.1932	642	2
59	256	860	.1925	623	1
60	.64279	.83910	1.1918	.76604	**0**
	Cos	Ctn	Tan	Sin	'

50°

′	Sin	Tan	Ctn	Cos	
0	.64279	.83910	1.1918	.76604	**60**
1	301	.83960	.1910	586	59
2	323	.84009	.1903	567	58
3	346	059	.1896	548	57
4	368	108	.1889	530	56
5	.64390	.84158	1.1882	.76511	**55**
6	412	208	.1875	492	54
7	435	258	.1868	473	53
8	457	307	.1861	455	52
9	479	357	.1854	436	51
10	.64501	.84407	1.1847	.76417	**50**
11	524	457	.1840	398	49
12	546	507	.1833	380	48
13	568	556	.1826	361	47
14	590	606	.1819	342	46
15	.64612	.84656	1.1812	.76323	**45**
16	635	706	.1806	304	44
17	657	756	.1799	286	43
18	679	806	.1792	267	42
19	701	856	.1785	248	41
20	.64723	.84906	1.1778	.76229	**40**
21	746	.84956	.1771	210	39
22	768	.85006	.1764	192	38
23	790	057	.1757	173	37
24	812	107	.1750	154	36
25	.64834	.85157	1.1743	.76135	**35**
26	856	207	.1736	116	34
27	878	257	.1729	097	33
28	901	308	.1722	078	32
29	923	358	.1715	059	31
30	.64945	.85408	1.1708	.76041	**30**
31	967	458	.1702	022	29
32	.64989	509	.1695	.76003	28
33	.65011	559	.1688	.75984	27
34	033	609	.1681	965	26
35	.65055	.85660	1.1674	.75946	**25**
36	077	710	.1667	927	24
37	100	761	.1660	908	23
38	122	811	.1653	889	22
39	144	862	.1647	870	21
40	.65166	.85912	1.1640	.75851	**20**
41	188	.85963	.1633	832	19
42	210	.86014	.1626	813	18
43	232	064	.1619	794	17
44	254	115	.1612	775	16
45	.65276	.86166	1.1606	.75756	**15**
46	298	216	.1599	738	14
47	320	267	.1592	719	13
48	342	318	.1585	700	12
49	364	368	.1578	680	11
50	.65386	.86419	1.1571	.75661	**10**
51	408	470	.1565	642	9
52	430	521	.1558	623	8
53	452	572	.1551	604	7
54	474	623	.1544	585	6
55	.65496	.86674	1.1538	.75566	**5**
56	518	725	.1531	547	4
57	540	776	.1524	528	3
58	562	827	.1517	509	2
59	584	878	.1510	490	1
60	.65606	.86929	1.1504	.75471	**0**
	Cos	Ctn	Tan	Sin	′

49°

′	Sin	Tan	Ctn	Cos	
0	.65606	.86929	1.1504	.75471	**60**
1	628	.86980	.1497	452	59
2	650	.87031	.1490	433	58
3	672	082	.1483	414	57
4	694	133	.1477	395	56
5	.65716	.87184	1.1470	.75375	**55**
6	738	236	.1463	356	54
7	759	287	.1456	337	53
8	781	338	.1450	318	52
9	803	389	.1443	299	51
10	.65825	.87441	1.1436	.75280	**50**
11	847	492	.1430	261	49
12	869	543	.1423	241	48
13	891	595	.1416	222	47
14	913	646	.1410	203	46
15	.65935	.87698	1.1403	.75184	**45**
16	956	749	.1396	165	44
17	.65978	801	.1389	146	43
18	.66000	852	.1383	126	42
19	022	904	.1376	107	41
20	.66044	.87955	1.1369	.75088	**40**
21	066	.88007	.1363	069	39
22	088	059	.1356	050	38
23	109	110	.1349	030	37
24	131	162	.1343	.75011	36
25	.66153	.88214	1.1336	.74992	**35**
26	175	265	.1329	973	34
27	197	317	.1323	953	33
28	218	369	.1316	934	32
29	240	421	.1310	915	31
30	.66262	.88473	1.1303	.74896	**30**
31	284	524	.1296	876	29
32	306	576	.1290	857	28
33	327	628	.1283	838	27
34	349	680	.1276	818	26
35	.66371	.88732	1.1270	.74799	**25**
36	393	784	.1263	780	24
37	414	836	.1257	760	23
38	436	888	.1250	741	22
39	458	940	.1243	722	21
40	.66480	.88992	1.1237	.74703	**20**
41	501	.89045	.1230	683	19
42	523	097	.1224	664	18
43	545	149	.1217	644	17
44	566	201	.1211	625	16
45	.66588	.89253	1.1204	.74606	**15**
46	610	306	.1197	586	14
47	632	358	.1191	567	13
48	653	410	.1184	548	12
49	675	463	.1178	528	11
50	.66697	.89515	1.1171	.74509	**10**
51	718	567	.1165	489	9
52	740	620	.1158	470	8
53	762	672	.1152	451	7
54	783	725	.1145	431	6
55	.66805	.89777	1.1139	.74412	**5**
56	827	830	.1132	392	4
57	848	883	.1126	373	3
58	870	935	.1119	353	2
59	891	.89988	.1113	334	1
60	.66913	.90040	1.1106	.74314	**0**
	Cos	Ctn	Tan	Sin	′

48°

′	Sin	Tan	Ctn	Cos	
0	.66913	.90040	1.1106	.74314	**60**
1	935	093	.1100	295	59
2	956	146	.1093	276	58
3	978	199	.1087	256	57
4	.66999	251	.1080	237	56
5	.67021	.90304	1.1074	.74217	**55**
6	043	357	.1067	198	54
7	064	410	.1061	178	53
8	086	463	.1054	159	52
9	107	516	.1048	139	51
10	.67129	.90569	1.1041	.74120	**50**
11	151	621	.1035	100	49
12	172	674	.1028	080	48
13	194	727	.1022	061	47
14	215	781	.1016	041	46
15	.67237	.90834	1.1009	.74022	**45**
16	258	887	.1003	.74002	44
17	280	940	.0996	.73983	43
18	301	.90993	.0990	963	42
19	323	.91046	.0983	944	41
20	.67344	.91099	1.0977	.73924	**40**
21	366	153	.0971	904	39
22	387	206	.0964	885	38
23	409	259	.0958	865	37
24	430	313	.0951	846	36
25	.67452	.91366	1.0945	.73826	**35**
26	473	419	.0939	806	34
27	495	473	.0932	787	33
28	516	526	.0926	767	32
29	538	580	.0919	747	31
30	.67559	.91633	1.0913	.73728	**30**
31	580	687	.0907	708	29
32	602	740	.0900	688	28
33	623	794	.0894	669	27
34	645	847	.0888	649	26
35	.67666	.91901	1.0881	.73629	**25**
36	688	.91955	.0875	610	24
37	709	.92008	.0869	590	23
38	730	062	.0862	570	22
39	752	116	.0856	551	21
40	.67773	.92170	1.0850	.73531	**20**
41	795	224	.0843	511	19
42	816	277	.0837	491	18
43	837	331	.0831	472	17
44	859	385	.0824	452	16
45	.67880	.92439	1.0818	.73432	**15**
46	901	493	.0812	413	14
47	923	547	.0805	393	13
48	944	601	.0799	373	12
49	965	655	.0793	353	11
50	.67987	.92709	1.0786	.73333	**10**
51	.68008	763	.0780	314	9
52	029	817	.0774	294	8
53	051	872	.0768	274	7
54	072	926	.0761	254	6
55	.68093	.92980	1.0755	.73234	**5**
56	115	.93034	.0749	215	4
57	136	088	.0742	195	3
58	157	143	.0736	175	2
59	179	197	.0730	155	1
60	.68200	.93252	1.0724	.73135	**0**
	Cos	Ctn	Tan	Sin	′

47°

′	Sin	Tan	Ctn	Cos	
0	.68200	.93252	1.0724	.73135	**60**
1	221	306	.0717	116	59
2	242	360	.0711	096	58
3	264	415	.0705	076	57
4	285	469	.0699	056	56
5	.68306	.93524	1.0692	.73036	**55**
6	327	578	.0686	.73016	54
7	349	633	.0680	.72996	53
8	370	688	.0674	976	52
9	391	742	.0668	957	51
10	.68412	.93797	1.0661	.72937	**50**
11	434	852	.0655	917	49
12	455	906	.0649	897	48
13	476	.93961	.0643	877	47
14	497	.94016	.0637	857	46
15	.68518	.94071	1.0630	.72837	**45**
16	539	125	.0624	817	44
17	561	180	.0618	797	43
18	582	235	.0612	777	42
19	603	290	.0606	757	41
20	.68624	.94345	1.0599	.72737	**40**
21	645	400	.0593	717	39
22	666	455	.0587	697	38
23	688	510	.0581	677	37
24	709	565	.0575	657	36
25	.68730	.94620	1.0569	.72637	**35**
26	751	676	.0562	617	34
27	772	731	.0556	597	33
28	793	786	.0550	577	32
29	814	841	.0544	557	31
30	.68835	.94896	1.0538	.72537	**30**
31	857	.94952	.0532	517	29
32	878	.95007	.0526	497	28
33	899	062	.0519	477	27
34	920	118	.0513	457	26
35	.68941	.95173	1.0507	.72437	**25**
36	962	229	.0501	417	24
37	.68983	284	.0495	397	23
38	.69004	340	.0489	377	22
39	025	395	.0483	357	21
40	.69046	.95451	1.0477	.72337	**20**
41	067	506	.0470	317	19
42	088	562	.0464	297	18
43	109	618	.0458	277	17
44	130	673	.0452	257	16
45	.69151	.95729	1.0446	.72236	**15**
46	172	785	.0440	216	14
47	193	841	.0434	196	13
48	214	897	.0428	176	12
49	235	.95952	.0422	156	11
50	.69256	.96008	1.0416	.72136	**10**
51	277	064	.0410	116	9
52	298	120	.0404	095	8
53	319	176	.0398	075	7
54	340	232	.0392	055	6
55	.69361	.96288	1.0385	.72035	**5**
56	382	344	.0379	.72015	4
57	403	400	.0373	.71995	3
58	424	457	.0367	974	2
59	445	513	.0361	954	1
60	.69466	.96569	1.0355	.71934	**0**
	Cos	Ctn	Tan	Sin	′

46°

44° — Values of Trigonometric Functions

′	Sin	Tan	Ctn	Cos	
0	.69466	.96569	1.0355	.71934	**60**
1	487	625	.0349	914	59
2	508	681	.0343	894	58
3	529	738	.0337	873	57
4	549	794	.0331	853	56
5	.69570	.96850	1.0325	.71833	**55**
6	591	907	.0319	813	54
7	612	.96963	.0313	792	53
8	633	.97020	.0307	772	52
9	654	076	.0301	752	51
10	.69675	.97133	1.0295	.71732	**50**
11	696	189	.0289	711	49
12	717	246	.0283	691	48
13	737	302	.0277	671	47
14	758	359	.0271	650	46
15	.69779	.97416	1.0265	.71630	**45**
16	800	472	.0259	610	44
17	821	529	.0253	590	43
18	842	586	.0247	569	42
19	862	643	.0241	549	41
20	.69883	.97700	1.0235	.71529	**40**
21	904	756	.0230	508	39
22	925	813	.0224	488	38
23	946	870	.0218	468	37
24	966	927	.0212	447	36
25	.69987	.97984	1.0206	.71427	**35**
26	.70008	.98041	.0200	407	34
27	029	098	.0194	386	33
28	049	155	.0188	366	32
29	070	213	.0182	345	31
30	.70091	.98270	1.0176	.71325	**30**
31	112	327	.0170	305	29
32	132	384	.0164	284	28
33	153	441	.0158	264	27
34	174	499	.0152	243	26
35	.70195	.98556	1.0147	.71223	**25**
36	215	613	.0141	203	24
37	236	671	.0135	182	23
38	257	728	.0129	162	22
39	277	786	.0123	141	21
40	.70298	.98843	1.0117	.71121	**20**
41	319	901	.0111	100	19
42	339	.98958	.0105	080	18
43	360	.99016	.0099	059	17
44	381	073	.0094	039	16
45	.70401	.99131	1.0088	.71019	**15**
46	422	189	.0082	.70998	14
47	443	247	.0076	978	13
48	463	304	.0070	957	12
49	484	362	.0064	937	11
50	.70505	.99420	1.0058	.70916	**10**
51	525	478	.0052	896	9
52	546	536	.0047	875	8
53	567	594	.0041	855	7
54	587	652	.0035	834	6
55	.70608	.99710	1.0029	.70813	**5**
56	628	768	.0023	793	4
57	649	826	.0017	772	3
58	670	884	.0012	752	2
59	690	.99942	.0006	731	1
60	.70711	1.0000	1.0000	.70711	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

45°

TABLE III

COMMON LOGARITHMS
OF THE
TRIGONOMETRIC FUNCTIONS
FROM
0° TO 90° AT INTERVALS OF ONE MINUTE
TO
FIVE DECIMAL PLACES

TABLE IIIa — AUXILIARY TABLE OF S AND T FOR A IN MINUTES

$S = \log \sin A - \log A'$ and $T = \log \tan A - \log A'$

A'	$S + 10$
0′ — 13′	6.46373
14′ — 42′	72
43′ — 58′	71
59′ — 71′	6.46370
72′ — 81′	69
82′ — 91′	68
92′ — 99′	6.46367
100′ — 107′	66
108′ — 115′	65
116′ — 121′	6.46364
122′ — 128′	63
129′ — 134′	62
135′ — 140′	6.46361
141′ — 146′	60
147′ — 151′	59
152′ — 157′	6.46358
158′ — 162′	57
163′ — 167′	56
168′ — 171′	6.46355
172′ — 176′	54
177′ — 181′	53

A'	$T + 10$
0′ — 26′	6.46373
27′ — 39′	74
40′ — 48′	75
49′ — 56′	6.46376
57′ — 63′	77
64′ — 69′	78
70′ — 74′	6.46379
75′ — 80′	80
81′ — 85′	81
86′ — 89′	6.46382
90′ — 94′	83
95′ — 98′	84
99′ — 102′	6.46385
103′ — 106′	86
107′ — 110′	87
111′ — 113′	6.46388
114′ — 117′	89
118′ — 120′	90
121′ — 124′	6.46391
125′ — 127′	92
128′ — 130′	93
131′ — 133′	6.46394
134′ — 136′	95
137′ — 139′	96
140′ — 142′	6.46397
143′ — 145′	98
146′ — 148′	99
149′ — 150′	6.46400
151′ — 153′	01
154′ — 156′	02
157′ — 158′	6.46403
159′ — 161′	04
162′ — 163′	05
164′ — 166′	6.46406
167′ — 168′	07
169′ — 171′	08
172′ — 173′	6.46409
174′ — 175′	10
176′ — 178′	11
179′ — 180′	6.46412
181′ — 182′	13
183′ — 184′	14

For small angles: $\log \sin A = \log A' + S$ and $\log \tan A = A' + T$
For angles near 90°: $\log \cos A = \log\ (90° - A)' + S$, $\log \text{ctn}\ A = \log\ (90° - A)' + T$
where A' = number of minutes in A, and $(90° - A)'$ = number of minutes in $90° - A$

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	——		——		——	0.00 000	**60**
1	6.46 373	30103	6.46 373	30103	3.53 627	0.00 000	59
2	6.76 476	17609	6.76 476	17609	3.23 524	0.00 000	58
3	6.94 085	12494	6.94 085	12494	3.05 915	0.00 000	57
4	7.06 579	9691	7.06 579	9691	2.93 421	0.00 000	56
5	7.16 270	7918	7.16 270	7918	2.83 730	0.00 000	**55**
6	7.24 188	6694	7.24 188	6694	2.75 812	0.00 000	54
7	7.30 882	5800	7.30 882	5800	2.69 118	0.00 000	53
8	7.36 682	5115	7.36 682	5115	2.63 318	0.00 000	52
9	7.41 797	4576	7.41 797	4576	2.58 203	0.00 000	51
10	7.46 373	4139	7.46 373	4139	2.53 627	0.00 000	**50**
11	7.50 512	3779	7.50 512	3779	2.49 488	0.00 000	49
12	7.54 291	3476	7.54 291	3476	2.45 709	0.00 000	48
13	7.57 767	3218	7.57 767	3219	2.42 233	0.00 000	47
14	7.60 985	2997	7.60 986	2996	2.39 014	0.00 000	46
15	7.63 982	2802	7.63 982	2803	2.36 018	0.00 000	**45**
16	7.66 784	2633	7.66 785	2633	2.33 215	0.00 000	44
17	7.69 417	2483	7.69 418	2482	2.30 582	9.99 999	43
18	7.71 900	2348	7.71 900	2348	2.28 100	9.99 999	42
19	7.74 248	2227	7.74 248	2228	2.25 752	9.99 999	41
20	7.76 475	2119	7.76 476	2119	2.23 524	9.99 999	**40**
21	7.78 594	2021	7.78 595	2020	2.21 405	9.99 999	39
22	7.80 615	1930	7.80 615	1931	2.19 385	9.99 999	38
23	7.82 545	1848	7.82 546	1848	2.17 454	9.99 999	37
24	7.84 393	1773	7.84 394	1773	2.15 606	9.99 999	36
25	7.86 166	1704	7.86 167	1704	2.13 833	9.99 999	**35**
26	7.87 870	1639	7.87 871	1639	2.12 129	9.99 999	34
27	7.89 509	1579	7.89 510	1579	2.10 490	9.99 999	33
28	7.91 088	1524	7.91 089	1524	2.08 911	9.99 999	32
29	7.92 612	1472	7.92 613	1473	2.07 387	9.99 998	31
30	7.94 084	1424	7.94 086	1424	2 05 914	9.99 998	**30**
31	7.95 508	1379	7.95 510	1379	2.04 490	9.99 998	29
32	7.96 887	1336	7.96 889	1336	2.03 111	9.99 998	28
33	7 98 223	1297	7.98 225	1297	2.01 775	9.99 998	27
34	7.99 520	1259	7.99 522	1259	2.00 478	9.99 998	26
35	8.00 779	1223	8.00 781	1223	1.99 219	9.99 998	**25**
36	8.02 002	1190	8.02 004	1190	1.97 996	9.99 998	24
37	8.03 192	1158	8.03 194	1159	1.96 806	9.99 997	23
38	8.04 350	1128	8.04 353	1128	1.95 647	9.99 997	22
39	8.05 478	1100	8.05 481	1100	1.94 519	9.99 997	21
40	8.06 578	1072	8.06 581	1072	1.93 419	9.99 997	**20**
41	8.07 650	1046	8.07 653	1047	1.92 347	9.99 997	19
42	8.08 696	1022	8.08 700	1022	1.91 300	9.99 997	18
43	8.09 718	999	8.09 722	998	1.90 278	9.99 997	17
44	8.10 717	976	8.10 720	976	1.89 280	9.99 996	16
45	8.11 693	954	8.11 696	955	1.88 304	9.99 996	**15**
46	8.12 647	934	8.12 651	934	1.87 349	9.99 996	14
47	8.13 581	914	8.13 585	915	1.86 415	9.99 996	13
48	8.14 495	896	8.14 500	895	1.85 500	9.99 996	12
49	8.15 391	877	8.15 395	878	1.84 605	9.99 996	11
50	8.16 268	860	8.16 273	860	1.83 727	9.99 995	**10**
51	8.17 128	843	8.17 133	843	1.82 867	9.99 995	9
52	8.17 971	827	8.17 976	828	1.82 024	9.99 995	8
53	8.18 798	812	8.18 804	812	1.81 196	9 99 995	7
54	8.19 610	797	8.19 616	797	1.80 384	9.99 995	6
55	8.20 407	782	8.20 413	782	1.79 587	9.99 994	**5**
56	8.21 189	769	8.21 195	769	1.78 805	9.99 994	4
57	8.21 958	755	8.21 964	756	1.78 036	9.99 994	3
58	8.22 713	743	8.22 720	742	1.77 280	9.99 994	2
59	8.23 456	730	8.23 462	730	1.76 538	9.99 994	1
60	8.24 186		8.24 192		1.75 808	9.99 993	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

For logarithms of sines or tangents of angles less than 3° (or logarithms of cosines or cotangents of angles greater than 87°), see Table IIIa, p. 45.

When the tabular differences are large, that method is usually better. The proportional parts stated for 1° and 2° in this table are sufficient when great accuracy is not required, even if the ordinary method of interpolation is used.

1° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.24 186	717	8.24 192	718	1.75 808	9.99 993	**60**
1	8.24 903	706	8.24 910	706	1.75 090	9.99 993	59
2	8.25 609	695	8.25 616	696	1.74 384	9.99 993	58
3	8.26 304	684	8.26 312	684	1.73 688	9.99 993	57
4	8.26 988	673	8.26 996	673	1.73 004	9.99 992	56
5	8.27 661	663	8.27 669	663	1.72 331	9.99 992	**55**
6	8.28 324	653	8.28 332	654	1.71 668	9.99 992	54
7	8.28 977	644	8.28 986	643	1.71 014	9.99 992	53
8	8.29 621	634	8.29 629	634	1.70 371	9.99 992	52
9	8.30 255	624	8.30 263	625	1.69 737	9.99 991	51
10	8.30 879	616	8.30 888	617	1.69 112	9.99 991	**50**
11	8.31 495	608	8.31 505	607	1.68 495	9.99 991	49
12	8.32 103	599	8.32 112	599	1.67 888	9.99 990	48
13	8.32 702	590	8.32 711	591	1.67 289	9.99 990	47
14	8.33 292	583	8.33 302	584	1.66 698	9.99 990	46
15	8.33 875	575	8.33 886	575	1.66 114	9.99 990	**45**
16	8.34 450	568	8.34 461	568	1.65 539	9.99 989	44
17	8.35 018	560	8.35 029	561	1.64 971	9.99 989	43
18	8.35 578	553	8.35 590	553	1.64 410	9.99 989	42
19	8.36 131	547	8.36 143	546	1.63 857	9.99 989	41
20	8.36 678	539	8.36 689	540	1.63 311	9.99 988	**40**
21	8.37 217	533	8.37 229	533	1.62 771	9.99 988	39
22	8.37 750	526	8.37 762	527	1.62 238	9.99 988	38
23	8.38 276	520	8.38 289	520	1.61 711	9.99 987	37
24	8.38 796	514	8.38 809	514	1.61 191	9.99 987	36
25	8.39 310	508	8.39 323	509	1.60 677	9.99 987	**35**
26	8.39 818	502	8.39 832	502	1.60 168	9.99 986	34
27	8.40 320	496	8.40 334	496	1.59 666	9.99 986	33
28	8.40 816	491	8.40 830	491	1.59 170	9.99 986	32
29	8.41 307	485	8.41 321	486	1.58 679	9.99 985	31
30	8.41 792	480	8.41 807	480	1.58 193	9.99 985	**30**
31	8.42 272	474	8.42 287	475	1.57 713	9.99 985	29
32	8.42 746	470	8.42 762	470	1.57 238	9.99 984	28
33	8.43 216	464	8.43 232	464	1.56 768	9.99 984	27
34	8.43 680	459	8.43 696	460	1.56 304	9.99 984	26
35	8.44 139	455	8.44 156	455	1.55 844	9.99 983	**25**
36	8.44 594	450	8.44 611	450	1.55 389	9.99 983	24
37	8.45 044	445	8.45 061	446	1.54 939	9.99 983	23
38	8.45 489	441	8.45 507	441	1.54 493	9.99 982	22
39	8.45 930	436	8.45 948	437	1.54 052	9.99 982	21
40	8.46 366	433	8.46 385	432	1.53 615	9.99 982	**20**
41	8.46 799	427	8.46 817	428	1.53 183	9.99 981	19
42	8.47 226	424	8.47 245	424	1.52 755	9.99 981	18
43	8.47 650	419	8.47 669	420	1.52 331	9.99 981	17
44	8.48 069	416	8.48 089	416	1.51 911	9.99 980	16
45	8.48 485	411	8.48 505	412	1.51 495	9.99 980	**15**
46	8.48 896	408	8.48 917	408	1.51 083	9.99 979	14
47	8.49 304	404	8.49 325	404	1.50 675	9.99 979	13
48	8.49 708	400	8.49 729	401	1.50 271	9.99 979	12
49	8.50 108	396	8.50 130	397	1.49 870	9.99 978	11
50	8.50 504	393	8.50 527	393	1.49 473	9.99 978	**10**
51	8.50 897	390	8.50 920	390	1.49 080	9.99 977	9
52	8.51 287	386	8.51 310	386	1.48 690	9.99 977	8
53	8.51 673	382	8.51 696	383	1.48 304	9.99 977	7
54	8.52 055	379	8.52 079	380	1.47 921	9.99 976	6
55	8.52 434	376	8.52 459	376	1.47 541	9.99 976	**5**
56	8.52 810	373	8.52 835	373	1.47 165	9.99 975	4
57	8.53 183	369	8.53 208	370	1.46 792	9.99 975	3
58	8.53 552	367	8.53 578	367	1.46 422	9.99 974	2
59	8.53 919	363	8.53 945	363	1.46 055	9.99 974	1
60	8.54 282		8.54 308		1.45 692	9.99 974	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	720	710	690	680	670
2	144	142	138	136	134
3	216	213	207	204	201
4	288	284	276	272	268
5	360	355	345	340	335
6	432	426	414	408	402
7	504	497	483	476	469
8	576	568	552	544	536
9	648	639	621	612	603

	660	650	640	630	620
2	132	130	128	126	124
3	198	195	192	189	186
4	264	260	256	252	248
5	330	325	320	315	310
6	396	390	384	378	372
7	462	455	448	441	434
8	528	520	512	504	496
9	594	585	576	567	558

	610	600	590	580	570
2	122	120	118	116	114
3	183	180	177	174	171
4	244	240	236	232	228
5	305	300	295	290	285
6	366	360	354	348	342
7	427	420	413	406	399
8	488	480	472	464	456
9	549	540	531	522	513

	560	550	540	530	520
2	112	110	108	106	104
3	168	165	162	159	156
4	224	220	216	212	208
5	280	275	270	265	260
6	336	330	324	318	312
7	392	385	378	371	364
8	448	440	432	424	416
9	504	495	486	477	468

	510	500	490	480	470
2	102	100	98	96	94
3	153	150	147	144	141
4	204	200	196	192	188
5	255	250	245	240	235
6	306	300	294	288	282
7	357	350	343	336	329
8	408	400	392	384	376
9	459	450	441	432	423

	460	450	440	430	420
2	92	90	88	86	84
3	138	135	132	129	126
4	184	180	176	172	168
5	230	225	220	215	210
6	276	270	264	258	252
7	322	315	308	301	294
8	368	360	352	344	336
9	414	405	396	387	378

	410	400	395	390	385
2	82	80	79.0	78	77.0
3	123	120	118.5	117	115.5
4	164	160	158.0	156	154.0
5	205	200	197.5	195	192.5
6	246	240	237.0	234	231.0
7	287	280	276.5	273	269.5
8	328	320	316.0	312	308.0
9	369	360	355.5	351	346.5

	380	375	370	365	360
2	76	75.0	74	73.0	72
3	114	112.5	111	109.5	108
4	152	150.0	148	146.0	144
5	190	187.5	185	182.5	180
6	228	225.0	222	219.0	216
7	266	262.5	259	255.5	252
8	304	300.0	296	292.0	288
9	342	337.5	333	328.5	324

Prop. Pts.

88° — Logarithms of Trigonometric Functions

2° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.54 282	360	8.54 308	361	1.45 692	9.99 974	**60**
1	8.54 642	357	8.54 669	358	1.45 331	9.99 973	59
2	8.54 999	355	8.55 027	355	1.44 973	9.99 973	58
3	8.55 354	351	8.55 382	352	1.44 618	9.99 972	57
4	8.55 705	349	8.55 734	349	1.44 266	9.99 972	56
5	8.56 054	346	8.56 083	346	1.43 917	9.99 971	**55**
6	8.56 400	343	8.56 429	344	1.43 571	9.99 971	54
7	8.56 743	341	8.56 773	341	1.43 227	9.99 970	53
8	8.57 084	337	8.57 114	338	1.42 886	9.99 970	52
9	8.57 421	336	8.57 452	336	1.42 548	9.99 969	51
10	8.57 757	332	8.57 788	333	1.42 212	9.99 969	**50**
11	8.58 089	330	8.58 121	330	1.41 879	9.99 968	49
12	8.58 419	328	8.58 451	328	1.41 549	9.99 968	48
13	8.58 747	325	8.58 779	326	1.41 221	9.99 967	47
14	8.59 072	323	8.59 105	323	1.40 895	9.99 967	46
15	8.59 395	320	8.59 428	321	1.40 572	9.99 967	**45**
16	8.59 715	318	8.59 749	319	1.40 251	9.99 966	44
17	8.60 033	316	8.60 068	316	1.39 932	9.99 966	43
18	8.60 349	313	8.60 384	314	1.39 616	9.99 965	42
19	8.60 662	311	8.60 698	311	1.39 302	9.99 964	41
20	8.60 973	309	8.61 009	310	1.38 991	9.99 964	**40**
21	8.61 282	307	8.61 319	307	1.38 681	9.99 963	39
22	8.61 589	305	8.61 626	305	1.38 374	9.99 963	38
23	8.61 894	302	8.61 931	303	1.38 069	9.99 962	37
24	8.62 196	301	8.62 234	301	1.37 766	9.99 962	36
25	8.62 497	298	8.62 535	299	1.37 465	9.99 961	**35**
26	8.62 795	296	8.62 834	297	1.37 166	9.99 961	34
27	8.63 091	294	8.63 131	295	1.36 869	9.99 960	33
28	8.63 385	293	8.63 426	292	1.36 574	9.99 960	32
29	8.63 678	290	8.63 718	291	1.36 282	9.99 959	31
30	8.63 968	288	8.64 009	289	1.35 991	9.99 959	**30**
31	8.64 256	287	8.64 298	287	1.35 702	9.99 958	29
32	8.64 543	284	8.64 585	285	1.35 415	9.99 958	28
33	8.64 827	283	8.64 870	284	1.35 130	9.99 957	27
34	8.65 110	281	8.65 154	281	1.34 846	9.99 956	26
35	8.65 391	279	8.65 435	280	1.34 565	9.99 956	**25**
36	8.65 670	277	8.65 715	278	1.34 285	9.99 955	24
37	8.65 947	276	8.65 993	276	1.34 007	9.99 955	23
38	8.66 223	274	8.66 269	274	1.33 731	9.99 954	22
39	8.66 497	272	8.66 543	273	1.33 457	9.99 954	21
40	8.66 769	270	8.66 816	271	1.33 184	9.99 953	**20**
41	8.67 039	269	8.67 087	269	1.32 913	9.99 952	19
42	8.67 308	267	8.67 356	268	1.32 644	9.99 952	18
43	8.67 575	266	8.67 624	266	1.32 376	9.99 951	17
44	8.67 841	263	8.67 890	264	1.32 110	9.99 951	16
45	8.68 104	263	8.68 154	263	1.31 846	9.99 950	**15**
46	8.68 367	260	8.68 417	261	1.31 583	9.99 949	14
47	8.68 627	259	8.68 678	260	1.31 322	9.99 949	13
48	8.68 886	258	8.68 938	258	1.31 062	9.99 948	12
49	8.69 144	256	8.69 196	257	1.30 804	9.99 948	11
50	8.69 400	254	8.69 453	255	1.30 547	9.99 947	**10**
51	8.69 654	253	8.69 708	254	1.30 292	9.99 946	9
52	8.69 907	252	8.69 962	252	1.30 038	9.99 946	8
53	8.70 159	250	8.70 214	251	1.29 786	9.99 945	7
54	8.70 409	249	8.70 465	249	1.29 535	9.99 944	6
55	8.70 658	247	8.70 714	248	1.29 286	9.99 944	**5**
56	8.70 905	246	8.70 962	246	1.29 038	9.99 943	4
57	8.71 151	244	8.71 208	245	1.28 792	9.99 942	3
58	8.71 395	243	8.71 453	244	1.28 547	9.99 942	2
59	8.71 638	242	8.71 697	243	1.28 303	9.99 941	1
60	8.71 880		8.71 940		1.28 060	9.99 940	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	360	**355**	**350**	**345**
2	72	71.0	70	69.0
3	108	106.5	105	103.5
4	144	142.0	140	138.0
5	180	177.5	175	172.5
6	216	213.0	210	207.0
7	252	248.5	245	241.5
8	288	284.0	280	276.0
9	324	319.5	315	310.5
	340	**335**	**330**	**325**
2	68	67.0	66	65.0
3	102	100.5	99	97.5
4	136	134.0	132	130.0
5	170	167.5	165	162.5
6	204	201.0	198	195.0
7	238	234.5	231	227.5
8	272	268.0	264	260.0
9	306	301.5	297	292,5
	320	**315**	**310**	**305**
2	64	63.0	62	61.0
3	96	94.5	93	91.5
4	128	126.0	124	122.0
5	160	157.5	155	152.5
6	192	189.0	186	183.0
7	224	220.5	217	213.5
8	256	252.0	248	244.0
9	288	283.5	279	274.5
	300	**295**	**290**	**285**
2	60	59.0	58	57.0
3	90	88.5	87	85.5
4	120	118.0	116	114.0
5	150	147.5	145	142.5
6	180	177.0	174	171.0
7	210	206.5	203	199.5
8	240	236.0	232	228.0
9	270	265.5	261	256.5
	280	**275**	**270**	**265**
2	56	55.0	54	53.0
3	84	82.5	81	79.5
4	112	110.0	108	106.0
5	140	137.5	135	132.5
6	168	165.0	162	159.0
7	196	192.5	189	185.5
8	224	220.0	216	212.0
9	252	247.5	243	238.5
	260	**255**	**250**	**245**
2	52	51.0	50	49.0
3	78	76.5	75	73.5
4	104	102.0	100	98.0
5	130	127.5	125	122.5
6	156	153.0	150	147.0
7	182	178.5	175	171.5
8	208	204.0	200	196.0
9	234	229.5	225	220.5

Prop. Pts.

87° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.71 880	240	8.71 940	241	1.28 060	9.99 940	**60**
1	8.72 120	239	8.72 181	239	1.27 819	9.99 940	59
2	8.72 359	238	8.72 420	239	1.27 580	9.99 939	58
3	8.72 597	237	8.72 659	237	1.27 341	9.99 938	57
4	8.72 834	235	8.72 896	236	1.27 104	9.99 938	56
5	8.73 069	234	8.73 132	234	1.26 868	9.99 937	**55**
6	8.73 303	232	8.73 366	234	1.26 634	9.99 936	54
7	8.73 535	232	8.73 600	232	1.26 400	9.99 936	53
8	8.73 767	230	8.73 832	231	1.26 168	9.99 935	52
9	8.73 997	229	8.74 063	229	1.25 937	9.99 934	51
10	8.74 226	228	8.74 292	229	1.25 708	9.99 934	**50**
11	8.74 454	226	8.74 521	227	1.25 479	9.99 933	49
12	8.74 680	226	8.74 748	226	1.25 252	9.99 932	48
13	8.74 906	224	8.74 974	225	1.25 026	9.99 932	47
14	8.75 130	223	8.75 199	224	1.24 801	9.99 931	46
15	8.75 353	222	8.75 423	222	1.24 577	9.99 930	**45**
16	8.75 575	220	8.75 645	222	1.24 355	9.99 929	44
17	8.75 795	220	8.75 867	220	1.24 133	9.99 929	43
18	8.76 015	219	8.76 087	219	1.23 913	9.99 928	42
19	8.76 234	217	8.76 306	219	1.23 694	9.99 927	41
20	8.76 451	216	8.76 525	217	1.23 475	9.99 926	**40**
21	8.76 667	216	8.76 742	216	1.23 258	9.99 926	39
22	8.76 883	214	8.76 958	215	1.23 042	9.99 925	38
23	8.77 097	213	8.77 173	214	1.22 827	9.99 924	37
24	8.77 310	212	8.77 387	213	1.22 613	9.99 923	36
25	8.77 522	211	8.77 600	211	1.22 400	9.99 923	**35**
26	8.77 733	210	8.77 811	211	1.22 189	9.99 922	34
27	8.77 943	209	8.78 022	210	1.21 978	9.99 921	33
28	8.78 152	208	8.78 232	209	1.21 768	9.99 920	32
29	8.78 360	208	8.78 441	208	1.21 559	9.99 920	31
30	8.78 568	206	8.78 649	206	1.21 351	9.99 919	**30**
31	8.78 774	205	8.78 855	206	1.21 145	9.99 918	29
32	8.78 979	204	8.79 061	205	1.20 939	9.99 917	28
33	8.79 183	203	8.79 266	204	1.20 734	9.99 917	27
34	8.79 386	202	8.79 470	203	1.20 530	9.99 916	26
35	8.79 588	201	8.79 673	202	1.20 327	9.99 915	**25**
36	8.79 789	201	8.79 875	201	1.20 125	9.99 914	24
37	8.79 990	199	8.80 076	201	1.19 924	9.99 913	23
38	8.80 189	199	8.80 277	199	1.19 723	9.99 913	22
39	8.80 388	197	8.80 476	198	1.19 524	9.99 912	21
40	8.80 585	197	8.80 674	198	1.19 326	9.99 911	**20**
41	8.80 782	196	8.80 872	196	1.19 128	9.99 910	19
42	8.80 978	195	8.81 068	196	1.18 932	9.99 909	18
43	8.81 173	194	8.81 264	195	1.18 736	9.99 909	17
44	8.81 367	193	8.81 459	194	1.18 541	9.99 908	16
45	8.81 560	192	8.81 653	193	1.18 347	9.99 907	**15**
46	8.81 752	192	8.81 846	192	1.18 154	9.99 906	14
47	8.81 944	190	8.82 038	192	1.17 962	9.99 905	13
48	8.82 134	190	8.82 230	190	1.17 770	9.99 904	12
49	8.82 324	189	8.82 420	190	1.17 580	9.99 904	11
50	8.82 513	188	8.82 610	189	1.17 390	9.99 903	**10**
51	8.82 701	187	8.82 799	188	1.17 201	9.99 902	9
52	8.82 888	187	8.82 987	188	1.17 013	9.99 901	8
53	8.83 075	186	8.83 175	186	1.16 825	9.99 900	7
54	8.83 261	185	8.83 361	186	1.16 639	9.99 899	6
55	8.83 446	184	8.83 547	185	1.16 453	9.99 898	**5**
56	8.83 630	183	8.83 732	184	1.16 268	9.99 898	4
57	8.83 813	183	8.83 916	184	1.16 084	9.99 897	3
58	8.83 996	181	8.84 100	182	1.15 900	9.99 896	2
59	8.84 177	181	8.84 282	182	1.15 718	9.99 895	1
60	8.84 358		8.84 464		1.15 536	9.99 894	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	241	239	237	235
2	48.2	47.8	47.4	47.0
3	72.3	71.7	71.1	70.5
4	96.4	95.6	94.8	94.0
5	120.5	119.5	118.5	117.5
6	144.6	143.4	142.2	141.0
7	168.7	167.3	165.9	164.5
8	192.8	191.2	189.6	188.0
9	216.9	215.1	213.3	211.5

	234	232	229	227
2	46.8	46.4	45.8	45.4
3	70.2	69.6	68.7	68.1
4	93.6	92.8	91.6	90.8
5	117.0	116.0	114.5	113.5
6	140.4	139.2	137.4	136.2
7	163.8	162.4	160.3	158.9
8	187.2	185.6	183.2	181.6
9	210.6	208.8	206.1	204.3

	226	224	222	220
2	45.2	44.8	44.4	44.0
3	67.8	67.2	66.6	66.0
4	90.4	89.6	88.8	88.0
5	113.0	112.0	111.0	110.0
6	135.6	134.4	133.2	132.0
7	158.2	156.8	155.4	154.0
8	180.8	179.2	177.6	176.0
9	203.4	201.6	199.8	198.0

	219	217	215	213
2	43.8	43.4	43.0	42.6
3	65.7	65.1	64.5	63.9
4	87.6	86.8	86.0	85.2
5	109.5	108.5	107.5	106.5
6	131.4	130.2	129.0	127.8
7	153.3	151.9	150.5	149.1
8	175.2	173.6	172.0	170.4
9	197.1	195.3	193.5	191.7

	211	208	206	203
2	42.2	41.6	41.2	40.6
3	63.3	62.4	61.8	60.9
4	84.4	83.2	82.4	81.2
5	105.5	104.0	103.0	101.5
6	126.6	124.8	123.6	121.8
7	147.7	145.6	144.2	142.1
8	168.8	166.4	164.8	162.4
9	189.9	187.2	185.4	182.7

	201	199	197	195
2	40.2	39.8	39.4	39.0
3	60.3	59.7	59.1	58.5
4	80.4	79.6	78.8	78.0
5	100.5	99.5	98.5	97.5
6	120.6	119.4	118.2	117.0
7	140.7	139.3	137.9	136.5
8	160.8	159.2	157.6	156.0
9	180.9	179.1	177.3	175.5

	193	192	190	188
2	38.6	38.4	38.0	37.6
3	57.9	57.6	57.0	56.4
4	77.2	76.8	76.0	75.2
5	96.5	96.0	95.0	94.0
6	115.8	115.2	114.0	112.8
7	135.1	134.4	133.0	131.6
8	154.4	153.6	152.0	150.4
9	173.7	172.8	171.0	169.2

	186	184	182	181
2	37.2	36.8	36.4	36.2
3	55.8	55.2	54.6	54.3
4	74.4	73.6	72.8	72.4
5	93.0	92.0	91.0	90.5
6	111.6	110.4	109.2	108.6
7	130.2	128.8	127.4	126.7
8	148.8	147.2	145.6	144.8
9	167.4	165.6	163.8	162.9

Prop. Pts.

4° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.84 358	181	8.84 464	182	1.15 536	9.99 894	**60**
1	8.84 539	179	8.84 646	180	1.15 354	9.99 893	59
2	8.84 718	179	8.84 826	180	1.15 174	9.99 892	58
3	8.84 897	178	8.85 006	179	1.14 994	9.99 891	57
4	8.85 075	177	8.85 185	178	1.14 815	9.99 891	56
5	8.85 252	177	8.85 363	177	1.14 637	9.99 890	**55**
6	8.85 429	176	8.85 540	177	1.14 460	9.99 889	54
7	8.85 605	175	8.85 717	176	1.14 283	9.99 888	53
8	8 85 780	175	8.85 893	176	1.14 107	9.99 887	52
9	8.85 955	173	8.86 069	174	1.13 931	9.99 886	51
10	8.86 128	173	8.86 243	174	1.13 757	9.99 885	**50**
11	8.86 301	173	8.86 417	174	1.13 583	9.99 884	49
12	8.86 474	171	8.86 591	172	1.13 409	9.99 883	48
13	8.86 645	171	8.86 763	172	1.13 237	9.99 882	47
14	8.86 816	171	8.86 935	171	1.13 065	9.99 881	46
15	8.86 987	169	8.87 106	171	1.12 894	9.99 880	**45**
16	8.87 156	169	8.87 277	170	1.12 723	9.99 879	44
17	8.87 325	169	8.87 447	169	1.12 553	9.99 879	43
18	8.87 494	167	8.87 616	169	1.12 384	9.99 878	42
19	8.87 661	168	8.87 785	168	1.12 215	9.99 877	41
20	8.87 829	166	8.87 953	167	1.12 047	9.99 876	**40**
21	8.87 995	166	8.88 120	167	1.11 880	9.99 875	39
22	8.88 161	165	8.88 287	166	1.11 713	9.99 874	38
23	8.88 326	164	8.88 453	165	1.11 547	9.99 873	37
24	8.88 490	164	8.88 618	165	1.11 382	9.99 872	36
25	8.88 654	163	8.88 783	165	1.11 217	9.99 871	**35**
26	8.88 817	163	8.88 948	163	1.11 052	9.99 870	34
27	8.88 980	162	8.89 111	163	1.10 889	9.99 869	33
28	8.89 142	162	8.89 274	163	1.10 726	9.99 868	32
29	8.89 304	160	8.89 437	161	1.10 563	9.99 867	31
30	8.89 464	161	8.89 598	162	1.10 402	9.99 866	**30**
31	8.89 625	159	8.89 760	160	1.10 240	9.99 865	29
32	8.89 784	159	8.89 920	160	1.10 080	9.99 864	28
33	8.89 943	159	8.90 080	160	1.09 920	9.99 863	27
34	8.90 102	158	8.90 240	159	1.09 760	9.99 862	26
35	8.90 260	157	8.90 399	158	1.09 601	9.99 861	**25**
36	8.90 417	157	8.90 557	158	1.09 443	9.99 860	24
37	8.90 574	156	8.90 715	157	1.09 285	9.99 859	23
38	8.90 730	155	8.90 872	157	1.09 128	9.99 858	22
39	8.90 885	155	8.91 029	156	1.08 971	9.99 857	21
40	8.91 040	155	8.91 185	155	1.08 815	9.99 856	**20**
41	8.91 195	154	8.91 340	155	1.08 660	9.99 855	19
42	8.91 349	153	8.91 495	155	1.08 505	9.99 854	18
43	8.91 502	153	8.91 650	153	1.08 350	9.99 853	17
44	8.91 655	152	8.91 803	154	1.08 197	9.99 852	16
45	8.91 807	152	8.91 957	153	1.08 043	9.99 851	**15**
46	8.91 959	151	8.92 110	152	1.07 890	9.99 850	14
47	8.92 110	151	8.92 262	152	1.07 738	9.99 848	13
48	8.92 261	150	8.92 414	151	1.07 586	9.99 847	12
49	8.92 411	150	8.92 565	151	1.07 435	9.99 846	11
50	8.92 561	149	8.92 716	150	1.07 284	9.99 845	**10**
51	8.92 710	149	8.92 866	150	1.07 134	9.99 844	9
52	8.92 859	148	8.93 016	149	1.06 984	9.99 843	8
53	8.93 007	147	8.93 165	148	1.06 835	9.99 842	7
54	8.93 154	147	8.93 313	149	1.06 687	9.99 841	6
55	8.93 301	147	8.93 462	147	1.06 538	9.99 840	**5**
56	8.93 448	146	8.93 609	147	1.06 391	9.99 839	4
57	8.93 594	146	8.93 756	147	1.06 244	9.99 838	3
58	8.93 740	145	8.93 903	146	1.06 097	9.99 837	2
59	8.93 885	145	8.94 049	146	1.05 951	9.99 836	1
60	8.94 030		8.94 195		1.05 805	9.99 834	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	182	181	180	179
2	36.4	36.2	36.0	35.8
3	54.6	54.3	54.0	53.7
4	72.8	72.4	72.0	71.6
5	91.0	90.5	90.0	89.5
6	109.2	108.6	108.0	107.4
7	127.4	126.7	126.0	125.3
8	145.6	144.8	144.0	143.2
9	163.8	162.9	162.0	161.1

	178	177	176	175
2	35.6	35.4	35.2	35.0
3	53.4	53.1	52.8	52.5
4	71.2	70.8	70.4	70.0
5	89.0	88.5	88.0	87.5
6	106.8	106.2	105.6	105.0
7	124.6	123.9	123.2	122.5
8	142.4	141.6	140.8	140.0
9	160.2	159.3	158.4	157.5

	174	173	172	171
2	34.8	34.6	34.4	34.2
3	52.2	51.9	51.6	51.3
4	69.6	69.2	68.8	68.4
5	87.0	86.5	86.0	85.5
6	104.4	103.8	103.2	102.6
7	121.8	121.1	120.4	119.7
8	139.2	138.4	137.6	136.8
9	156.6	155.7	154.8	153.9

	170	169	168	167
2	34.0	33.8	33.6	33.4
3	51.0	50.7	50.4	50.1
4	68.0	67.6	67.2	66.8
5	85.0	84.5	84.0	83.5
6	102.0	101.4	100.8	100.2
7	119.0	118.3	117.6	116.9
8	136.0	135.2	134.4	133.6
9	153.0	152.1	151.2	150.3

	166	165	164	163
2	33.2	33.0	32.8	32.6
3	49.8	49.5	49.2	48.9
4	66.4	66.0	65.6	65.2
5	83.0	82.5	82.0	81.5
6	99.6	99.0	98.4	97.8
7	116.2	115.5	114.8	114.1
8	132.8	132.0	131.2	130.4
9	149.4	148.5	147.6	146.7

	162	161	160	159
2	32.4	32.2	32.0	31.8
3	48.6	48.3	48.0	47.7
4	64.8	64.4	64.0	63.6
5	81.0	80.5	80.0	79.5
6	97.2	96.6	96.0	95.4
7	113.4	112.7	112.0	111.3
8	129.6	128.8	128.0	127.2
9	145.8	144.9	144.0	143.1

	158	157	156	155
2	31.6	31.4	31.2	31.0
3	47.4	47.1	46.8	46.5
4	63.2	62.8	62.4	62.0
5	79.0	78.5	78.0	77.5
6	94.8	94.2	93.6	93.0
7	110.6	109.9	109.2	108.5
8	126.4	125.6	124.8	124.0
9	142.2	141.3	140.4	139.5

	154	153	152	151
2	30.8	30.6	30.4	30.2
3	46.2	45.9	45.6	45.3
4	61.6	61.2	60.8	60.4
5	77.0	76.5	76.0	75.5
6	92.4	91.8	91.2	90.6
7	107.8	107.1	106.4	105.7
8	123.2	122.4	121.6	120.8
9	138.6	137.7	136.8	135.9

Prop. Pts.

85° — Logarithms of Trigonometric Functions

5° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.94 030	144	8.94 195	145	1.05 805	9.99 834	60
1	8.94 174	143	8.94 340	145	1.05 660	9.99 833	59
2	8.94 317	144	8.94 485	145	1.05 515	9.99 832	58
3	8.94 461	142	8.94 630	143	1.05 370	9.99 831	57
4	8.94 603	143	8.94 773	144	1.05 227	9.99 830	56
5	8.94 746	141	8.94 917	143	1.05 083	9.99 829	55
6	8.94 887	142	8.95 060	142	1.04 940	9.99 828	54
7	8.95 029	141	8.95 202	142	1.04 798	9.99 827	53
8	8.95 170	140	8.95 344	142	1.04 656	9.99 825	52
9	8.95 310	140	8.95 486	141	1.04 514	9.99 824	51
10	8.95 450	139	8.95 627	140	1.04 373	9.99 823	50
11	8.95 589	139	8.95 767	141	1.04 233	9.99 822	49
12	8.95 728	139	8.95 908	139	1.04 092	9.99 821	48
13	8.95 867	138	8.96 047	140	1.03 953	9.99 820	47
14	8.96 005	138	8.96 187	138	1.03 813	9.99 819	46
15	8.96 143	137	8.96 325	139	1.03 675	9.99 817	45
16	8.96 280	137	8.96 464	138	1.03 536	9.99 816	44
17	8.96 417	136	8.96 602	137	1.03 398	9.99 815	43
18	8.96 553	136	8.96 739	138	1.03 261	9.99 814	42
19	8.96 689	136	8.96 877	136	1.03 123	9.99 813	41
20	8.96 825	135	8.97 013	137	1.02 987	9.99 812	40
21	8.96 960	135	8.97 150	135	1.02 850	9.99 810	39
22	8.97 095	134	8.97 285	136	1.02 715	9.99 809	38
23	8.97 229	134	8.97 421	135	1.02 579	9.99 808	37
24	8.97 363	133	8.97 556	135	1.02 444	9.99 807	36
25	8.97 496	133	8.97 691	134	1.02 309	9.99 806	35
26	8.97 629	133	8.97 825	134	1.02 175	9.99 804	34
27	8.97 762	132	8.97 959	133	1.02 041	9.99 803	33
28	8.97 894	132	8.98 092	133	1.01 908	9.99 802	32
29	8.98 026	131	8.98 225	133	1.01 775	9.99 801	31
30	8.98 157	131	8.98 358	132	1.01 642	9.99 800	30
31	8.98 288	131	8.98 490	132	1.01 510	9.99 798	29
32	8.98 419	130	8.98 622	131	1.01 378	9.99 797	28
33	8.98 549	130	8.98 753	131	1.01 247	9.99 796	27
34	8.98 679	129	8.98 884	131	1.01 116	9.99 795	26
35	8.98 808	129	8.99 015	130	1.00 985	9.99 793	25
36	8.98 937	129	8.99 145	130	1.00 855	9.99 792	24
37	8.99 066	128	8.99 275	130	1.00 725	9.99 791	23
38	8.99 194	128	8.99 405	129	1.00 595	9.99 790	22
39	8.99 322	128	8.99 534	128	1.00 466	9.99 788	21
40	8.99 450	127	8.99 662	129	1.00 338	9.99 787	20
41	8.99 577	127	8.99 791	128	1.00 209	9.99 786	19
42	8.99 704	126	8.99 919	127	1.00 081	9.99 785	18
43	8.99 830	126	9.00 046	128	0.99 954	9.99 783	17
44	8.99 956	126	9.00 174	127	0.99 826	9.99 782	16
45	9.00 082	125	9.00 301	126	0.99 699	9.99 781	15
46	9.00 207	125	9.00 427	126	0.99 573	9.99 780	14
47	9.00 332	124	9.00 553	126	0.99 447	9.99 778	13
48	9.00 456	125	9.00 679	126	0.99 321	9.99 777	12
49	9.00 581	123	9.00 805	125	0.99 195	9.99 776	11
50	9.00 704	124	9.00 930	125	0.99 070	9.99 775	10
51	9.00 828	123	9.01 055	124	0.98 945	9.99 773	9
52	9.00 951	123	9.01 179	124	0.98 821	9.99 772	8
53	9.01 074	122	9.01 303	124	0.98 697	9.99 771	7
54	9.01 196	122	9.01 427	123	0.98 573	9.99 769	6
55	9.01 318	122	9.01 550	123	0.98 450	9.99 768	5
56	9.01 440	121	9.01 673	123	0.98 327	9.99 767	4
57	9.01 561	121	9.01 796	122	0.98 204	9.99 765	3
58	9.01 682	121	9.01 918	122	0.98 082	9.99 764	2
59	9.01 803	120	9.02 040	122	0.97 960	9.99 763	1
60	9.01 923		9.02 162		0.97 838	9.99 761	0
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	150	149	148	147
2	30.0	29.8	29.6	29.4
3	45.0	44.7	44.4	44.1
4	60.0	59.6	59.2	58.8
5	75.0	74.5	74.0	73.5
6	90.0	89.4	88.8	88.2
7	105.0	104.3	103.6	102.9
8	120.0	119.2	118.4	117.6
9	135.0	134.1	133.2	132.3

	146	145	144	143
2	29.2	29.0	28.8	28.6
3	43.8	43.5	43.2	42.9
4	58.4	58.0	57.6	57.2
5	73.0	72.5	72.0	71.5
6	87.6	87.0	86.4	85.8
7	102.2	101.5	100.8	100.1
8	116.8	116.0	115.2	114.4
9	131.4	130.5	129.6	128.7

	142	141	140	139
2	28.4	28.2	28.0	27.8
3	42.6	42.3	42.0	41.7
4	56.8	56.4	56.0	55.6
5	71.0	70.5	70.0	69.5
6	85.2	84.6	84.0	83.4
7	99.4	98.7	98.0	97.3
8	113.6	112.8	112.0	111.2
9	127.8	126.9	126.0	125.1

	138	137	136	135
2	27.6	27.4	27.2	27.0
3	41.4	41.1	40.8	40.5
4	55.2	54.8	54.4	54.0
5	69.0	68.5	68.0	67.5
6	82.8	82.2	81.6	81.0
7	96.6	95.9	95.2	94.5
8	110.4	109.6	108.8	108.0
9	124.2	123.3	122.4	121.5

	134	133	132	131
2	26.8	26.6	26.4	26.2
3	40.2	39.9	39.6	39.3
4	53.6	53.2	52.8	52.4
5	67.0	66.5	66.0	65.5
6	80.4	79.8	79.2	78.6
7	93.8	93.1	92.4	91.7
8	107.2	106.4	105.6	104.8
9	120.6	119.7	118.8	117.9

	130	129	128	127
2	26.0	25.8	25.6	25.4
3	39.0	38.7	38.4	38.1
4	52.0	51.6	51.2	50.8
5	65.0	64.5	64.0	63.5
6	78.0	77.4	76.8	76.2
7	91.0	90.3	89.6	88.9
8	104.0	103.2	102.4	101.6
9	117.0	116.1	115.2	114.3

	126	125	124	123
2	25.2	25.0	24.8	24.6
3	37.8	37.5	37.2	36.9
4	50.4	50.0	49.6	49.2
5	63.0	62.5	62.0	61.5
6	75.6	75.0	74.4	73.8
7	88.2	87.5	86.8	86.1
8	100.8	100.0	99.2	98.4
9	113.4	112.5	111.6	110.7

	122	121	120
2	24.4	24.2	24.0
3	36.6	36.3	36.0
4	48.8	48.4	48.0
5	61.0	60.5	60.0
6	73.2	72.6	72.0
7	85.4	84.7	84.0
8	97.6	96.8	96.0
9	109.8	108.9	108.0

Prop. Pts.

84° — Logarithms of Trigonometric Functions

6° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	9.01 923	120	9.02 162	121	0.97 838	9.99 761	**60**
1	9.02 043	120	9.02 283	121	0.97 717	9.99 760	59
2	9.02 163	120	9.02 404	121	0.97 596	9.99 759	58
3	9.02 283	119	9.02 525	120	0.97 475	9.99 757	57
4	9.02 402	118	9.02 645	121	0.97 355	9.99 756	56
5	9.02 520	119	9.02 766	119	0.97 234	9.99 755	**55**
6	9.02 639	118	9.02 885	120	0.97 115	9.99 753	54
7	9.02 757	117	9.03 005	119	0.96 995	9.99 752	53
8	9.02 874	118	9.03 124	118	0.96 876	9.99 751	52
9	9.02 992	117	9.03 242	119	0.96 758	9.99 749	51
10	9.03 109	117	9.03 361	118	0.96 639	9.99 748	**50**
11	9.03 226	116	9.03 479	118	0.96 521	9.99 747	49
12	9.03 342	116	9.03 597	117	0.96 403	9.99 745	48
13	9.03 458	116	9.03 714	118	0.96 286	9.99 744	47
14	9.03 574	116	9.03 832	116	0.96 168	9.99 742	46
15	9.03 690	115	9.03 948	117	0.96 052	9.99 741	**45**
16	9.03 805	115	9.04 065	116	0.95 935	9.99 740	44
17	9.03 920	114	9.04 181	116	0.95 819	9.99 738	43
18	9.04 034	115	9.04 297	116	0.95 703	9.99 737	42
19	9.04 149	113	9.04 413	115	0.95 587	9.99 736	41
20	9.04 262	114	9.04 528	115	0.95 472	9.99 734	**40**
21	9.04 376	114	9.04 643	115	0.95 357	9.99 733	39
22	9.04 490	113	9.04 758	115	0.95 242	9.99 731	38
23	9.04 603	112	9.04 873	114	0.95 127	9.99 730	37
24	9.04 715	113	9.04 987	114	0.95 013	9.99 728	36
25	9.04 828	112	9.05 101	113	0.94 899	9.99 727	**35**
26	9.04 940	112	9.05 214	114	0.94 786	9.99 726	34
27	9.05 052	112	9.05 328	113	0.94 672	9.99 724	33
28	9.05 164	111	9.05 441	112	0.94 559	9.99 723	32
29	9.05 275	111	9.05 553	113	0.94 447	9.99 721	31
30	9.05 386	111	9.05 666	112	0.94 334	9.99 720	**30**
31	9.05 497	110	9.05 778	112	0.94 222	9.99 718	29
32	9.05 607	110	9.05 890	112	0.94 110	9.99 717	28
33	9.05 717	110	9.06 002	111	0.93 998	9.99 716	27
34	9.05 827	110	9.06 113	111	0.93 887	9.99 714	26
35	9.05 937	109	9.06 224	111	0.93 776	9.99 713	**25**
36	9.06 046	109	9.06 335	110	0.93 665	9.99 711	24
37	9.06 155	109	9.06 445	111	0.93 555	9.99 710	23
38	9.06 264	108	9.06 556	110	0.93 444	9.99 708	22
39	9.06 372	109	9.06 666	109	0.93 334	9.99 707	21
40	9.06 481	108	9.06 775	110	0.93 225	9.99 705	**20**
41	9.06 589	107	9.06 885	109	0.93 115	9.99 704	19
42	9.06 696	108	9.06 994	109	0.93 006	9.99 702	18
43	9.06 804	107	9.07 103	108	0.92 897	9.99 701	17
44	9.06 911	107	9.07 211	109	0.92 789	9.99 699	16
45	9.07 018	106	9.07 320	108	0.92 680	9.99 698	**15**
46	9.07 124	107	9.07 428	108	0.92 572	9.99 696	14
47	9.07 231	106	9.07 536	107	0.92 464	9.99 695	13
48	9.07 337	105	9.07 643	108	0.92 357	9.99 693	12
49	9.07 442	106	9.07 751	107	0.92 249	9.99 692	11
50	9.07 548	105	9.07 858	106	0.92 142	9.99 690	**10**
51	9.07 653	105	9.07 964	107	0.92 036	9.99 689	9
52	9.07 758	105	9.08 071	106	0.91 929	9.99 687	8
53	9.07 863	105	9.08 177	106	0.91 823	9.99 686	7
54	9.07 968	104	9.08 283	106	0.91 717	9.99 684	6
55	9.08 072	104	9.08 389	106	0.91 611	9.99 683	**5**
56	9.08 176	104	9.08 495	105	0.91 505	9.99 681	4
57	9.08 280	103	9.08 600	105	0.91 400	9.99 680	3
58	9.08 383	103	9.08 705	105	0.91 295	9.99 678	2
59	9.08 486	103	9.08 810	104	0.91 190	9.99 677	1
60	9.08 589		9.08 914		0.91 086	9.99 675	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	121	**120**	**119**	**118**
2	24.2	24.0	23.8	23.6
3	36.3	36.0	35.7	35.4
4	48.4	48.0	47.6	47.2
5	60.5	60.0	59.5	59.0
6	72.6	72.0	71.4	70.8
7	84.7	84.0	83.3	82.6
8	96.8	96.0	95.2	94.4
9	108.9	108.0	107.1	106.2

	117	**116**	**115**	**114**
2	23.4	23.2	23.0	22.8
3	35.1	34.8	34.5	34.2
4	46.8	46.4	46.0	45.6
5	58.5	58.0	57.5	57.0
6	70.2	69.6	69.0	68.4
7	81.9	81.2	80.5	79.8
8	93.6	92.8	92.0	91.2
9	105.3	104.4	103.5	102.6

	113	**112**	**111**	**110**
2	22.6	22.4	22.2	22.0
3	33.9	33.6	33.3	33.0
4	45.2	44.8	44.4	44.0
5	56.5	56.0	55.5	55.0
6	67.8	67.2	66.6	66.0
7	79.1	78.4	77.7	77.0
8	90.4	89.6	88.8	88.0
9	101.7	100.8	99.9	99.0

	109	**108**	**107**	**106**
2	21.8	21.6	21.4	21.2
3	32.7	32.4	32.1	31.8
4	43.6	43.2	42.8	42.4
5	54.5	54.0	53.5	53.0
6	65.4	64.8	64.2	63.6
7	76.3	75.6	74.9	74.2
8	87.2	86.4	85.6	84.8
9	98.1	97.2	96.3	95.4

From the top:

For **6°+** or **186°+**, read as printed; for **96°+** or **276°+**, read co-function.

From the bottom:

For **83°+** or **263°+**, read as printed; for **173°+** or **353°+**, read co-function.

Prop. Pts.

83° — Logarithms of Trigonometric Functions

7° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	9.08 589	103	9.08 914	105	0.91 086	9.99 675	**60**
1	9.08 692	103	9.09 019	104	0.90 981	9.99 674	59
2	9.08 795	102	9.09 123	104	0.90 877	9.99 672	58
3	9.08 897	102	9.09 227	103	0.90 773	9.99 670	57
4	9.08 999	102	9.09 330	104	0.90 670	9.99 669	56
5	9.09 101	101	9.09 434	103	0.90 566	9.99 667	**55**
6	9.09 202	102	9.09 537	103	0.90 463	9.99 666	54
7	9.09 304	101	9.09 640	102	0.90 360	9.99 664	53
8	9.09 405	101	9.09 742	103	0.90 258	9.99 663	52
9	9.09 506	100	9.09 845	102	0.90 155	9.99 661	51
10	9.09 606	101	9.09 947	102	0.90 053	9.99 659	**50**
11	9.09 707	100	9.10 049	101	0.89 951	9.99 658	49
12	9.09 807	100	9.10 150	102	0.89 850	9.99 656	48
13	9.09 907	99	9.10 252	101	0.89 748	9.99 655	47
14	9.10 006	100	9.10 353	101	0.89 647	9.99 653	46
15	9.10 106	99	9.10 454	101	0.89 546	9.99 651	**45**
16	9.10 205	99	9.10 555	101	0.89 445	9.99 650	44
17	9.10 304	98	9.10 656	100	0.89 344	9.99 648	43
18	9.10 402	99	9.10 756	100	0.89 244	9.99 647	42
19	9.10 501	98	9.10 856	100	0.89 144	9.99 645	41
20	9.10 599	98	9.10 956	100	0.89 044	9.99 643	**40**
21	9.10 697	98	9.11 056	99	0.88 944	9.99 642	39
22	9.10 795	98	9.11 155	99	0.88 845	9.99 640	38
23	9.10 893	97	9.11 254	99	0.88 746	9.99 638	37
24	9.10 990	97	9.11 353	99	0.88 647	9.99 637	36
25	9.11 087	97	9.11 452	99	0.88 548	9.99 635	**35**
26	9.11 184	97	9.11 551	98	0.88 449	9.99 633	34
27	9.11 281	96	9.11 649	98	0.88 351	9.99 632	33
28	9.11 377	97	9.11 747	98	0.88 253	9.99 630	32
29	9.11 474	96	9.11 845	98	0.88 155	9.99 629	31
30	9.11 570	96	9.11 943	97	0.88 057	9.99 627	**30**
31	9.11 666	95	9.12 040	98	0.87 960	9.99 625	29
32	9.11 761	96	9.12 138	97	0.87 862	9.99 624	28
33	9.11 857	95	9.12 235	97	0.87 765	9.99 622	27
34	9.11 952	95	9.12 332	96	0.87 668	9.99 620	26
35	9.12 047	95	9.12 428	97	0.87 572	9.99 618	**25**
36	9.12 142	94	9.12 525	96	0.87 475	9.99 617	24
37	9.12 236	95	9.12 621	96	0.87 379	9.99 615	23
38	9.12 331	94	9.12 717	96	0.87 283	9.99 613	22
39	9.12 425	94	9.12 813	96	0.87 187	9.99 612	21
40	9.12 519	93	9.12 909	95	0.87 091	9.99 610	**20**
41	9.12 612	94	9.13 004	95	0.86 996	9.99 608	19
42	9.12 706	93	9.13 099	95	0.86 901	9.99 607	18
43	9.12 799	93	9.13 194	95	0.86 806	9.99 605	17
44	9.12 892	93	9.13 289	95	0.86 711	9.99 603	16
45	9.12 985	93	9.13 384	94	0.86 616	9.99 601	**15**
46	9.13 078	93	9.13 478	95	0.86 522	9.99 600	14
47	9.13 171	92	9.13 573	94	0.86 427	9.99 598	13
48	9.13 263	92	9.13 667	94	0.86 333	9.99 596	12
49	9.13 355	92	9.13 761	93	0.86 239	9.99 595	11
50	9.13 447	92	9.13 854	94	0.86 146	9.99 593	**10**
51	9.13 539	91	9.13 948	93	0.86 052	9.99 591	9
52	9.13 630	92	9.14 041	93	0 85 959	9.99 589	8
53	9.13 722	91	9.14 134	93	0.85 866	9.99 588	7
54	9.13 813	91	9.14 227	93	0.85 773	9.99 586	6
55	9.13 904	90	9.14 320	92	0.85 680	9.99 584	**5**
56	9.13 994	91	9.14 412	92	0.85 588	9.99 582	4
57	9.14 085	90	9.14 504	93	0.85 496	9.99 581	3
58	9.14 175	91	9.14 597	91	0.85 403	9.99 579	2
59	9.14 266	90	9.14 688	92	0.85 312	9.99 577	1
60	9.14 356		9.14 780		0.85 220	9.99 575	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	105	**104**	**103**	**102**
2	21.0	20.8	20.6	20.4
3	31.5	31.2	30.9	30.6
4	42.0	41.6	41.2	40.8
5	52.5	52.0	51.5	51.0
6	63.0	62.4	61.8	61.2
7	73.5	72.8	72.1	71.4
8	84.0	83.2	82.4	81.6
9	94.5	93.6	92.7	91.8

	101	**99**	**98**	**97**
2	20.2	19.8	19.6	19.4
3	30.3	29.7	29.4	29.1
4	40.4	39.6	39.2	38.8
5	50.5	49.5	49.0	48.5
6	60.6	59.4	58.8	58.2
7	70.7	69.3	68.6	67.9
8	80.8	79.2	78.4	77.6
9	90.9	89.1	88.2	87.3

	96	**95**	**94**	**93**
2	19.2	19.0	18.8	18.6
3	28.8	28.5	28.2	27.9
4	38.4	38.0	37.6	37.2
5	48.0	47.5	47.0	46.5
6	57.6	57.0	56.4	55.8
7	67.2	66.5	65.8	65.1
8	76.8	76.0	75.2	74.4
9	86.4	85.5	84.6	83.7

	92	**91**	**90**
2	18.4	18.2	18.0
3	27.6	27.3	27.0
4	36.8	36.4	36.0
5	46.0	45.5	45.0
6	55.2	54.6	54.0
7	64.4	63.7	63.0
8	73.6	72.8	72.0
9	82.8	81.9	81.0

From the top:

For **7°+** or **187°+**, read as printed; for **97°+** or **277°+**, read co-function.

From the bottom:

For **82°+** or **262°+**, read as printed; for **172°+** or **352°+**, read co-function.

Prop. Pts.

82° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	9.14 356	89	9.14 780	92	0.85 220	9.99 575	**60**
1	9.14 445	90	9.14 872	91	0.85 128	9.99 574	59
2	9.14 535	89	9.14 963	91	0.85 037	9.99 572	58
3	9.14 624	90	9.15 054	91	0.84 946	9.99 570	57
4	9.14 714	89	9.15 145	91	0.84 855	9.99 568	56
5	9.14 803	88	9.15 236	91	0.84 764	9.99 566	**55**
6	9.14 891	89	9.15 327	90	0.84 673	9.99 565	54
7	9.14 980	89	9.15 417	91	0.84 583	9.99 563	53
8	9.15 069	88	9.15 508	90	0.84 492	9.99 561	52
9	9.15 157	88	9.15 598	90	0.84 402	9.99 559	51
10	9.15 245	88	9.15 688	89	0.84 312	9.99 557	**50**
11	9.15 333	88	9.15 777	90	0.84 223	9.99 556	49
12	9.15 421	87	9.15 867	89	0.84 133	9.99 554	48
13	9.15 508	88	9.15 956	90	0.84 044	9.99 552	47
14	9.15 596	87	9.16 046	89	0.83 954	9.99 550	46
15	9.15 683	87	9.16 135	89	0.83 865	9.99 548	**45**
16	9.15 770	87	9.16 224	88	0.83 776	9.99 546	44
17	9.15 857	87	9.16 312	89	0.83 688	9.99 545	43
18	9.15 944	86	9.16 401	88	0.83 599	9.99 543	42
19	9.16 030	86	9.16 489	88	0.83 511	9.99 541	41
20	9.16 116	87	9.16 577	88	0.83 423	9.99 539	**40**
21	9.16 203	86	9.16 665	88	0.83 335	9.99 537	39
22	9.16 289	85	9.16 753	88	0.83 247	9.99 535	38
23	9.16 374	86	9.16 841	87	0.83 159	9.99 533	37
24	9.16 460	85	9.16 928	88	0.83 072	9.99 532	36
25	9.16 545	86	9.17 016	87	0.82 984	9.99 530	**35**
26	9.16 631	85	9.17 103	87	0.82 897	9.99 528	34
27	9.16 716	85	9.17 190	87	0.82 810	9.99 526	33
28	9.16 801	85	9.17 277	86	0.82 723	9.99 524	32
29	9.16 886	84	9.17 363	87	0.82 637	9.99 522	31
30	9.16 970	85	9.17 450	86	0.82 550	9.99 520	**30**
31	9.17 055	84	9.17 536	86	0.82 464	9.99 518	29
32	9.17 139	84	9.17 622	86	0.82 378	9.99 517	28
33	9.17 223	84	9.17 708	86	0.82 292	9.99 515	27
34	9.17 307	84	9.17 794	86	0.82 206	9.99 513	26
35	9.17 391	83	9.17 880	85	0.82 120	9.99 511	**25**
36	9.17 474	84	9.17 965	86	0.82 035	9.99 509	24
37	9.17 558	83	9.18 051	85	0.81 949	9.99 507	23
38	9.17 641	83	9.18 136	85	0.81 864	9.99 505	22
39	9.17 724	83	9.18 221	85	0.81 779	9.99 503	21
40	9.17 807	83	9.18 306	85	0.81 694	9.99 501	**20**
41	9.17 890	83	9.18 391	84	0.81 609	9.99 499	19
42	9.17 973	82	9.18 475	85	0.81 525	9.99 497	18
43	9.18 055	82	9.18 560	84	0.81 440	9.99 495	17
44	9.18 137	83	9.18 644	84	0.81 356	9.99 494	16
45	9.18 220	82	9.18 728	84	0.81 272	9.99 492	**15**
46	9.18 302	81	9.18 812	84	0.81 188	9.99 490	14
47	9.18 383	82	9.18 896	83	0.81 104	9.99 488	13
48	9.18 465	82	9.18 979	84	0.81 021	9.99 486	12
49	9.18 547	81	9.19 063	83	0.80 937	9.99 484	11
50	9.18 628	81	9.19 146	83	0.80 854	9.99 482	**10**
51	9.18 709	81	9.19 229	83	0.80 771	9.99 480	9
52	9.18 790	81	9.19 312	83	0.80 688	9.99 478	8
53	9.18 871	81	9.19 395	83	0.80 605	9.99 476	7
54	9.18 952	81	9.19 478	83	0.80 522	9.99 474	6
55	9.19 033	80	9.19 561	82	0.80 439	9.99 472	**5**
56	9.19 113	80	9.19 643	82	0.80 357	9.99 470	4
57	9.19 193	80	9.19 725	82	0.80 275	9.99 468	3
58	9.19 273	80	9.19 807	82	0.80 193	9.99 466	2
59	9.19 353	80	9.19 889	82	0.80 111	9.99 464	1
60	9.19 433		9.19 971		0.80 029	9.99 462	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	92	91	90	89
2	18.4	18.2	18.0	17.8
3	27.6	27.3	27.0	26.7
4	36.8	36.4	36.0	35.6
5	46.0	45.5	45.0	44.5
6	55.2	54.6	54.0	53.4
7	64.4	63.7	63.0	62.3
8	73.6	72.8	72.0	71.2
9	82.8	81.9	81.0	80.1

	88	87	86
2	17.6	17.4	17.2
3	26.4	26.1	25.8
4	35.2	34.8	34.4
5	44.0	43.5	43.0
6	52.8	52.2	51.6
7	61.6	60.9	60.2
8	70.4	69.6	68.8
9	79.2	78.3	77.4

	85	84	83
2	17.0	16.8	16.6
3	25.5	25.2	24.9
4	34.0	33.6	33.2
5	42.5	42.0	41.5
6	51.0	50.4	49.8
7	59.5	58.8	58.1
8	68.0	67.2	66.4
9	76.5	75.6	74.7

	82	81	80
2	16.4	16.2	16.0
3	24.6	24.3	24.0
4	32.8	32.4	32.0
5	41.0	40.5	40.0
6	49.2	48.6	48.0
7	57.4	56.7	56.0
8	65.6	64.8	64.0
9	73.8	72.9	72.0

From the top :

For **8°+** or **188°+**, read as printed ; for **98°+** or **278°+**, read co-function.

From the bottom :

For **81°+** or **261°+**, read as printed ; for **171°+** or **351°+**, read co-function.

Prop. Pts.

9° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	9.19 433	80	9.19 971	82	0.80 029	9.99 462	**60**
1	9.19 513	79	9.20 053	81	0.79 947	9.99 460	59
2	9.19 592	80	9.20 134	82	0.79 866	9.99 458	58
3	9.19 672	79	9.20 216	81	0.79 784	9.99 456	57
4	9.19 751	79	9.20 297	81	0.79 703	9.99 454	56
5	9.19 830	79	9.20 378	81	0.79 622	9.99 452	**55**
6	9.19 909	79	9.20 459	81	0.79 541	9.99 450	54
7	9.19 988	79	9.20 540	81	0.79 460	9.99 448	53
8	9.20 067	78	9.20 621	80	0.79 379	9.99 446	52
9	9.20 145	78	9.20 701	81	0.79 299	9.99 444	51
10	9.20 223	79	9.20 782	80	0.79 218	9.99 442	**50**
11	9.20 302	78	9.20 862	80	0.79 138	9.99 440	49
12	9.20 380	78	9.20 942	80	0.79 058	9.99 438	48
13	9.20 458	77	9.21 022	80	0.78 978	9.99 436	47
14	9.20 535	78	9.21 102	80	0.78 898	9.99 434	46
15	9.20 613	78	9.21 182	79	0.78 818	9.99 432	**45**
16	9.20 691	77	9.21 261	80	0.78 739	9.99 429	44
17	9.20 768	77	9.21 341	79	0.78 659	9.99 427	43
18	9.20 845	77	9.21 420	79	0.78 580	9.99 425	42
19	9.20 922	77	9.21 499	79	0.78 501	9.99 423	41
20	9.20 999	77	9.21 578	79	0.78 422	9.99 421	**40**
21	9.21 076	77	9.21 657	79	0.78 343	9.99 419	39
22	9.21 153	76	9.21 736	78	0.78 264	9.99 417	38
23	9.21 229	77	9.21 814	79	0.78 186	9.99 415	37
24	9.21 306	76	9.21 893	78	0.78 107	9.99 413	36
25	9.21 382	76	9.21 971	78	0.78 029	9.99 411	**35**
26	9.21 458	76	9.22 049	78	0.77 951	9.99 409	34
27	9.21 534	76	9.22 127	78	0.77 873	9.99 407	33
28	9.21 610	75	9.22 205	78	0.77 795	9.99 404	32
29	9.21 685	76	9.22 283	78	0.77 717	9.99 402	31
30	9.21 761	75	9.22 361	77	0.77 639	9.99 400	**30**
31	9.21 836	76	9.22 438	78	0.77 562	9.99 398	29
32	9.21 912	75	9.22 516	77	0.77 484	9.99 396	28
33	9.21 987	75	9.22 593	77	0.77 407	9.99 394	27
34	9.22 062	75	9.22 670	77	0.77 330	9.99 392	26
35	9.22 137	74	9.22 747	77	0.77 253	9.99 390	**25**
36	9.22 211	75	9.22 824	77	0.77 176	9.99 388	24
37	9.22 286	75	9.22 901	76	0.77 099	9.99 385	23
38	9.22 361	74	9.22 977	77	0.77 023	9.99 383	22
39	9.22 435	74	9.23 054	76	0.76 946	9.99 381	21
40	9.22 509	74	9.23 130	76	0.76 870	9.99 379	**20**
41	9.22 583	74	9.23 206	77	0.76 794	9.99 377	19
42	9.22 657	74	9.23 283	76	0.76 717	9.99 375	18
43	9.22 731	74	9.23 359	76	0.76 641	9.99 372	17
44	9.22 805	73	9.23 435	75	0.76 565	9.99 370	16
45	9.22 878	74	9.23 510	76	0.76 490	9.99 368	**15**
46	9.22 952	73	9.23 586	75	0.76 414	9.99 366	14
47	9.23 025	73	9.23 661	76	0.76 339	9.99 364	13
48	9.23 098	73	9.23 737	75	0.76 263	9.99 362	12
49	9.23 171	73	9.23 812	75	0.76 188	9.99 359	11
50	9.23 244	73	9.23 887	75	0.76 113	9.99 357	**10**
51	9.23 317	73	9.23 962	75	0.76 038	9.99 355	9
52	9.23 390	72	9.24 037	75	0.75 963	9.99 353	8
53	9.23 462	73	9.24 112	74	0.75 888	9.99 351	7
54	9.23 535	72	9.24 186	75	0.75 814	9.99 348	6
55	9.23 607	72	9.24 261	74	0.75 739	9.99 346	**5**
56	9.23 679	73	9.24 335	75	0.75 665	9.99 344	4
57	9.23 752	71	9.24 410	74	0.75 590	9.99 342	3
58	9.23 823	72	9.24 484	74	0.75 516	9.99 340	2
59	9.23 895	72	9.24 558	74	0.75 442	9.99 337	1
60	9.23 967		9.24 632		0.75 368	9.99 335	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	82	**81**	**80**	**79**
2	16.4	16.2	16.0	15.8
3	24.6	24.3	24.0	23.7
4	32.8	32.4	32.0	31.6
5	41.0	40.5	40.0	39.5
6	49.2	48.6	48.0	47.4
7	57.4	56.7	56.0	55.3
8	65.6	64.8	64.0	63.2
9	73.8	72.9	72.0	71.1

	78	**77**	**76**	**75**
2	15.6	15.4	15.2	15.0
3	23.4	23.1	22.8	22.5
4	31.2	30.8	30.4	30.0
5	39.0	38.5	38.0	37.5
6	46.8	46.2	45.6	45.0
7	54.6	53.9	53.2	52.5
8	62.4	61.6	60.8	60.0
9	70.2	69.3	68.4	67.5

	74	**73**	**72**	**71**
2	14.8	14.6	14.4	14.2
3	22.2	21.9	21.6	21.3
4	29.6	29.2	28.8	28.4
5	37.0	36.5	36.0	35.5
6	44.4	43.8	43.2	42.6
7	51.8	51.1	50.4	49.7
8	59.2	58.4	57.6	56.8
9	66.6	65.7	64.8	63.9

From the top:

For **9°+**, or **189°+**, read as printed; for **99°+** or **279°+**, read co-function.

From the bottom:

For **80°+** or **260°+**, read as printed; for **170°+** or **350°+**, read co-function.

Prop. Pts.

80° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.23 967	72	9.24 632	74	0.75 368	9.99 335	2	**60**
1	9.24 039	71	9.24 706	73	0.75 294	9.99 333	2	59
2	9.24 110	71	9.24 779	74	0.75 221	9.99 331	3	58
3	9.24 181	72	9.24 853	73	0.75 147	9.99 328	2	57
4	9.24 253	71	9.24 926	74	0.75 074	9.99 326	2	56
5	9.24 324	71	9.25 000	73	0.75 000	9.99 324	2	**55**
6	9.24 395	71	9.25 073	73	0.74 927	9.99 322	3	54
7	9.24 466	70	9.25 146	73	0.74 854	9.99 319	2	53
8	9.24 536	71	9.25 219	73	0.74 781	9.99 317	2	52
9	9.24 607	70	9.25 292	73	0.74 708	9.99 315	2	51
10	9.24 677	71	9.25 365	72	0.74 635	9.99 313	3	**50**
11	9.24 748	70	9.25 437	73	0.74 563	9.99 310	2	49
12	9.24 818	70	9.25 510	72	0.74 490	9.99 308	2	48
13	9.24 888	70	9.25 582	73	0.74 418	9.99 306	2	47
14	9.24 958	70	9.25 655	72	0.74 345	9.99 304	3	46
15	9.25 028	70	9.25 727	72	0.74 273	9.99 301	2	**45**
16	9.25 098	70	9.25 799	72	0.74 201	9.99 299	2	44
17	9.25 168	69	9.25 871	72	0.74 129	9.99 297	3	43
18	9.25 237	70	9.25 943	72	0.74 057	9.99 294	2	42
19	9.25 307	69	9.26 015	71	0.73 985	9.99 292	2	41
20	9.25 376	69	9.26 086	72	0.73 914	9.99 290	2	**40**
21	9.25 445	69	9.26 158	71	0.73 842	9.99 288	3	39
22	9.25 514	69	9.26 229	72	0.73 771	9.99 285	2	38
23	9.25 583	69	9.26 301	71	0.73 699	9.99 283	2	37
24	9.25 652	69	6.26 372	71	0.73 628	9.99 281	3	36
25	9.25 721	69	9.26 443	71	0.73 557	9.99 278	2	**35**
26	9.25 790	68	9.26 514	71	0.73 486	9.99 276	2	34
27	9.25 858	69	9.26 585	70	0.73 415	9.99 274	3	33
28	9.25 927	68	9.26 655	71	0.73 345	9.99 271	2	32
29	9.25 995	68	9.26 726	71	0.73 274	9.99 269	2	31
30	9.26 063	68	9.26 797	70	0.73 203	9.99 267	3	**30**
31	9.26 131	68	9.26 867	70	0.73 133	9.99 264	2	29
32	9.26 199	68	9.26 937	71	0.73 063	9.99 262	2	28
33	9.26 267	68	9.27 008	70	0.72 992	9.99 260	3	27
34	9.26 335	68	9.27 078	70	0.72 922	9.99 257	2	26
35	9.26 403	67	9.27 148	70	0.72 852	9.99 255	3	**25**
36	9.26 470	68	9.27 218	70	0.72 782	9.99 252	2	24
37	9.26 538	67	9.27 288	69	0.72 712	9.99 250	2	23
38	9.26 605	67	9.27 357	70	0.72 643	9.99 248	3	22
39	9.26 672	67	9.27 427	69	0.72 573	9.99 245	2	21
40	9.26 739	67	9.27 496	70	0.72 504	9.99 243	2	**20**
41	9.26 806	67	9.27 566	69	0.72 434	9.99 241	3	19
42	9.26 873	67	9.27 635	69	0.72 365	9.99 238	2	18
43	9.26 940	67	9.27 704	69	0.72 296	9.99 236	3	17
44	9.27 007	66	9.27 773	69	0.72 227	9.99 233	2	16
45	9.27 073	67	9.27 842	69	0.72 158	9.99 231	2	**15**
46	9.27 140	66	9.27 911	69	0.72 089	9.99 229	3	14
47	9.27 206	67	9.27 980	69	0.72 020	9.99 226	2	13
48	9.27 273	66	9.28 049	68	0.71 951	9.99 224	3	12
49	9.27 339	66	9.28 117	69	0.71 883	9.99 221	2	11
50	9.27 405	66	9.28 186	68	0.71 814	9.99 219	2	**10**
51	9.27 471	66	9.28 254	69	0.71 746	9.99 217	3	9
52	9.27 537	65	9.28 323	68	0.71 677	9.99 214	2	8
53	9.27 602	66	9.28 391	68	0.71 609	9.99 212	3	7
54	9.27 668	66	9.28 459	68	0.71 541	9.99 209	2	6
55	9.27 734	65	9.28 527	68	0.71 473	9.99 207	3	**5**
56	9.27 799	65	9.28 595	67	0.71 405	9.99 204	2	4
57	9.27 864	66	9.28 662	68	0.71 338	9.99 202	2	3
58	9.27 930	65	9.28 730	68	0.71 270	9.99 200	3	2
59	9.27 995	65	9.28 798	67	0.71 202	9.99 197	2	1
60	9.28 060		9.28 865		0.71 135	9.99 195		**0**
	L Cos	**d**	**L Ctn**	**c d**	**L Tan**	**L Sin**	**d**	**′**

Prop. Pts.

	74	73	72
2	14.8	14.6	14.4
3	22.2	21.9	21.6
4	29.6	29.2	28.8
5	37.0	36.5	36.0
6	44.4	43.8	43.2
7	51.8	51.1	50.4
8	59.2	58.4	57.6
9	66.6	65.7	64.8

	71	70	69
2	14.2	14.0	13.8
3	21.3	21.0	20.7
4	28.4	28.0	27.6
5	35.5	35.0	34.5
6	42.6	42.0	41.4
7	49.7	49.0	48.3
8	56.8	56.0	55.2
9	63.9	63.0	62.1

	68	67	66
2	13.6	13.4	13.2
3	20.4	20.1	19.8
4	27.2	26.8	26.4
5	34.0	33.5	33.0
6	40.8	40.2	39.6
7	47.6	46.9	46.2
8	54.4	53.6	52.8
9	61.2	60.3	59.4

	65	3
2	13.0	0.6
3	19.5	0.9
4	26.0	1.2
5	32.5	1.5
6	39.0	1.8
7	45.5	2.1
8	52.0	2.4
9	58.5	2.7

From the top:

For **10°+** or **190°+**, read as printed; for **100°+** or **280°+**, read co-function.

From the bottom:

For **79°+** or **259°+**, read as printed; for **169°+** or **349°+**, read co-function.

Prop. Pts.

79° — Logarithms of Trigonometric Functions

11° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.28 060	65	9.28 865	68	0.71 135	9.99 195	3	**60**
1	9.28 125	65	9.28 933	67	0.71 067	9.99 192	2	59
2	9.28 190	64	9.29 000	67	0.71 000	9.99 190	3	58
3	9.28 254	65	9.29 067	67	0.70 933	9.99 187	2	57
4	9.28 319	65	9.29 134	67	0.70 866	9.99 185	3	56
5	9.28 384	64	9.29 201	67	0.70 799	9.99 182	2	**55**
6	9.28 448	64	9.29 268	67	0.70 732	9.99 180	3	54
7	9.28 512	65	9.29 335	67	0.70 665	9.99 177	2	53
8	9.28 577	64	9.29 402	66	0.70 598	9.99 175	3	52
9	9.28 641	64	9.29 468	67	0.70 532	9.99 172	2	51
10	9.28 705	64	9.29 535	66	0.70 465	9.99 170	3	**50**
11	9.28 769	64	9.29 601	67	0.70 399	9.99 167	2	49
12	9.28 833	63	9.29 668	66	0.70 332	9.99 165	3	48
13	9.28 896	64	9.29 734	66	0.70 266	9.99 162	2	47
14	9.28 960	64	9.29 800	66	0.70 200	9.99 160	3	46
15	9.29 024	63	9.29 866	66	0.70 134	9.99 157	2	**45**
16	9.29 087	63	9.29 932	66	0.70 068	9.99 155	3	44
17	9.29 150	64	9.29 998	66	0.70 002	9.99 152	2	43
18	9.29 214	63	9.30 064	66	0.69 936	9.99 150	3	42
19	9.29 277	63	9.30 130	65	0.69 870	9.99 147	2	41
20	9.29 340	63	9.30 195	66	0.69 805	9.99 145	3	**40**
21	9.29 403	63	9.30 261	65	0.69 739	9.99 142	2	39
22	9.29 466	63	9.30 326	65	0.69 674	9.99 140	3	38
23	9.29 529	62	9.30 391	66	0.69 609	9.99 137	2	37
24	9.29 591	63	9.30 457	65	0.69 543	9.99 135	3	36
25	9.29 654	62	9.30 522	65	0.69 478	9.99 132	2	**35**
26	9.29 716	63	9.30 587	65	0.69 413	9.99 130	3	34
27	9.29 779	62	9.30 652	65	0.69 348	9.99 127	3	33
28	9.29 841	62	9.30 717	65	0.69 283	9.99 124	2	32
29	9.29 903	63	9.30 782	64	0.69 218	9.99 122	3	31
30	9.29 966	62	9.30 846	65	0.69 154	9.99 119	2	**30**
31	9.30 028	62	9.30 911	64	0.69 089	9.99 117	3	29
32	9.30 090	61	9.30 975	65	0.69 025	9.99 114	2	28
33	9.30 151	62	9.31 040	64	0.68 960	9.99 112	3	27
34	9.30 213	62	9.31 104	64	0.68 896	9.99 109	3	26
35	9.30 275	61	9.31 168	65	0.68 832	9.99 106	2	**25**
36	9.30 336	62	9.31 233	64	0.68 767	9.99 104	3	24
37	9.30 398	61	9.31 297	64	0.68 703	9.99 101	2	23
38	9.30 459	62	9.31 361	64	0.68 639	9.99 099	3	22
39	9.30 521	61	9.31 425	64	0.68 575	9.99 096	3	21
40	9.30 582	61	9.31 489	63	0.68 511	9.99 093	2	**20**
41	9.30 643	61	9.31 552	64	0.68 448	9.99 091	3	19
42	9.30 704	61	9.31 616	63	0.68 384	9.99 088	2	18
43	9.30 765	61	9.31 679	64	0.68 321	9.99 086	3	17
44	9.30 826	61	9.31 743	63	0.68 257	9.99 083	3	16
45	9.30 887	60	9.31 806	64	0.68 194	9.99 080	2	**15**
46	9.30 947	61	9.31 870	63	0.68 130	9.99 078	3	14
47	9.31 008	60	9.31 933	63	0.68 067	9.99 075	3	13
48	9.31 068	61	9.31 996	63	0.68 004	9.99 072	2	12
49	9.31 129	60	9.32 059	63	0.67 941	9.99 070	3	11
50	9.31 189	61	9.32 122	63	0.67 878	9.99 067	3	**10**
51	9.31 250	60	9.32 185	63	0.67 815	9.99 064	2	9
52	9.31 310	60	9.32 248	63	0.67 752	9.99 062	3	8
53	9.31 370	60	9.32 311	62	0.67 689	9.99 059	3	7
54	9.31 430	60	9.32 373	63	0.67 627	9.99 056	2	6
55	9.31 490	59	9.32 436	62	0.67 564	9.99 054	3	**5**
56	9.31 549	60	9.32 498	63	0.67 502	9.99 051	3	4
57	9.31 609	60	9.32 561	62	0.67 439	9.99 048	2	3
58	9.31 669	59	9.32 623	62	0.67 377	9.99 046	3	2
59	9.31 728	60	9.32 685	62	0.67 315	9.99 043	3	1
60	9.31 788		9.32 747		0.67 253	9.99 040		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	68	67	66
2	13.6	13.4	13.2
3	20.4	20.1	19.8
4	27.2	26.8	26.4
5	34.0	33.5	33.0
6	40.8	40.2	39.6
7	47.6	46.9	46.2
8	54.4	53.6	52.8
9	61.2	60.3	59.4

	65	64	63
2	13.0	12.8	12.6
3	19.5	19.2	18.9
4	26.0	25.6	25.2
5	32.5	32.0	31.5
6	39.0	38.4	37.8
7	45.5	44.8	44.1
8	52.0	51.2	50.4
9	58.5	57.6	56.7

	62	61	60
2	12.4	12.2	12.0
3	18.6	18.3	18.0
4	24.8	24.4	24.0
5	31.0	30.5	30.0
6	37.2	36.6	36.0
7	43.4	42.7	42.0
8	49.6	48.8	48.0
9	55.8	54.9	54.0

	59	3
2	11.8	0.6
3	17.7	0.9
4	23.6	1.2
5	29.5	1.5
6	35.4	1.8
7	41.3	2.1
8	47.2	2.4
9	53.1	2.7

From the top:

For **11°+** or **191°+**, read as printed; for **101°+** or **281°+**, read co-function.

From the bottom:

For **78°+** or **258°+**, read as printed; for **168°+** or **348°+**, read co-function.

Prop. Pts.

78° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.31 788	59	9.32 747	63	0.67 253	9.99 040	2	**60**
1	9.31 847	60	9.32 810	62	0.67 190	9.99 038	3	59
2	9.31 907	59	9.32 872	61	0.67 128	9.99 035	3	58
3	9.31 966	59	9.32 933	62	0.67 067	9.99 032	2	57
4	9.32 025	59	9.32 995	62	0.67 005	9.99 030	3	56
5	9.32 084	59	9.33 057	62	0.66 943	9.99 027	3	**55**
6	9.32 143	59	9.33 119	61	0.66 881	9.99 024	2	54
7	9.32 202	59	9.33 180	62	0.66 820	9.99 022	3	53
8	9.32 261	58	9.33 242	61	0.66 758	9.99 019	3	52
9	9.32 319	59	9.33 303	62	0.66 697	9.99 016	3	51
10	9.32 378	59	9.33 365	61	0.66 635	9.99 013	2	**50**
11	9.32 437	58	9.33 426	61	0.66 574	9.99 011	3	49
12	9.32 495	58	9.33 487	61	0.66 513	9.99 008	3	48
13	9.32 553	59	9.33 548	61	0.66 452	9.99 005	3	47
14	9.32 612	58	9.33 609	61	0.66 391	9.99 002	2	46
15	9.32 670	58	9.33 670	61	0.66 330	9.99 000	3	**45**
16	9.32 728	58	9.33 731	61	0.66 269	9.98 997	3	44
17	9.32 786	58	9.33 792	61	0.66 208	9.98 994	3	43
18	9.32 844	58	9.33 853	60	0.66 147	9.98 991	2	42
19	9.32 902	58	9.33 913	61	0.66 087	9.98 989	3	41
20	9.32 960	58	9.33 974	60	0.66 026	9.98 986	3	**40**
21	9.33 018	57	9.34 034	61	0.65 966	9.98 983	3	39
22	9.33 075	58	9.34 095	60	0.65 905	9.98 980	2	38
23	9.33 133	57	9.34 155	60	0.65 845	9.98 978	3	37
24	9.33 190	58	9.34 215	61	0.65 785	9.98 975	3	36
25	9.33 248	57	9.34 276	60	0.65 724	9.98 972	3	**35**
26	9.33 305	57	9.34 336	60	0.65 664	9.98 969	2	34
27	9.33 362	58	9.34 396	60	0.65 604	9.98 967	3	33
28	9.33 420	57	9.34 456	60	0.65 544	9.98 964	3	32
29	9.33 477	57	9.34 516	60	0.65 484	9.98 961	3	31
30	9.33 534	57	9.34 576	59	0.65 424	9.98 958	3	**30**
31	9.33 591	56	9.34 635	60	0.65 365	9.98 955	2	29
32	9.33 647	57	9.34 695	60	0.65 305	9.98 953	3	28
33	9.33 704	57	9.34 755	59	0.65 245	9.98 950	3	27
34	9.33 761	57	9.34 814	60	0.65 186	9.98 947	3	26
35	9.33 818	56	9.34 874	59	0.65 126	9.98 944	3	**25**
36	9.33 874	57	9.34 933	59	0.65 067	9.98 941	3	24
37	9.33 931	56	9.34 992	59	0.65 008	9.98 938	2	23
38	9.33 987	56	9.35 051	60	0.64 949	9.98 936	3	22
39	9.34 043	57	9.35 111	59	0.64 889	9.98 933	3	21
40	9.34 100	56	9.35 170	59	0.64 830	9.98 930	3	**20**
41	9.34 156	56	9.35 229	59	0.64 771	9.98 927	3	19
42	9.34 212	56	9.35 288	59	0.64 712	9.98 924	3	18
43	9.34 268	56	9.35 347	58	0.64 653	9.98 921	2	17
44	9.34 324	56	9.35 405	59	0.64 595	9.98 919	3	16
45	9.34 380	56	9.35 464	59	0.64 536	9.98 916	3	**15**
46	9.34 436	55	9.35 523	58	0.64 477	9.98 913	3	14
47	9.34 491	56	9.35 581	59	0.64 419	9.98 910	3	13
48	9.34 547	55	9.35 640	58	0.64 360	9.98 907	3	12
49	9.34 602	56	9.35 698	59	0.64 302	9.98 904	3	11
50	9.34 658	55	9.35 757	58	0.64 243	9.98 901	3	**10**
51	9.34 713	56	9.35 815	58	0.64 185	9.98 898	2	9
52	9.34 769	55	9.35 873	58	0.64 127	9.98 896	3	8
53	9.34 824	55	9.35 931	58	0.64 069	9.98 893	3	7
54	9.34 879	55	9.35 989	58	0.64 011	9.98 890	3	6
55	9.34 934	55	9.36 047	58	0.63 953	9.98 887	3	**5**
56	9.34 989	55	9.36 105	58	0.63 895	9.98 884	3	4
57	9.35 044	55	9.36 163	58	0.63 837	9.98 881	3	3
58	9.35 099	55	9.36 221	58	0.63 779	9.98 878	3	2
59	9.35 154	55	9.36 279	57	0.63 721	9.98 875	3	1
60	9.35 209		9.36 336		0.63 664	9.98 872		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	63	62	61
2	12.6	12.4	12.2
3	18.9	18.6	18.3
4	25.2	24.8	24.4
5	31.5	31.0	30.5
6	37.8	37.2	36.6
7	44.1	43.4	42.7
8	50.4	49.6	48.8
9	56.7	55.8	54.9

	60	59	58
2	12.0	11.8	11.6
3	18.0	17.7	17.4
4	24.0	23.6	23.2
5	30.0	29.5	29.0
6	36.0	35.4	34.8
7	42.0	41.3	40.6
8	48.0	47.2	46.4
9	54.0	53.1	52.2

	57	56
2	11.4	11.2
3	17.1	16.8
4	22.8	22.4
5	28.5	28.0
6	34.2	33.6
7	39.9	39.2
8	45.6	44.8
9	51.3	50.4

	55	3
2	11.0	0.6
3	16.5	0.9
4	22.0	1.2
5	27.5	1.5
6	33.0	1.8
7	38.5	2.1
8	44.0	2.4
9	49.5	2.7

From the top :

For **12°+** or **192°+**, read as printed; for **102°+** or **282°+**, read co-function.

From the bottom ·

For **77°** or **257°**, read as printed ; for **167°** or **347°**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.35 209	54	9.36 336	58	0.63 664	9.98 872	3	**60**
1	9.35 263	55	9.36 394	58	0.63 606	9.98 869	2	59
2	9.35 318	55	9.36 452	57	0.63 548	9.98 867	3	58
3	9.35 373	54	9.36 509	57	0.63 491	9.98 864	3	57
4	9.35 427	54	9.36 566	58	0.63 434	9.98 861	3	56
5	9.35 481	55	9.36 624	57	0.63 376	9.98 858	3	**55**
6	9.35 536	54	9.36 681	57	0.63 319	9.98 855	3	54
7	9.35 590	54	9.36 738	57	0.63 262	9.98 852	3	53
8	9.35 644	54	9.36 795	57	0.63 205	9.98 849	3	52
9	9.35 698	54	9.36 852	57	0.63 148	9.98 846	3	51
10	9.35 752	54	9.36 909	57	0.63 091	9.98 843	3	**50**
11	9.35 806	54	9.36 966	57	0.63 034	9.98 840	3	49
12	9.35 860	54	9.37 023	57	0.62 977	9.98 837	3	48
13	9.35 914	54	9.37 080	57	0.62 920	9.98 834	3	47
14	9.35 968	54	9.37 137	56	0.62 863	9.98 831	3	46
15	9.36 022	53	9.37 193	57	0.62 807	9.98 828	3	**45**
16	9.36 075	54	9.37 250	56	0.62 750	9.98 825	3	44
17	9.36 129	53	9.37 306	57	0.62 694	9.98 822	3	43
18	9.36 182	54	9.37 363	56	0.62 637	9.98 819	3	42
19	9.36 236	53	9.37 419	57	0.62 581	9.98 816	3	41
20	9.36 289	53	9.37 476	56	0.62 524	9.98 813	3	**40**
21	9.36 342	53	9.37 532	56	0.62 468	9.98 810	3	39
22	9.36 395	54	9.37 588	56	0.62 412	9.98 807	3	38
23	9.36 449	53	9.37 644	56	0.62 356	9.98 804	3	37
24	9.36 502	53	9.37 700	56	0.62 300	9.98 801	3	36
25	9.36 555	53	9.37 756	56	0.62 244	9.98 798	3	**35**
26	9.36 608	52	9.37 812	56	0.62 188	9.98 795	3	34
27	9.36 660	53	9.37 868	56	0.62 132	9.98 792	3	33
28	9.36 713	53	9.37 924	56	0.62 076	9.98 789	3	32
29	9.36 766	53	9.37 980	55	0.62 020	9.98 786	3	31
30	9.36 819	52	9.38 035	56	0.61 965	9.98 783	3	**30**
31	9.36 871	53	9.38 091	56	0.61 909	9.98 780	3	29
32	9.36 924	52	9.38 147	55	0.61 853	9.98 777	3	28
33	9.36 976	52	9.38 202	55	0.61 798	9.98 774	3	27
34	9.37 028	53	9.38 257	56	0.61 743	9.98 771	3	26
35	9.37 081	52	9.38 313	55	0.61 687	9.98 768	3	**25**
36	9.37 133	52	9.38 368	55	0.61 632	9.98 765	3	24
37	9.37 185	52	9.38 423	56	0.61 577	9.98 762	3	23
38	9.37 237	52	9.38 479	55	0.61 521	9.98 759	3	22
39	9.37 289	52	9 38 534	55	0.61 466	9.98 756	3	21
40	9.37 341	52	9.38 589	55	0.61 411	9.98 753	3	**20**
41	9.37 393	52	9.38 644	55	0.61 356	9.98 750	4	19
42	9.37 445	52	9.38 699	55	0.61 301	9.98 746	3	18
43	9.37 497	52	9.38 754	54	0.61 246	9.98 743	3	17
44	9.37 549	51	9.38 808	55	0.61 192	9.98 740	3	16
45	9.37 600	52	9.38 863	55	0.61 137	9.98 737	3	**15**
46	9.37 652	51	9.38 918	54	0.61 082	9.98 734	3	14
47	9.37 703	52	9.38 972	55	0.61 028	9.98 731	3	13
48	9.37 755	51	9.39 027	55	0.60 973	9.98 728	3	12
49	9.37 806	52	9.39 082	54	0.60 918	9.98 725	3	11
50	9.37 858	51	9.39 136	54	0.60 864	9.98 722	3	**10**
51	9.37 909	51	9.39 190	55	0.60 810	9.98 719	4	9
52	9.37 960	51	9.39 245	54	0.60 755	9.98 715	3	8
53	9.38 011	51	9.39 299	54	0.60 701	9.98 712	3	7
54	9.38 062	51	9.39 353	54	0.60 647	9.98 709	3	6
55	9.38 113	51	9.39 407	54	0.60 593	9.98 706	3	**5**
56	9.38 164	51	9.39 461	54	0.60 539	9.98 703	3	4
57	9.38 215	51	9.39 515	54	0.60 485	9.98 700	3	3
58	9.38 266	51	9.39 569	54	0.60 431	9.98 697	3	2
59	9.38 317	51	9.39 623	54	0.60 377	9.98 694	4	1
60	9.38 368		9.39 677		0.60 323	9.98 690		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	58	57	56
2	11.6	11.4	11.2
3	17.4	17.1	16.8
4	23.2	22.8	22.4
5	29.0	28.5	28.0
6	34.8	34.2	33.6
7	40.6	39.9	39.2
8	46.4	45.6	44.8
9	52.2	51.3	50.4

	55	54	53
2	11.0	10.8	10.6
3	16.5	16.2	15.9
4	22.0	21.6	21.2
5	27.5	27.0	26.5
6	33.0	32.4	31.8
7	38.5	37.8	37.1
8	44.0	43.2	42.4
9	49.5	48.6	47.7

	52	51
2	10.4	10.2
3	15.6	15.3
4	20.8	20.4
5	26.0	25.5
6	31.2	30.6
7	36.4	35.7
8	41.6	40.8
9	46.8	45.9

	4	3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

From the top:

For **13°+** or **193°+**, read as printed; for **103°+** or **283°+**, read co-function.

From the bottom:

For **76°** or **256°**, read as printed; for **166°+** or **346°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.38 368	50	9.39 677	54	0.60 323	9.98 690	3	**60**
1	9.38 418	51	9.39 731	54	0.60 269	9.98 687	3	59
2	9.38 469	50	9.39 785	53	0.60 215	9.98 684	3	58
3	9.38 519	51	9.39 838	54	0.60 162	9.98 681	3	57
4	9.38 570	50	9.39 892	53	0.60 108	9.98 678	3	56
5	9.38 620	50	9.39 945	54	0.60 055	9.98 675	4	**55**
6	9.38 670	51	9.39 999	53	0.60 001	9.98 671	3	54
7	9.38 721	50	9.40 052	54	0.59 948	9.98 668	3	53
8	9.38 771	50	9.40 106	53	0.59 894	9.98 665	3	52
9	9.38 821	50	9.40 159	53	0.59 841	9.98 662	3	51
10	9.38 871	50	9.40 212	54	0.59 788	9.98 659	3	**50**
11	9.38 921	50	9.40 266	53	0.59 734	9.98 656	4	49
12	9.38 971	50	9.40 319	53	0.59 681	9.98 652	3	48
13	9.39 021	50	9.40 372	53	0.59 628	9.98 649	3	47
14	9.39 071	50	9.40 425	53	0.59 575	9.98 646	3	46
15	9.39 121	49	9.40 478	53	0.59 522	9.98 643	3	**45**
16	9.39 170	50	9.40 531	53	0.59 469	9.98 640	4	44
17	9.39 220	50	9.40 584	52	0.59 416	9.98 636	3	43
18	9.39 270	49	9.40 636	53	0.59 364	9.98 633	3	42
19	9.39 319	50	9.40 689	53	0.59 311	9.98 630	3	41
20	9.39 369	49	9.40 742	53	0.59 258	9.98 627	4	**40**
21	9.39 418	49	9.40 795	52	0.59 205	9.98 623	3	39
22	9.39 467	50	9.40 847	53	0.59 153	9.98 620	3	38
23	9.39 517	49	9.40 900	52	0.59 100	9.98 617	3	37
24	9.39 566	49	9.40 952	53	0.59 048	9.98 614	4	36
25	9.39 615	49	9.41 005	52	0.58 995	9.98 610	3	**35**
26	9.39 664	49	9.41 057	52	0.58 943	9.98 607	3	34
27	9.39 713	49	9.41 109	52	0.58 891	9.98 604	3	33
28	9.39 762	49	9.41 161	53	0.58 839	9.98 601	4	32
29	9.39 811	49	9.41 214	52	0.58 786	9.98 597	3	31
30	9.39 860	49	9.41 266	52	0.58 734	9.98 594	3	**30**
31	9.39 909	49	9.41 318	52	0.58 682	9.98 591	3	29
32	9.39 958	48	9.41 370	52	0.58 630	9.98 588	4	28
33	9.40 006	49	9.41 422	52	0.58 578	9.98 584	3	27
34	9.40 055	48	9.41 474	52	0.58 526	9.98 581	3	26
35	9.40 103	49	9.41 526	52	0.58 474	9.98 578	4	**25**
36	9.40 152	48	9.41 578	51	0.58 422	9.98 574	3	24
37	9.40 200	49	9.41 629	52	0.58 371	9.98 571	3	23
38	9.40 249	48	9.41 681	52	0.58 319	9.98 568	3	22
39	9.40 297	49	9.41 733	51	0.58 267	9.98 565	4	21
40	9.40 346	48	9.41 784	52	0.58 216	9.98 561	3	**20**
41	9.40 394	48	9.41 836	51	0.58 164	9.98 558	3	19
42	9.40 442	48	9.41 887	52	0.58 113	9.98 555	4	18
43	9.40 490	48	9.41 939	51	0.58 061	9.98 551	3	17
44	9.40 538	48	9.41 990	51	0.58 010	9.98 548	3	16
45	9.40 586	48	9.42 041	52	0.57 959	9.98 545	4	**15**
46	9.40 634	48	9.42 093	51	0.57 907	9.98 541	3	14
47	9.40 682	48	9.42 144	51	0.57 856	9.98 538	3	13
48	9.40 730	48	9.42 195	51	0.57 805	9.98 535	4	12
49	9.40 778	47	9.42 246	51	0.57 754	9.98 531	3	11
50	9.40 825	48	9.42 297	51	0.57 703	9.98 528	3	**10**
51	9.40 873	48	9.42 348	51	0.57 652	9.98 525	4	9
52	9.40 921	47	9.42 399	51	0.57 601	9.98 521	3	8
53	9.40 968	48	9.42 450	51	0.57 550	9.98 518	3	7
54	9.41 016	47	9.42 501	51	0.57 499	9.98 515	4	6
55	9.41 063	48	9.42 552	51	0.57 448	9.98 511	3	**5**
56	9.41 111	47	9.42 603	50	0.57 397	9.98 508	3	4
57	9.41 158	47	9.42 653	51	0.57 347	9.98 505	4	3
58	9.41 205	47	9.42 704	51	0.57 296	9.98 501	3	2
59	9.41 252	48	9.42 755	50	0.57 245	9.98 498	4	1
60	9.41 300		9.42 805		0.57 195	9.98 494		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	54	53	52
2	10.8	10.6	10.4
3	16.2	15.9	15.6
4	21.6	21.2	20.8
5	27.0	26.5	26.0
6	32.4	31.8	31.2
7	37.8	37.1	36.4
8	43.2	42.4	41.6
9	48.6	47.7	46.8

	51	50	49
2	10.2	10.0	9.8
3	15.3	15.0	14.7
4	20.4	20.0	19.6
5	25.5	25.0	24.5
6	30.6	30.0	29.4
7	35.7	35.0	34.3
8	40.8	40.0	39.2
9	45.9	45.0	44.1

	48	47
2	9.6	9.4
3	14.4	14.1
4	19.2	18.8
5	24.0	23.5
6	28.8	28.2
7	33.6	32.9
8	38.4	37.6
9	43.2	42.3

	4	3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

From the top:

For **14°**+ or **194°**+, read as printed; for **104°**+ or **284°**+, read co-function.

From the bottom:

For **75°**+ or **255°**+, read as printed; for **165°**+ or **345°**+, read co-function.

Prop. Pts.

15° — Logarithms of Trigonometric Functions

'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.41 300		9.42 805		0.57 195	9.98 494		**60**
1	9.41 347	47	9.42 856	51	0.57 144	9.98 491	3	59
2	9.41 394	47	9.42 906	50	0.57 094	9.98 488	3	58
3	9.41 441	47	9.42 957	51	0.57 043	9.98 484	4	57
4	9.41 488	47	9.43 007	50	0.56 993	9.98 481	3	56
5	9.41 535	47	9.43 057	50	0.56 943	9.98 477	4	**55**
6	9.41 582	47	9.43 108	51	0.56 892	9.98 474	3	54
7	9.41 628	46	9.43 158	50	0.56 842	9.98 471	3	53
8	9.41 675	47	9.43 208	50	0.56 792	9.98 467	4	52
9	9.41 722	47	9.43 258	50	0.56 742	9.98 464	3	51
10	9.41 768	46	9.43 308	50	0.56 692	9.98 460	4	**50**
11	9.41 815	47	9.43 358	50	0.56 642	9.98 457	3	49
12	9.41 861	46	9.43 408	50	0.56 592	9.98 453	4	48
13	9.41 908	47	9.43 458	50	0.56 542	9.98 450	3	47
14	9.41 954	46	9.43 508	50	0.56 492	9.98 447	3	46
15	9.42 001	47	9.43 558	50	0.56 442	9.98 443	4	**45**
16	9.42 047	46	9.43 607	49	0.56 393	9.98 440	3	44
17	9.42 093	46	9.43 657	50	0.56 343	9.98 436	4	43
18	9.42 140	47	9.43 707	50	0.56 293	9.98 433	3	42
19	9.42 186	46	9.43 756	49	0.56 244	9.98 429	4	41
20	9.42 232	46	9.43 806	50	0.56 194	9.98 426	3	**40**
21	9.42 278	46	9.43 855	49	0.56 145	9.98 422	4	39
22	9.42 324	46	9.43 905	50	0.56 095	9.98 419	3	38
23	9.42 370	46	9.43 954	49	0.56 046	9.98 415	4	37
24	9.42 416	46	9.44 004	50	0.55 996	9.98 412	3	36
25	9.42 461	45	9.44 053	49	0.55 947	9.98 409	3	**35**
26	9.42 507	46	9.44 102	49	0.55 898	9.98 405	4	34
27	9.42 553	46	9.44 151	49	0.55 849	9.98 402	3	33
28	9.42 599	46	9.44 201	50	0.55 799	9.98 398	4	32
29	9.42 644	45	9.44 250	49	0.55 750	9.98 395	3	31
30	9.42 690	46	9.44 299	49	0.55 701	9.98 391	4	**30**
31	9.42 735	45	9.44 348	49	0.55 652	9.98 388	3	29
32	9.42 781	46	9.44 397	49	0.55 603	9.98 384	4	28
33	9.42 826	45	9.44 446	49	0.55 554	9.98 381	3	27
34	9.42 872	46	9.44 495	49	0.55 505	9.98 377	4	26
35	9.42 917	45	9.44 544	49	0.55 456	9.98 373	4	**25**
36	9.42 962	45	9.44 592	48	0.55 408	9.98 370	3	24
37	9.43 008	46	9.44 641	49	0.55 359	9.98 366	4	23
38	9.43 053	45	9.44 690	49	0.55 310	9.98 363	3	22
39	9.43 098	45	9.44 738	48	0.55 262	9.98 359	4	21
40	9.43 143	45	9.44 787	49	0.55 213	9.98 356	3	**20**
41	9.43 188	45	9.44 836	49	0.55 164	9.98 352	4	19
42	9.43 233	45	9.44 884	48	0.55 116	9.98 349	3	18
43	9.43 278	45	9.44 933	49	0.55 067	9.98 345	4	17
44	9.43 323	45	9.44 981	48	0.55 019	9.98 342	3	16
45	9.43 367	44	9.45 029	48	0.54 971	9.98 338	4	**15**
46	9.43 412	45	9.45 078	49	0.54 922	9.98 334	4	14
47	9.43 457	45	9.45 126	48	0.54 874	9.98 331	3	13
48	9.43 502	45	9.45 174	48	0.54 826	9.98 327	4	12
49	9.43 546	44	9.45 222	48	0.54 778	9.98 324	3	11
50	9.43 591	45	9.45 271	49	0.54 729	9.98 320	4	**10**
51	9.43 635	44	9.45 319	48	0.54 681	9.98 317	3	9
52	9.43 680	45	9.45 367	48	0.54 633	9.98 313	4	8
53	9.43 724	44	9.45 415	48	0.54 585	9.98 309	4	7
54	9.43 769	45	9.45 463	48	0.54 537	9.98 306	3	6
55	9.43 813	44	9.45 511	48	0.54 489	9.98 302	4	**5**
56	9.43 857	44	9.45 559	48	0.54 441	9.98 299	3	4
57	9.43 901	44	9.45 606	47	0.54 394	9.98 295	4	3
58	9.43 946	45	9.45 654	48	0.54 346	9.98 291	4	2
59	9.43 990	44	9.45 702	48	0.54 298	9.98 288	3	1
60	9.44 034	44	9.45 750	48	0.54 250	9.98 284	4	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	'

Prop. Pts.

	51	50	49
2	10.2	10.0	9.8
3	15.3	15.0	14.7
4	20.4	20.0	19.6
5	25.5	25.0	24.5
6	30.6	30.0	29.4
7	35.7	35.0	34.3
8	40.8	40.0	39.2
9	45.9	45.0	44.1

	48	47	46
2	9.6	9.4	9.2
3	14.4	14.1	13.8
4	19.2	18.8	18.4
5	24.0	23.5	23.0
6	28.8	28.2	27.6
7	33.6	32.9	32.2
8	38.4	37.6	36.8
9	43.2	42.3	41.4

	45	44
2	9.0	8.8
3	13.5	13.2
4	18.0	17.6
5	22.5	22.0
6	27.0	26.4
7	31.5	30.8
8	36.0	35.2
9	40.5	39.6

	4	3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

From the top :

For **15°**+ or **195°**+, read as printed ; for **105°**+ or **285°**+, read co-function.

From the bottom :

For **74°**+ or **254°**+, read as printed ; for **164°**+ or **344°**+, read co-function.

Prop. Pts.

74° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.44 034	44	9.45 750	47	0.54 250	9.98 284	3	**60**
1	9.44 078	44	9.45 797	48	0.54 203	9.98 281	4	59
2	9.44 122	44	9.45 845	47	0.54 155	9.98 277	4	58
3	9.44 166	44	9.45 892	48	0.54 108	9.98 273	3	57
4	9.44 210	43	9.45 940	47	0.54 060	9.98 270	4	56
5	9.44 253	44	9.45 987	48	0.54 013	9.98 266	4	**55**
6	9.44 297	44	9.46 035	47	0.53 965	9.98 262	3	54
7	9.44 341	44	9.46 082	48	0.53 918	9.98 259	4	53
8	9.44 385	43	9.46 130	47	0.53 870	9.98 255	4	52
9	9.44 428	44	9.46 177	47	0.53 823	9.98 251	3	51
10	9.44 472	44	9.46 224	47	0.53 776	9.98 248	4	**50**
11	9.44 516	43	9.46 271	48	0.53 729	9.98 244	4	49
12	9.44 559	43	9.46 319	47	0.53 681	9.98 240	3	48
13	9.44 602	44	9.46 366	47	0.53 634	9.98 237	4	47
14	9.44 646	43	9.46 413	47	0.53 587	9.98 233	4	46
15	9.44 689	44	9.46 460	47	0.53 540	9.98 229	3	**45**
16	9.44 733	43	9.46 507	47	0.53 493	9.98 226	4	44
17	9.44 776	43	9.46 554	47	0.53 446	9.98 222	4	43
18	9.44 819	43	9.46 601	47	0.53 399	9.98 218	3	42
19	9.44 862	43	9.46 648	46	0.53 352	9.98 215	4	41
20	9.44 905	43	9.46 694	47	0.53 306	9.98 211	4	**40**
21	9.44 948	44	9.46 741	47	0.53 259	9.98 207	3	39
22	9.44 992	43	9.46 788	47	0.53 212	9.98 204	4	38
23	9.45 035	42	9.46 835	46	0.53 165	9.98 200	4	37
24	9.45 077	43	9.46 881	47	0.53 119	9.98 196	4	36
25	9.45 120	43	9.46 928	47	0.53 072	9.98 192	3	**35**
26	9.45 163	43	9.46 975	46	0.53 025	9.98 189	4	34
27	9.45 206	43	9.47 021	47	0.52 979	9.98 185	4	33
28	9.45 249	43	9.47 068	46	0.52 932	9.98 181	4	32
29	9.45 292	42	9.47 114	46	0.52 886	9.98 177	3	31
30	9.45 334	43	9.47 160	47	0.52 840	9.98 174	4	**30**
31	9.45 377	42	9.47 207	46	0.52 793	9.98 170	4	29
32	9.45 419	43	9.47 253	46	0.52 747	9.98 166	4	28
33	9.45 462	42	9.47 299	47	0.52 701	9.98 162	3	27
34	9.45 504	43	9.47 346	46	0.52 654	9.98 159	4	26
35	9.45 547	42	9.47 392	46	0.52 608	9.98 155	4	**25**
36	9.45 589	43	9.47 438	46	0.52 562	9.98 151	4	24
37	9.45 632	42	9.47 484	46	0.52 516	9.98 147	3	23
38	9.45 674	42	9.47 530	46	0.52 470	9.98 144	4	22
39	9.45 716	42	9.47 576	46	0.52 424	9.98 140	4	21
40	9.45 758	43	9.47 622	46	0.52 378	9.98 136	4	**20**
41	9.45 801	42	9.47 668	46	0.52 332	9.98 132	3	19
42	9.45 843	42	9.47 714	46	0.52 286	9.98 129	4	18
43	9.45 885	42	9.47 760	46	0.52 240	9.98 125	4	17
44	9.45 927	42	9.47 806	46	0.52 194	9.98 121	4	16
45	9.45 969	42	9.47 852	45	0.52 148	9.98 117	4	**15**
46	9.46 011	42	9.47 897	46	0.52 103	9.98 113	3	14
47	9.46 053	42	9.47 943	46	0.52 057	9.98 110	4	13
48	9.46 095	41	9.47 989	46	0.52 011	9.98 106	4	12
49	9.46 136	42	9.48 035	45	0.51 965	9.98 102	4	11
50	9.46 178	42	9.48 080	46	0.51 920	9.98 098	4	**10**
51	9.46 220	42	9.48 126	45	0.51 874	9.98 094	4	9
52	9.46 262	41	9.48 171	46	0.51 829	9.98 090	3	8
53	9.46 303	42	9.48 217	45	0.51 783	9.98 087	4	7
54	9.46 345	41	9.48 262	45	0.51 738	9.98 083	4	6
55	9.46 386	42	9.48 307	46	0.51 693	9.98 079	4	**5**
56	9.46 428	41	9.48 353	45	0.51 647	9.98 075	4	4
57	9.46 469	42	9.48 398	45	0.51 602	9.98 071	4	3
58	9.46 511	41	9.48 443	46	0.51 557	9.98 067	4	2
59	9.46 552	42	9.48 489	45	0.51 511	9.98 063	3	1
60	9.46 594		9.48 534		0.51 466	9.98 060		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	48	47	46
2	9.6	9.4	9.2
3	14.4	14.1	13.8
4	19.2	18.8	18.4
5	24.0	23.5	23.0
6	28.8	28.2	27.6
7	33.6	32.9	32.2
8	38.4	37.6	36.8
9	43.2	42.3	41.4

	45	44	43
2	9.0	8.8	8.6
3	13.5	13.2	12.9
4	18.0	17.6	17.2
5	22.5	22.0	21.5
6	27.0	26.4	25.8
7	31.5	30.8	30.1
8	36.0	35.2	34.4
9	40.5	39.6	38.7

	42	41
2	8.4	8.2
3	12.6	12.3
4	16.8	16.4
5	21.0	20.5
6	25.2	24.6
7	29.4	28.7
8	33.6	32.8
9	37.8	36.9

	4	3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

From the top :

For **16°+** or **196°+**, read as printed; for **106°+** or **286°+**, read co-function.

From the bottom :

For **73°+** or **253°+**, read as printed ; for **163°+** or **343°+**, read co-function.

Prop. Pts.

'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.46 594	41	9.48 534	45	0.51 466	9.98 060	4	**60**
1	9.46 635	41	9.48 579	45	0.51 421	9.98 056	4	59
2	9.46 676	41	9.48 624	45	0.51 376	9.98 052	4	58
3	9.46 717	41	9.48 669	45	0.51 331	9.98 048	4	57
4	9.46 758	42	9.48 714	45	0.51 286	9.98 044	4	56
5	9.46 800	41	9.48 759	45	0.51 241	9.98 040	4	**55**
6	9.46 841	41	9.48 804	45	0.51 196	9.98 036	4	54
7	9.46 882	41	9.48 849	45	0.51 151	9.98 032	3	53
8	9.46 923	41	9.48 894	45	0.51 106	9.98 029	4	52
9	9.46 964	41	9.48 939	45	0.51 061	9.98 025	4	51
10	9.47 005	40	9.48 984	45	0.51 016	9.98 021	4	**50**
11	9.47 045	41	9.49 029	44	0.50 971	9.98 017	4	49
12	9.47 086	41	9.49 073	45	0.50 927	9.98 013	4	48
13	9.47 127	41	9.49 118	45	0.50 882	9.98 009	4	47
14	9.47 168	41	9.49 163	44	0.50 837	9.98 005	4	46
15	9.47 209	40	9.49 207	45	0.50 793	9.98 001	4	**45**
16	9.47 249	41	9.49 252	44	0.50 748	9.97 997	4	44
17	9.47 290	40	9.49 296	45	0.50 704	9.97 993	4	43
18	9.47 330	41	9.49 341	44	0.50 659	9.97 989	3	42
19	9.47 371	40	9.49 385	45	0.50 615	9.97 986	4	41
20	9.47 411	41	9.49 430	44	0.50 570	9.97 982	4	**40**
21	9.47 452	40	9.49 474	45	0.50 526	9.97 978	4	39
22	9.47 492	41	9.49 519	44	0.50 481	9.97 974	4	38
23	9.47 533	40	9.49 563	44	0.50 437	9.97 970	4	37
24	9.47 573	40	9.49 607	45	0.50 393	9.97 966	4	36
25	9.47 613	41	9.49 652	44	0.50 348	9.97 962	4	**35**
26	9.47 654	40	9.49 696	44	0.50 304	9.97 958	4	34
27	9.47 694	40	9.49 740	44	0.50 260	9.97 954	4	33
28	9.47 734	40	9.49 784	44	0.50 216	9.97 950	4	32
29	9.47 774	40	9.49 828	44	0.50 172	9.97 946	4	31
30	9.47 814	40	9.49 872	44	0.50 128	9.97 942	4	**30**
31	9.47 854	40	9.49 916	44	0.50 084	9.97 938	4	29
32	9.47 894	40	9.49 960	44	0.50 040	9.97 934	4	28
33	9.47 934	40	9.50 004	44	0.49 996	9.97 930	4	27
34	9.47 974	40	9.50 048	44	0.49 952	9.97 926	4	26
35	9.48 014	40	9.50 092	44	0.49 908	9.97 922	4	**25**
36	9.48 054	40	9.50 136	44	0.49 864	9.97 918	4	24
37	9.48 094	39	9.50 180	43	0.49 820	9.97 914	4	23
38	9.48 133	40	9.50 223	44	0.49 777	9.97 910	4	22
39	9.48 173	40	9.50 267	44	0.49 733	9.97 906	4	21
40	9.48 213	39	9.50 311	44	0.49 689	9.97 902	4	**20**
41	9.48 252	40	9.50 355	43	0.49 645	9.97 898	4	19
42	9.48 292	40	9.50 398	44	0.49 602	9.97 894	4	18
43	9.48 332	39	9.50 442	43	0.49 558	9.97 890	4	17
44	9.48 371	40	9.50 485	44	0.49 515	9.97 886	4	16
45	9.48 411	39	9.50 529	43	0.49 471	9.97 882	4	**15**
46	9.48 450	40	9.50 572	44	0.49 428	9.97 878	4	14
47	9.48 490	39	9.50 616	43	0.49 384	9.97 874	4	13
48	9.48 529	39	9.50 659	44	0.49 341	9.97 870	4	12
49	9.48 568	39	9.50 703	43	0.49 297	9.97 866	5	11
50	9.48 607	40	9.50 746	43	0.49 254	9.97 861	4	**10**
51	9.48 647	39	9.50 789	44	0.49 211	9.97 857	4	9
52	9.48 686	39	9.50 833	43	0.49 167	9.97 853	4	8
53	9.48 725	39	9.50 876	43	0.49 124	9.97 849	4	7
54	9.48 764	39	9.50 919	43	0.49 081	9.97 845	4	6
55	9.48 803	39	9.50 962	43	0.49 038	9.97 841	4	**5**
56	9.48 842	39	9.51 005	43	0.48 995	9.97 837	4	4
57	9.48 881	39	9.51 048	44	0.48 952	9.97 833	4	3
58	9.48 920	39	9.51 092	43	0.48 908	9.97 829	4	2
59	9.48 959	39	9.51 135	43	0.48 865	9.97 825	4	1
60	9.48 998		9.51 178		0.48 822	9.97 821		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	'

Prop. Pts.

	45	44	43
2	9.0	8.8	8.6
3	13.5	13.2	12.9
4	18.0	17.6	17.2
5	22.5	22.0	21.5
6	27.0	26.4	25.8
7	31.5	30.8	30.1
8	36.0	35.2	34.4
9	40.5	39.6	38.7

	42	41	40
2	8.4	8.2	8.0
3	12.6	12.3	12.0
4	16.8	16.4	16.0
5	21.0	20.5	20.0
6	25.2	24.6	24.0
7	29.4	28.7	28.0
8	33.6	32.8	32.0
9	37.8	36.9	36.0

	39	5
2	7.8	1.0
3	11.7	1.5
4	15.6	2.0
5	19.5	2.5
6	23.4	3.0
7	27.3	3.5
8	31.2	4.0
9	35.1	4.5

	4	3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

From the top:

For **17°**+ or **197°**+, read as printed; for **107°**+ or **287°**+, read co-function.

From the bottom:

For **72°**+ or **252°**+, read as printed; for **162°**+ or **342°**+, read co-function.

Prop. Pts.

18° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.48 998	39	9.51 178	43	0.48 822	9.97 821	4	**60**
1	9.49 037	39	9.51 221	43	0.48 779	9.97 817	5	59
2	9.49 076	39	9.51 264	42	0.48 736	9.97 812	4	58
3	9.49 115	38	9.51 306	43	0.48 694	9.97 808	4	57
4	9.49 153	39	9.51 349	43	0.48 651	9.97 804	4	56
5	9.49 192	39	9.51 392	43	0.48 608	9.97 800	4	**55**
6	9.49 231	38	9.51 435	43	0.48 565	9.97 796	4	54
7	9.49 269	39	9.51 478	42	0.48 522	9.97 792	4	53
8	9.49 308	39	9.51 520	43	0.48 480	9.97 788	4	52
9	9.49 347	38	9.51 563	43	0.48 437	9.97 784	5	51
10	9.49 385	39	9.51 606	42	0.48 394	9.97 779	4	**50**
11	9.49 424	38	9.51 648	43	0.48 352	9.97 775	4	49
12	9.49 462	38	9.51 691	43	0.48 309	9.97 771	4	48
13	9.49 500	39	9.51 734	42	0.48 266	9.97 767	4	47
14	9.49 539	38	9.51 776	43	0.48 224	9.97 763	4	46
15	9.49 577	38	9.51 819	42	0.48 181	9.97 759	5	**45**
16	9.49 615	39	9.51 861	42	0.48 139	9.97 754	4	44
17	9.49 654	38	9.51 903	43	0.48 097	9.97 750	4	43
18	9.49 692	38	9.51 946	42	0.48 054	9.97 746	4	42
19	9.49 730	38	9.51 988	43	0.48 012	9.97 742	4	41
20	9.49 768	38	9.52 031	42	0.47 969	9.97 738	4	**40**
21	9.49 806	38	9.52 073	42	0.47 927	9.97 734	5	39
22	9.49 844	38	9.52 115	42	0.47 885	9.97 729	4	38
23	9.49 882	38	9.52 157	43	0.47 843	9.97 725	4	37
24	9.49 920	38	9.52 200	42	0.47 800	9.97 721	4	36
25	9.49 958	38	9.52 242	42	0.47 758	9.97 717	4	**35**
26	9.49 996	38	9.52 284	42	0.47 716	9.97 713	5	34
27	9.50 034	38	9.52 326	42	0.47 674	9.97 708	4	33
28	9.50 072	38	9.52 368	42	0.47 632	9.97 704	4	32
29	9.50 110	38	9.52 410	42	0.47 590	9.97 700	4	31
30	9.50 148	37	9.52 452	42	0.47 548	9.97 696	5	**30**
31	9.50 185	38	9.52 494	42	0.47 506	9.97 691	4	29
32	9.50 223	38	9.52 536	42	0.47 464	9.97 687	4	28
33	9.50 261	37	9.52 578	42	0.47 422	9.97 683	4	27
34	9.50 298	38	9.52 620	41	0.47 380	9.97 679	5	26
35	9.50 336	38	9.52 661	42	0.47 339	9.97 674	4	**25**
36	9.50 374	37	9.52 703	42	0.47 297	9.97 670	4	24
37	9.50 411	38	9.52 745	42	0.47 255	9.97 666	4	23
38	9.50 449	37	9.52 787	42	0.47 213	9.97 662	5	22
39	9.50 486	37	9.52 829	41	0.47 171	9.97 657	4	21
40	9.50 523	38	9.52 870	42	0.47 130	9.97 653	4	**20**
41	9.50 561	37	9.52 912	41	0.47 088	9.97 649	4	19
42	9.50 598	37	9.52 953	42	0.47 047	9.97 645	5	18
43	9.50 635	38	9.52 995	42	0.47 005	9.97 640	4	17
44	9.50 673	37	9.53 037	41	0.46 963	9.97 636	4	16
45	9.50 710	37	9.53 078	42	0.46 922	9.97 632	4	**15**
46	9.50 747	37	9.53 120	41	0.46 880	9.97 628	5	14
47	9.50 784	37	9.53 161	41	0.46 839	9.97 623	4	13
48	9.50 821	37	9.53 202	42	0.46 798	9.97 619	4	12
49	9.50 858	38	9.53 244	41	0.46 756	9.97 615	5	11
50	9.50 896	37	9.53 285	42	0.46 715	9.97 610	4	**10**
51	9.50 933	37	9.53 327	41	0.46 673	9.97 606	4	9
52	9.50 970	37	9.53 368	41	0.46 632	9.97 602	5	8
53	9.51 007	36	9.53 409	41	0.46 591	9.97 597	4	7
54	9.51 043	37	9.53 450	42	0.46 550	9.97 593	4	6
55	9.51 080	37	9.53 492	41	0.46 508	9.97 589	5	**5**
56	9.51 117	37	9.53 533	41	0.46 467	9.97 584	4	4
57	9.51 154	37	9.53 574	41	0.46 426	9.97 580	4	3
58	9.51 191	36	9.53 615	41	0.46 385	9.97 576	5	2
59	9.51 227	37	9.53 656	41	0.46 344	9.97 571	4	1
60	9.51 264		9.53 697		0.46 303	9.97 567		**0**
	L Cos	**d**	**L Ctn**	**c d**	**L Tan**	**L Sin**	**d**	**′**

Prop. Pts.

	43	**42**	**41**
2	8.6	8.4	8.2
3	12.9	12.6	12.3
4	17.2	16.8	16.4
5	21.5	21.0	20.5
6	25.8	25.2	24.6
7	30.1	29.4	28.7
8	34.4	33.6	32.8
9	38.7	37.8	36.9

	39	**38**	**37**
2	7.8	7.6	7.4
3	11.7	11.4	11.1
4	15.6	15.2	14.8
5	19.5	19.0	18.5
6	23.4	22.8	22.2
7	27.3	26.6	25.9
8	31.2	30.4	29.6
9	35.1	34.2	33.3

	36	**5**	**4**
2	7.2	1.0	0.8
3	10.8	1.5	1.2
4	14.4	2.0	1.6
5	18.0	2.5	2.0
6	21.6	3.0	2.4
7	25.2	3.5	2.8
8	28.8	4.0	3.2
9	32.4	4.5	3.6

From the top:

For **18°+** or **198°+**, read as printed; for **108°+** or **288°+**, read co-function.

From the bottom:

For **71°+** or **251°+**, read as printed; for **161°+** or **341°+**, read co-function.

Prop. Pts.

71° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.51 264	37	9.53 697	41	0.46 303	9.97 567	4	**60**
1	9.51 301	37	9.53 738	41	0.46 262	9.97 563	5	59
2	9.51 338	36	9.53 779	41	0.46 221	9.97 558	4	58
3	9.51 374	37	9.53 820	41	0.46 180	9.97 554	4	57
4	9.51 411	36	9.53 861	41	0.46 139	9.97 550	5	56
5	9.51 447	37	9.53 902	41	0.46 098	9.97 545	4	**55**
6	9.51 484	36	9.53 943	41	0.46 057	9.97 541	5	54
7	9.51 520	37	9.53 984	41	0.46 016	9.97 536	4	53
8	9.51 557	36	9.54 025	40	0.45 975	9.97 532	4	52
9	9.51 593	36	9.54 065	41	0.45 935	9.97 528	5	51
10	9.51 629	37	9.54 106	41	0.45 894	9.97 523	4	**50**
11	9.51 666	36	9.54 147	40	0.45 853	9.97 519	4	49
12	9.51 702	36	9.54 187	41	0.45 813	9.97 515	5	48
13	9.51 738	36	9.54 228	41	0.45 772	9.97 510	4	47
14	9.51 774	37	9.54 269	40	0.45 731	9.97 506	5	46
15	9.51 811	36	9.54 309	41	0.45 691	9.97 501	4	**45**
16	9.51 847	36	9.54 350	40	0.45 650	9.97 497	5	44
17	9.51 883	36	9.54 390	41	0.45 610	9.97 492	4	43
18	9.51 919	36	9.54 431	40	0.45 569	9.97 488	4	42
19	9.51 955	36	9.54 471	41	0.45 529	9.97 484	5	41
20	9.51 991	36	9.54 512	40	0.45 488	9.97 479	4	**40**
21	9.52 027	36	9.54 552	41	0.45 448	9.97 475	5	39
22	9.52 063	36	9.54 593	40	0.45 407	9.97 470	4	38
23	9.52 099	36	9.54 633	40	0.45 367	9.97 466	5	37
24	9.52 135	36	9.54 673	41	0.45 327	9.97 461	4	36
25	9.52 171	36	9.54 714	40	0.45 286	9.97 457	4	**35**
26	9.52 207	35	9.54 754	40	0.45 246	9.97 453	5	34
27	9.52 242	36	9.54 794	41	0.45 206	9.97 448	4	33
28	9.52 278	36	9.54 835	40	0.45 165	9.97 444	5	32
29	9.52 314	36	9.54 875	40	0.45 125	9.97 439	4	31
30	9.52 350	35	9.54 915	40	0.45 085	9.97 435	5	**30**
31	9.52 385	36	9.54 955	40	0.45 045	9.97 430	4	29
32	9.52 421	35	9.54 995	40	0.45 005	9.97 426	5	28
33	9.52 456	36	9.55 035	40	0.44 965	9.97 421	4	27
34	9.52 492	35	9.55 075	40	0.44 925	9.97 417	5	26
35	9.52 527	36	9.55 115	40	0.44 885	9.97 412	4	**25**
36	9.52 563	35	9.55 155	40	0.44 845	9.97 408	5	24
37	9.52 598	36	9.55 195	40	0.44 805	9.97 403	4	23
38	9.52 634	35	9.55 235	40	0.44 765	9.97 399	5	22
39	9.52 669	36	9.55 275	40	0.44 725	9.97 394	4	21
40	9.52 705	35	9.55 315	40	0.44 685	9.97 390	5	**20**
41	9.52 740	35	9.55 355	40	0.44 645	9.97 385	4	19
42	9.52 775	36	9.55 395	39	0.44 605	9.97 381	5	18
43	9.52 811	35	9.55 434	40	0.44 566	9.97 376	4	17
44	9.52 846	35	9.55 474	40	0.44 526	9.97 372	5	16
45	9.52 881	35	9.55 514	40	0.44 486	9.97 367	4	**15**
46	9.52 916	35	9.55 554	39	0.44 446	9.97 363	5	14
47	9.52 951	35	9.55 593	40	0.44 407	9.97 358	5	13
48	9.52 986	35	9.55 633	40	0.44 367	9.97 353	4	12
49	9.53 021	35	9.55 673	39	0.44 327	9.97 349	5	11
50	9.53 056	36	9.55 712	40	0.44 288	9.97 344	4	**10**
51	9.53 092	34	9.55 752	39	0.44 248	9.97 340	5	9
52	9.53 126	35	9.55 791	40	0.44 209	9.97 335	4	8
53	9.53 161	35	9.55 831	39	0.44 169	9.97 331	5	7
54	9.53 196	35	9.55 870	40	0.44 130	9.97 326	4	6
55	9.53 231	35	9.55 910	39	0.44 090	9.97 322	5	**5**
56	9.53 266	35	9.55 949	40	0.44 051	9.97 317	5	4
57	9.53 301	35	9.55 989	39	0.44 011	9.97 312	4	3
58	9.53 336	34	9.56 028	39	0.43 972	9.97 308	5	2
59	9.53 370	35	9.56 067	40	0.43 933	9.97 303	4	1
60	9.53 405		9.56 107		0.43 893	9.97 299		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	41	40	39
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	37	36	35
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

	34	5	4
2	6.8	1.0	0.8
3	10.2	1.5	1.2
4	13.6	2.0	1.6
5	17.0	2.5	2.0
6	20.4	3.0	2.4
7	23.8	3.5	2.8
8	27.2	4.0	3.2
9	30.6	4.5	3.6

From the top:

For **19°+** or **199°+**, read as printed; for **109°+** or **289°+**, read co-function.

From the bottom:

For **70°+** or **250°+**, read as printed; for **160°+** or **340°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.53 405	35	9.56 107	39	0.43 893	9.97 299	5	**60**
1	9.53 440	35	9.56 146	39	0.43 854	9.97 294	5	59
2	9.53 475	34	9.56 185	39	0.43 815	9.97 289	4	58
3	9.53 509	35	9.56 224	40	0.43 776	9.97 285	5	57
4	9.53 544	34	9.56 264	39	0.43 736	9.97 280	4	56
5	9.53 578	35	9.56 303	39	0.43 697	9.97 276	5	**55**
6	9.53 613	34	9.56 342	39	0.43 658	9.97 271	5	54
7	9.53 647	35	9.56 381	39	0.43 619	9.97 266	4	53
8	9.53 682	34	9.56 420	39	0.43 580	9.97 262	5	52
9	9.53 716	35	9.56 459	39	0.43 541	9.97 257	5	51
10	9.53 751	34	9.56 498	39	0.43 502	9.97 252	4	**50**
11	9.53 785	34	9.56 537	39	0.43 463	9.97 248	5	49
12	9.53 819	35	9.56 576	39	0.43 424	9.97 243	5	48
13	9.53 854	34	9.56 615	39	0.43 385	9.97 238	4	47
14	9.53 888	34	9.56 654	39	0.43 346	9.97 234	5	46
15	9.53 922	35	9.56 693	39	0.43 307	9.97 229	5	**45**
16	9.53 957	34	9.56 732	39	0.43 268	9.97 224	4	44
17	9.53 991	34	9.56 771	39	0.43 229	9.97 220	5	43
18	9.54 025	34	9.56 810	39	0.43 190	9.97 215	5	42
19	9.54 059	34	9.56 849	38	0.43 151	9.97 210	4	41
20	9.54 093	34	9.56 887	39	0.43 113	9.97 206	5	**40**
21	9.54 127	34	9.56 926	39	0.43 074	9.97 201	5	39
22	9.54 161	34	9.56 965	39	0.43 035	9.97 196	4	38
23	9.54 195	34	9.57 004	38	0.42 996	9.97 192	5	37
24	9.54 229	34	9.57 042	39	0.42 958	9.97 187	5	36
25	9.54 263	34	9.57 081	39	0.42 919	9.97 182	4	**35**
26	9.54 297	34	9.57 120	38	0.42 880	9.97 178	5	34
27	9.54 331	34	9.57 158	39	0.42 842	9.97 173	5	33
28	9.54 365	34	9.57 197	38	0.42 803	9.97 168	5	32
29	9.54 399	34	9.57 235	39	0.42 765	9.97 163	4	31
30	9.54 433	33	9.57 274	38	0.42 726	9.97 159	5	**30**
31	9.54 466	34	9.57 312	39	0.42 688	9.97 154	5	29
32	9.54 500	34	9.57 351	38	0.42 649	9.97 149	4	28
33	9.54 534	33	9.57 389	39	0.42 611	9.97 145	5	27
34	9.54 567	34	9.57 428	38	0.42 572	9.97 140	5	26
35	9.54 601	34	9.57 466	38	0.42 534	9.97 135	5	**25**
36	9.54 635	33	9.57 504	39	0.42 496	9.97 130	4	24
37	9.54 668	34	9.57 543	38	0.42 457	9.97 126	5	23
38	9.54 702	33	9.57 581	38	0.42 419	9.97 121	5	22
39	9.54 735	34	9.57 619	39	0.42 381	9.97 116	5	21
40	9.54 769	33	9.57 658	38	0.42 342	9.97 111	4	**20**
41	9.54 802	34	9.57 696	38	0.42 304	9.97 107	5	19
42	9.54 836	33	9.57 734	38	0.42 266	9.97 102	5	18
43	9.54 869	34	9.57 772	38	0.42 228	9.97 097	5	17
44	9.54 903	33	9.57 810	39	0.42 190	9.97 092	5	16
45	9.54 936	33	9.57 849	38	0.42 151	9.97 087	4	**15**
46	9.54 969	34	9.57 887	38	0.42 113	9.97 083	5	14
47	9.55 003	33	9.57 925	38	0.42 075	9.97 078	5	13
48	9.55 036	33	9.57 963	38	0.42 037	9.97 073	5	12
49	9.55 069	33	9.58 001	38	0.41 999	9.97 068	5	11
50	9.55 102	34	9.58 039	38	0.41 961	9.97 063	4	**10**
51	9.55 136	33	9.58 077	38	0.41 923	9.97 059	5	9
52	9.55 169	33	9.58 115	38	0.41 885	9.97 054	5	8
53	9.55 202	33	9.58 153	38	0.41 847	9.97 049	5	7
54	9.55 235	33	9.58 191	38	0.41 809	9.97 044	5	6
55	9.55 268	33	9.58 229	38	0.41 771	9.97 039	4	**5**
56	9.55 301	33	9.58 267	37	0.41 733	9.97 035	5	4
57	9.55 334	33	9.58 304	38	0.41 696	9.97 030	5	3
58	9.55 367	33	9.58 342	38	0.41 658	9.97 025	5	2
59	9.55 400	33	9.58 380	38	0.41 620	9.97 020	5	1
60	9.55 433		9.58 418		0.41 582	9.97 015		**0**
	L Cos	**d**	**L Ctn**	**c d**	**L Tan**	**L Sin**	**d**	**′**

Prop. Pts.

	40	**39**	**38**
2	8.0	7.8	7.6
3	12.0	11.7	11.4
4	16.0	15.6	15.2
5	20.0	19.5	19.0
6	24.0	23.4	22.8
7	28.0	27.3	26.6
8	32.0	31.2	30.4
9	36.0	35.1	34.2

	37	**35**	**34**
2	7.4	7.0	6.8
3	11.1	10.5	10.2
4	14.8	14.0	13.6
5	18.5	17.5	17.0
6	22.2	21.0	20.4
7	25.9	24.5	23.8
8	29.6	28.0	27.2
9	33.3	31.5	30.6

	33	**5**	**4**
2	6.6	1.0	0.8
3	9.9	1.5	1.2
4	13.2	2.0	1.6
5	16.5	2.5	2.0
6	19.8	3.0	2.4
7	23.1	3.5	2.8
8	26.4	4.0	3.2
9	29.7	4.5	3.6

From the top :

For **20°+** or **200°+**, read as printed; for **110°+** or **290°+**, read co-function.

From the bottom .

For **69°+** or **249°+**, read as printed ; for **159°+** or **339°+**, read co-function.

Prop. Pts.

69° — Logarithms of Trigonometric Functions

21° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.55 433	33	9.58 418	37	0.41 582	9.97 015	5	**60**
1	9.55 466	33	9.58 455	38	0.41 545	9.97 010	5	59
2	9.55 499	33	9.58 493	38	0.41 507	9.97 005	4	58
3	9.55 532	32	9.58 531	38	0.41 469	9.97 001	5	57
4	9.55 564	33	9.58 569	37	0.41 431	9.96 996	5	56
5	9.55 597	33	9.58 606	38	0.41 394	9.96 991	5	**55**
6	9.55 630	33	9.58 644	37	0.41 356	9.96 986	5	54
7	9.55 663	32	9.58 681	38	0.41 319	9.96 981	5	53
8	9.55 695	33	9.58 719	38	0.41 281	9.96 976	5	52
9	9.55 728	33	9.58 757	37	0.41 243	9.96 971	5	51
10	9.55 761	32	9.58 794	38	0.41 206	9.96 966	4	**50**
11	9.55 793	33	9.58 832	37	0.41 168	9.96 962	5	49
12	9.55 826	32	9.58 869	38	0.41 131	9.96 957	5	48
13	9.55 858	33	9.58 907	37	0.41 093	9.96 952	5	47
14	9.55 891	32	9.58 944	37	0.41 056	9.96 947	5	46
15	9.55 923	33	9.58 981	38	0.41 019	9.96 942	5	**45**
16	9.55 956	32	9.59 019	37	0.40 981	9.96 937	5	44
17	9.55 988	33	9.59 056	38	0.40 944	9.96 932	5	43
18	9.56 021	32	9.59 094	37	0.40 906	9.96 927	5	42
19	9.56 053	32	9.59 131	37	0.40 869	9.96 922	5	41
20	9.56 085	33	9.59 168	37	0.40 832	9.96 917	5	**40**
21	9.56 118	32	9.59 205	38	0.40 795	9.96 912	5	39
22	9.56 150	32	9.59 243	37	0.40 757	9.96 907	4	38
23	9.56 182	33	9.59 280	37	0.40 720	9.96 903	5	37
24	9.56 215	32	9.59 317	37	0.40 683	9.96 898	5	36
25	9.56 247	32	9.59 354	37	0.40 646	9.96 893	5	**35**
26	9.56 279	32	9.59 391	38	0.40 609	9.96 888	5	34
27	9.56 311	32	9.59 429	37	0.40 571	9.96 883	5	33
28	9.56 343	32	9.59 466	37	0.40 534	9.96 878	5	32
29	9.56 375	33	9.59 503	37	0.40 497	9.96 873	5	31
30	9.56 408	32	9.59 540	37	0.40 460	9.96 868	5	**30**
31	9.56 440	32	9.59 577	37	0.40 423	9.96 863	5	29
32	9.56 472	32	9.59 614	37	0.40 386	9.96 858	5	28
33	9.56 504	32	9.59 651	37	0.40 349	9.96 853	5	27
34	9.56 536	32	9.59 688	37	0.40 312	9.96 848	5	26
35	9.56 568	31	9.59 725	37	0.40 275	9.96 843	5	**25**
36	9.56 599	32	9.59 762	37	0.40 238	9.96 838	5	24
37	9.56 631	32	9.59 799	36	0.40 201	9.96 833	5	23
38	9.56 663	32	9.59 835	37	0.40 165	9.96 828	5	22
39	9.56 695	32	9.59 872	37	0.40 128	9.96 823	5	21
40	9.56 727	32	9.59 909	37	0.40 091	9.96 818	5	**20**
41	9.56 759	31	9.59 946	37	0.40 054	9.96 813	5	19
42	9.56 790	32	9.59 983	36	0.40 017	9.96 808	5	18
43	9.56 822	32	9.60 019	37	0.39 981	9.96 803	5	17
44	9.56 854	32	9.60 056	37	0.39 944	9.96 798	5	16
45	9.56 886	31	9.60 093	37	0.39 907	9.96 793	5	**15**
46	9.56 917	32	9.60 130	36	0.39 870	9.96 788	5	14
47	9.56 949	31	9.60 166	37	0.39 834	9.96 783	5	13
48	9.56 980	32	9.60 203	37	0.39 797	9.96 778	6	12
49	9.57 012	32	9.60 240	36	0.39 760	9.96 772	5	11
50	9.57 044	31	9.60 276	37	0.39 724	9.96 767	5	**10**
51	9.57 075	32	9.60 313	36	0.39 687	9.96 762	5	9
52	9.57 107	31	9.60 349	37	0.39 651	9.96 757	5	8
53	9.57 138	31	9.60 386	36	0.39 614	9.96 752	5	7
54	9.57 169	32	9.60 422	37	0.39 578	9.96 747	5	6
55	9.57 201	31	9.60 459	36	0.39 541	9.96 742	5	**5**
56	9.57 232	32	9.60 495	37	0.39 505	9.96 737	5	4
57	9.57 264	31	9.60 532	36	0.39 468	9.96 732	5	3
58	9.57 295	31	9.60 568	37	0.39 432	9.96 727	5	2
59	9.57 326	32	9.60 605	36	0.39 395	9.96 722	5	1
60	9.57 358		9.60 641		0.39 359	9.96 717		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	38	37	36
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	33	32	31
2	6.6	6.4	6.2
3	9.9	9.6	9.3
4	13.2	12.8	12.4
5	16.5	16.0	15.5
6	19.8	19.2	18.6
7	23.1	22.4	21.7
8	26.4	25.6	24.8
9	29.7	28.8	27.9

	6	5	4
2	1.2	1.0	0.8
3	1.8	1.5	1.2
4	2.4	2.0	1.6
5	3.0	2.5	2.0
6	3.6	3.0	2.4
7	4.2	3.5	2.8
8	4.8	4.0	3.2
9	5.4	4.5	3.6

From the top:

For **21°**+ or **201°**+, read as printed; for **111°**+ or **291°**+, read co-function.

From the bottom:

For **68°**+ or **248°**+, read as printed; for **158°**+ or **338°**+, read co-function.

Prop. Pts.

68° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.57 358	31	9.60 641	36	0.39 359	9.96 717	6	**60**
1	9.57 389	31	9.60 677	37	0.39 323	9.96 711	5	59
2	9.57 420	31	9.60 714	36	0.39 286	9.96 706	5	58
3	9.57 451	31	9.60 750	36	0.39 250	9.96 701	5	57
4	9.57 482	32	9.60 786	37	0.39 214	9.96 696	5	56
5	9.57 514	31	9.60 823	36	0.39 177	9.96 691	5	**55**
6	9.57 545	31	9.60 859	36	0.39 141	9.96 686	5	54
7	9.57 576	31	9.60 895	36	0.39 105	9.96 681	5	53
8	9.57 607	31	9.60 931	36	0.39 069	9.96 676	6	52
9	9.57 638	31	9.60 967	37	0.39 033	9.96 670	5	51
10	9.57 669	31	9.61 004	36	0.38 996	9.96 665	5	**50**
11	9.57 700	31	9.61 040	36	0.38 960	9.96 660	5	49
12	9.57 731	31	9.61 076	36	0.38 924	9.96 655	5	48
13	9.57 762	31	9.61 112	36	0.38 888	9.96 650	5	47
14	9.57 793	31	9.61 148	36	0.38 852	9.96 645	5	46
15	9.57 824	31	9.61 184	36	0.38 816	9.96 640	6	**45**
16	9.57 855	30	9.61 220	36	0.38 780	9.96 634	5	44
17	9.57 885	31	9.61 256	36	0.38 744	9.96 629	5	43
18	9.57 916	31	9.61 292	36	0.38 708	9.96 624	5	42
19	9.57 947	31	9.61 328	36	0.38 672	9.96 619	5	41
20	9.57 978	30	9.61 364	36	0.38 636	9.96 614	6	**40**
21	9.58 008	31	9.61 400	36	0.38 600	9.96 608	5	39
22	9.58 039	31	9.61 436	36	0.38 564	9.96 603	5	38
23	9.58 070	31	9.61 472	36	0.38 528	9.96 598	5	37
24	9.58 101	30	9.61 508	36	0.38 492	9.96 593	5	36
25	9.58 131	31	9.61 544	35	0.38 456	9.96 588	6	**35**
26	9.58 162	30	9.61 579	36	0.38 421	9.96 582	5	34
27	9.58 192	31	9.61 615	36	0.38 385	9.96 577	5	33
28	9.58 223	30	9.61 651	36	0.38 349	9.96 572	5	32
29	9.58 253	31	9.61 687	35	0.38 313	9.96 567	5	31
30	9.58 284	30	9.61 722	36	0.38 278	9.96 562	6	**30**
31	9.58 314	31	9.61 758	36	0.38 242	9.96 556	5	29
32	9.58 345	30	9.61 794	36	0.38 206	9.96 551	5	28
33	9.58 375	31	9.61 830	35	0.38 170	9.96 546	5	27
34	9.58 406	30	9.61 865	36	0.38 135	9.96 541	6	26
35	9.58 436	31	9.61 901	35	0.38 099	9.96 535	5	**25**
36	9.58 467	30	9.61 936	36	0.38 064	9.96 530	5	24
37	9.58 497	30	9.61 972	36	0.38 028	9.96 525	5	23
38	9.58 527	30	9.62 008	35	0.37 992	9.96 520	6	22
39	9.58 557	31	9.62 043	36	0.37 957	9.96 514	5	21
40	9.58 588	30	9.62 079	35	0.37 921	9.96 509	5	**20**
41	9.58 618	30	9.62 114	36	0.37 886	9.96 504	6	19
42	9.58 648	30	9.62 150	35	0.37 850	9.96 498	5	18
43	9.58 678	31	9.62 185	36	0.37 815	9.96 493	5	17
44	9.58 709	30	9.62 221	35	0.37 779	9.96 488	5	16
45	9.58 739	30	9.62 256	36	0.37 744	9.96 483	6	**15**
46	9.58 769	30	9.62 292	35	0.37 708	9.96 477	5	14
47	9.58 799	30	9.62 327	35	0.37 673	9.96 472	5	13
48	9.58 829	30	9.62 362	36	0.37 638	9.96 467	6	12
49	9.58 859	30	9.62 398	35	0.37 602	9.96 461	5	11
50	9.58 889	30	9.62 433	35	0.37 567	9.96 456	5	**10**
51	9.58 919	30	9.62 468	36	0.37 532	9.96 451	6	9
52	9 58 949	30	9.62 504	35	0.37 496	9.96 445	5	8
53	9.58 979	30	9.62 539	35	0.37 461	9.96 440	5	7
54	9.59 009	30	9.62 574	35	0.37 426	9.96 435	6	6
55	9.59 039	30	9.62 609	36	0.37 391	9.96 429	5	**5**
56	9.59 069	29	9.62 645	35	0.37 355	9.96 424	5	4
57	9.59 098	30	9.62 680	35	0.37 320	9.96 419	6	3
58	9.59 128	30	9.62 715	35	0.37 285	9.96 413	5	2
59	9.59 158	30	9.62 750	35	0.37 250	9.96 408	5	1
60	9.59 188		9.62 785		0.37 215	9.96 403		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	37	36	35
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

	32	31	30
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

	29	6	5
2	5.8	1.2	1.0
3	8.7	1.8	1.5
4	11.6	2.4	2.0
5	14.5	3.0	2.5
6	17.4	3.6	3.0
7	20.3	4.2	3.5
8	23.2	4.8	4.0
9	26.1	5.4	4.5

From the top:

For **22°**+ or **202°**+, read as printed; for **112°**+ or **292°**+, read co-function.

From the bottom:

For **67°**+ or **247°**+, read as printed; for **157°**+ or **337°**+, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.59 188	30	9.62 785	35	0.37 215	9.96 403	6	**60**
1	9.59 218	29	9.62 820	35	0.37 180	9.96 397	5	59
2	9.59 247	30	9.62 855	35	0.37 145	9.96 392	5	58
3	9.59 277	30	9.62 890	36	0.37 110	9.96 387	6	57
4	9.59 307	29	9.62 926	35	0.37 074	9.96 381	5	56
5	9.59 336	30	9.62 961	35	0.37 039	9.96 376	6	**55**
6	9.59 366	30	9.62 996	35	0.37 004	9.96 370	5	54
7	9.59 396	29	9.63 031	35	0.36 969	9.96 365	5	53
8	9.59 425	30	9.63 066	35	0.36 934	9.96 360	6	52
9	9.59 455	29	9.63 101	34	0.36 899	9.96 354	5	51
10	9.59 484	30	9.63 135	35	0.36 865	9.96 349	6	**50**
11	9.59 514	29	9.63 170	35	0.36 830	9.96 343	5	49
12	9.59 543	30	9.63 205	35	0.36 795	9.96 338	5	48
13	9.59 573	29	9.63 240	35	0.36 760	9.96 333	6	47
14	9.59 602	30	9.63 275	35	0.36 725	9.96 327	5	46
15	9.59 632	29	9.63 310	35	0.36 690	9.96 322	6	**45**
16	9.59 661	29	9.63 345	34	0.36 655	9.96 316	5	44
17	9.59 690	30	9.63 379	35	0.36 621	9.96 311	6	43
18	9.59 720	29	9.63 414	35	0.36 586	9.96 305	5	42
19	9.59 749	29	9.63 449	35	0.36 551	9.96 300	6	41
20	9.59 778	30	9.63 484	35	0.36 516	9.96 294	5	**40**
21	9.59 808	29	9.63 519	34	0.36 481	9.96 289	5	39
22	9.59 837	29	9.63 553	35	0.36 447	9.96 284	6	38
23	9.59 866	29	9.63 588	35	0.36 412	9.96 278	5	37
24	9.59 895	29	9.63 623	34	0.36 377	9.96 273	6	36
25	9.59 924	30	9.63 657	35	0.36 343	9.96 267	5	**35**
26	9.59 954	29	9.63 692	34	0.36 308	9.96 262	6	34
27	9.59 983	29	9.63 726	35	0.36 274	9.96 256	5	33
28	9.60 012	29	9.63 761	35	0.36 239	9.96 251	6	32
29	9.60 041	29	9.63 796	34	0.36 204	9.96 245	5	31
30	9.60 070	29	9.63 830	35	0.36 170	9.96 240	6	**30**
31	9.60 099	29	9.63 865	34	0.36 135	9.96 234	5	29
32	9.60 128	29	9.63 899	35	0.36 101	9.96 229	6	28
33	9.60 157	29	9.63 934	34	0.36 066	9.96 223	5	27
34	9.60 186	29	9.63 968	35	0.36 032	9.96 218	6	26
35	9.60 215	29	9.64 003	34	0.35 997	9.96 212	5	**25**
36	9.60 244	29	9.64 037	35	0.35 963	9.96 207	6	24
37	9.60 273	29	9.64 072	34	0.35 928	9.96 201	5	23
38	9.60 302	29	9.64 106	34	0.35 894	9.96 196	6	22
39	9.60 331	28	9.64 140	35	0.35 860	9.96 190	5	21
40	9.60 359	29	9.64 175	34	0.35 825	9.96 185	6	**20**
41	9.60 388	29	9.64 209	34	0.35 791	9.96 179	5	19
42	9.60 417	29	9.64 243	35	0.35 757	9.96 174	6	18
43	9.60 446	28	9.64 278	34	0.35 722	9.96 168	6	17
44	9.60 474	29	9.64 312	34	0.35 688	9.96 162	5	16
45	9.60 503	29	9.64 346	35	0.35 654	9.96 157	6	**15**
46	9.60 532	29	9.64 381	34	0.35 619	9.96 151	5	14
47	9.60 561	28	9.64 415	34	0.35 585	9.96 146	6	13
48	9.60 589	29	9.64 449	34	0.35 551	9.96 140	5	12
49	9.60 618	28	9.64 483	34	0.35 517	9.96 135	6	11
50	9.60 646	29	9.64 517	35	0.35 483	9.96 129	6	**10**
51	9.60 675	29	9.64 552	34	0.35 448	9.96 123	5	9
52	9.60 704	28	9.64 586	34	0.35 414	9.96 118	6	8
53	9.60 732	29	9.64 620	34	0.35 380	9.96 112	5	7
54	9.60 761	28	9.64 654	34	0.35 346	9.96 107	6	6
55	9.60 789	29	9.64 688	34	0.35 312	9.96 101	6	**5**
56	9.60 818	28	9.64 722	34	0.35 278	9.96 095	5	4
57	9.60 846	29	9.64 756	34	0.35 244	9.96 090	6	3
58	9.60 875	28	9.64 790	34	0.35 210	9.96 084	5	2
59	9.60 903	28	9.64 824	34	0.35 176	9.96 079	6	1
60	9.60 931		9.64 858		0.35 142	9.96 073		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	36	**35**	**34**
2	7.2	7.0	6.8
3	10.8	10.5	10.2
4	14.4	14.0	13.6
5	18.0	17.5	17.0
6	21.6	21.0	20.4
7	25.2	24.5	23.8
8	28.8	28.0	27.2
9	32.4	31.5	30.6

	30	**29**	**28**
2	6.0	5.8	5.6
3	9.0	8.7	8.4
4	12.0	11.6	11.2
5	15.0	14.5	14.0
6	18.0	17.4	16.8
7	21.0	20.3	19.6
8	24.0	23.2	22.4
9	27.0	26.1	25.2

	6	**5**
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

From the top:

For **23°**+ or **203°**+, read as printed; for **113°**+ or **293°**+, read co-function.

From the bottom:

For **66°**+ or **246°**+, read as printed; for **156°**+ or **336°**+, read co-function.

Prop. Pts.

66° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.60 931	29	9.64 858	34	0.35 142	9.96 073	6	60
1	9.60 960	28	9.64 892	34	0.35 108	9.96 067	5	59
2	9.60 988	28	9.64 926	34	0.35 074	9.96 062	6	58
3	9.61 016	29	9.64 960	34	0.35 040	9.96 056	6	57
4	9.61 045	28	9.64 994	34	0.35 006	9.96 050	5	56
5	9.61 073	28	9.65 028	34	0.34 972	9.96 045	6	55
6	9.61 101	28	9.65 062	34	0.34 938	9.96 039	5	54
7	9.61 129	29	9.65 096	34	0.34 904	9.96 034	6	53
8	9.61 158	28	9.65 130	34	0.34 870	9.96 028	6	52
9	9.61 186	28	9.65 164	33	0.34 836	9.96 022	5	51
10	9.61 214	28	9.65 197	34	0.34 803	9.96 017	6	50
11	9.61 242	28	9.65 231	34	0.34 769	9.96 011	6	49
12	9.61 270	28	9.65 265	34	0.34 735	9.96 005	5	48
13	9.61 298	28	9.65 299	34	0.34 701	9.96 000	6	47
14	9.61 326	28	9.65 333	33	0.34 667	9.95 994	6	46
15	9.61 354	28	9.65 366	34	0.34 634	9.95 988	6	45
16	9.61 382	29	9.65 400	34	0.34 600	9.95 982	5	44
17	9.61 411	27	9.65 434	33	0.34 566	9.95 977	6	43
18	9.61 438	28	9.65 467	34	0.34 533	9.95 971	6	42
19	9.61 466	28	9.65 501	34	0.34 499	9.95 965	5	41
20	9.61 494	28	9.65 535	33	0.34 465	9.95 960	6	40
21	9.61 522	28	9.65 568	34	0.34 432	9.95 954	6	39
22	9.61 550	28	9.65 602	34	0.34 398	9.95 948	6	38
23	9.61 578	28	9.65 636	33	0.34 364	9.95 942	5	37
24	9.61 606	28	9.65 669	34	0.34 331	9.95 937	6	36
25	9.61 634	28	9.65 703	33	0.34 297	9.95 931	6	35
26	9.61 662	27	9.65 736	34	0.34 264	9.95 925	5	34
27	9.61 689	28	9.65 770	33	0.34 230	9.95 920	6	33
28	9.61 717	28	9.65 803	34	0.34 197	9.95 914	6	32
29	9.61 745	28	9.65 837	33	0.34 163	9.95 908	6	31
30	9.61 773	27	9.65 870	34	0.34 130	9.95 902	5	30
31	9.61 800	28	9.65 904	33	0.34 096	9.95 897	6	29
32	9.61 828	28	9.65 937	34	0.34 063	9.95 891	6	28
33	9.61 856	27	9.65 971	33	0.34 029	9.95 885	6	27
34	9.61 883	28	9.66 004	34	0.33 996	9.95 879	6	26
35	9.61 911	28	9.66 038	33	0.33 962	9.95 873	5	25
36	9.61 939	27	9.66 071	33	0.33 929	9.95 868	6	24
37	9.61 966	28	9.66 104	34	0.33 896	9.95 862	6	23
38	9.61 994	27	9.66 138	33	0.33 862	9.95 856	6	22
39	9.62 021	28	9.66 171	33	0.33 829	9.95 850	6	21
40	9.62.049	27	9.66 204	34	0.33 796	9.95 844	5	20
41	9.62 076	28	9.66 238	33	0.33 762	9.95 839	6	19
42	9.62 104	27	9.66 271	33	0.33 729	9.95 833	6	18
43	9.62 131	28	9.66 304	33	0.33 696	9.95 827	6	17
44	9.62 159	27	9.66 337	34	0.33 663	9.95 821	6	16
45	9.62 186	28	9.66 371	33	0.33 629	9.95 815	5	15
46	9.62 214	27	9.66 404	33	0.33 596	9.95 810	6	14
47	9.62 241	27	9.66 437	33	0.33 563	9.95 804	6	13
48	9.62 268	28	9.66 470	33	0.33 530	9.95 798	6	12
49	9.62 296	27	9.66 503	34	0.33 497	9.95 792	6	11
50	9.62 323	27	9.66 537	33	0.33 463	9.95 786	6	10
51	9.62 350	27	9.66 570	33	0.33 430	9.95 780	5	9
52	9.62 377	28	9.66 603	33	0.33 397	9.95 775	6	8
53	9.62 405	27	9.66 636	33	0.33 364	9.95 769	6	7
54	9.62 432	27	9.66 669	33	0.33 331	9.95 763	6	6
55	9.62 459	27	9.66 702	33	0.33 298	9.95 757	6	5
56	9.62 486	27	9.66 735	33	0.33 265	9.95 751	6	4
57	9.62 513	28	9.66 768	33	0.33 232	9.95 745	6	3
58	9.62 541	27	9.66 801	33	0.33 199	9.95 739	6	2
59	9.62 568	27	9.66 834	33	0.33 166	9.95 733	5	1
60	9.62 595		9.66 867		0.33 133	9.95 728		0
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	34	33	29
2	6.8	6.6	5.8
3	10.2	9.9	8.7
4	13.6	13.2	11.6
5	17.0	16.5	14.5
6	20.4	19.8	17.4
7	23.8	23.1	20.3
8	27.2	26.4	23.2
9	30.6	29.7	26.1

	28	27
2	5.6	5.4
3	8.4	8.1
4	11.2	10.8
5	14.0	13.5
6	16.8	16.2
7	19.6	18.9
8	22.4	21.6
9	25.2	24.3

	6	5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

From the top:

For **24°**+ or **204°**+, read as printed; for **114°**+ or **294°**+, read co-function.

From the bottom:

For **65°**+ or **245°**+, read as printed; for **155°**+ or **335°**+, read co-function.

Prop. Pts.

25° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.62 595	27	9.66 867	33	0.33 133	9.95 728	6	**60**
1	9.62 622	27	9.66 900	33	0.33 100	9.95 722	6	59
2	9.62 649	27	9.66 933	33	0.33 067	9.95 716	6	58
3	9.62 676	27	9.66 966	33	0.33 034	9.95 710	6	57
4	9.62 703	27	9.66 999	33	0.33 001	9.95 704	6	56
5	9.62 730	27	9.67 032	33	0.32 968	9.95 698	6	**55**
6	9.62 757	27	9.67 065	33	0.32 935	9.95 692	6	54
7	9.62 784	27	9.67 098	33	0.32 902	9.95 686	6	53
8	9.62 811	27	9.67 131	32	0.32 869	9.95 680	6	52
9	9.62 838	27	9.67 163	33	0.32 837	9.95 674	6	51
10	9.62 865	27	9.67 196	33	0.32 804	9.95 668	5	**50**
11	9.62 892	26	9.67 229	33	0.32 771	9.95 663	6	49
12	9.62 918	27	9.67 262	33	0.32 738	9.95 657	6	48
13	9.62 945	27	9.67 295	32	0.32 705	9.95 651	6	47
14	9.62 972	27	9.67 327	33	0.32 673	9.95 645	6	46
15	9.62 999	27	9.67 360	33	0.32 640	9.95 639	6	**45**
16	9.63 026	26	9.67 393	33	0.32 607	9.95 633	6	44
17	9.63 052	27	9.67 426	32	0.32 574	9.95 627	6	43
18	9.63 079	27	9.67 458	33	0.32 542	9.95 621	6	42
19	9.63 106	27	9.67 491	33	0.32 509	9.95 615	6	41
20	9.63 133	26	9.67 524	32	0.32 476	9.95 609	6	**40**
21	9.63 159	27	9.67 556	33	0.32 444	9.95 603	6	39
22	9.63 186	27	9.67 589	33	0.32 411	9.95 597	6	38
23	9.63 213	26	9.67 622	32	0.32 378	9.95 591	6	37
24	9.63 239	27	9.67 654	33	0.32 346	9.95 585	6	36
25	9.63 266	26	9.67 687	32	0.32 313	9.95 579	6	**35**
26	9.63 292	27	9.67 719	33	0.32 281	9.95 573	6	34
27	9.63 319	26	9.67 752	33	0.32 248	9.95 567	6	33
28	9.63 345	27	9.67 785	32	0.32 215	9.95 561	6	32
29	9.63 372	26	9.67 817	33	0.32 183	9.95 555	6	31
30	9.63 398	27	9.67 850	32	0.32 150	9.95 549	6	**30**
31	9.63 425	26	9.67 882	33	0.32 118	9.95 543	6	29
32	9.63 451	27	9.67 915	32	0.32 085	9.95 537	6	28
33	9.63 478	26	9.67 947	33	0.32 053	9.95 531	6	27
34	9.63 504	27	9.67 980	32	0.32 020	9.95 525	6	26
35	9.63 531	26	9.68 012	32	0.31 988	9.95 519	6	**25**
36	9.63 557	26	9.68 044	33	0.31 956	9.95 513	6	24
37	9.63 583	27	9.68 077	32	0.31 923	9.95 507	7	23
38	9.63 610	26	9.68 109	33	0.31 891	9.95 500	6	22
39	9.63 636	26	9.68 142	32	0.31 858	9.95 494	6	21
40	9.63 662	27	9.68 174	32	0.31 826	9.95 488	6	**20**
41	9.63 689	26	9.68 206	33	0.31 794	9.95 482	6	19
42	9.63 715	26	9.68 239	32	0.31 761	9.95 476	6	18
43	9.63 741	26	9.68 271	32	0.31 729	9.95 470	6	17
44	9.63 767	27	9.68 303	33	0.31 697	9.95 464	6	16
45	9.63 794	26	9.68 336	32	0.31 664	9.95 458	6	**15**
46	9.63 820	26	9.68 368	32	0.31 632	9.95 452	6	14
47	9.63 846	26	9.68 400	32	0.31 600	9.95 446	6	13
48	9.63 872	26	9.68 432	33	0.31 568	9.95 440	6	12
49	9.63 898	26	9.68 465	32	0.31 535	9.95 434	7	11
50	9.63 924	26	9.68 497	32	0.31 503	9.95 427	6	**10**
51	9.63 950	26	9.68 529	32	0.31 471	9.95 421	6	9
52	9.63 976	26	9.68 561	32	0.31 439	9.95 415	6	8
53	9.64 002	26	9.68 593	33	0.31 407	9.95 409	6	7
54	9.64 028	26	9.68 626	32	0.31 374	9.95 403	6	6
55	9.64 054	26	9.68 658	32	0.31 342	9.95 397	6	**5**
56	9.64 080	26	9.68 690	32	0.31 310	9.95 391	7	4
57	9.64 106	26	9.68 722	32	0.31 278	9.95 384	6	3
58	9.64 132	26	9.68 754	32	0.31 246	9.95 378	6	2
59	9.64 158	26	9.68 786	32	0.31 214	9.95 372	6	1
60	9.64 184		9.68 818		0.31 182	9.95 366		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	33	**32**	**27**
2	6.6	6.4	5.4
3	9.9	9.6	8.1
4	13.2	12.8	10.8
5	16.5	16.0	13.5
6	19.8	19.2	16.2
7	23.1	22.4	18.9
8	26.4	25.6	21.6
9	29.7	28.8	24.3

	26	**7**
2	5.2	1.4
3	7.8	2.1
4	10.4	2.8
5	13.0	3.5
6	15.6	4.2
7	18.2	4.9
8	20.8	5.6
9	23.4	6.3

	6	**5**
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

From the top.

For **25°+** or **205°+**, read as printed; for **115°+** or **295°+**, read co-function.

From the bottom:

For **64°+** or **244°+**, read as printed; for **154°+** or **334°+**, read co-function.

Prop. Pts.

64° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.64 184		9.68 818		0.31 182	9.95 366		**60**
1	9.64 210	26	9.68 850	32	0.31 150	9.95 360	6	59
2	9.64 236	26	9.68 882	32	0.31 118	9.95 354	6	58
3	9.64 262	26	9.68 914	32	0.31 086	9.95 348	6	57
4	9.64 288	26	9.68 946	32	0.31 054	9.95 341	7	56
5	9.64 313	25	9.68 978	32	0.31 022	9.95 335	6	**55**
6	9.64 339	26	9.69 010	32	0.30 990	9.95 329	6	54
7	9.64 365	26	9.69 042	32	0.30 958	9.95 323	6	53
8	9.64 391	26	9.69 074	32	0.30 926	9.95 317	6	52
9	9.64 417	26	9.69 106	32	0.30 894	9.95 310	7	51
10	9.64 442	25	9.69 138	32	0.30 862	9.95 304	6	**50**
11	9.64 468	26	9.69 170	32	0.30 830	9.95 298	6	49
12	9.64 494	26	9.69 202	32	0.30 798	9.95 292	6	48
13	9.64 519	25	9.69 234	32	0.30 766	9.95 286	6	47
14	9.64 545	26	9.69 266	32	0.30 734	9.95 279	7	46
15	9.64 571	26	9.69 298	32	0.30 702	9.95 273	6	**45**
16	9.64 596	25	9.69 329	31	0.30 671	9.95 267	6	44
17	9.64 622	26	9.69 361	32	0.30 639	9.95 261	6	43
18	9.64 647	25	9.69 393	32	0.30 607	9.95 254	7	42
19	9.64 673	26	9.69 425	32	0.30 575	9.95 248	6	41
20	9.64 698	25	9.69 457	32	0.30 543	9.95 242	6	**40**
21	9.64 724	26	9.69 488	31	0.30 512	9.95 236	6	39
22	9.64 749	25	9.69 520	32	0.30 480	9.95 229	7	38
23	9.64 775	26	9.69 552	32	0.30 448	9.95 223	6	37
24	9.64 800	25	9.69 584	32	0.30 416	9.95 217	6	36
25	9.64 826	26	9.69 615	31	0.30 385	9.95 211	6	**35**
26	9.64 851	25	9.69 647	32	0.30 353	9.95 204	7	34
27	9.64 877	26	9.69 679	32	0.30 321	9.95 198	6	33
28	9.64 902	25	9.69 710	31	0.30 290	9.95 192	6	32
29	9.64 927	25	9.69 742	32	0.30 258	9.95 185	7	31
30	9.64 953	26	9.69 774	32	0.30 226	9.95 179	6	**30**
31	9.64 978	25	9.69 805	31	0.30 195	9.95 173	6	29
32	9.65 003	25	9.69 837	32	0.30 163	9.95 167	6	28
33	9.65 029	26	9.69 868	31	0.30 132	9.95 160	7	27
34	9.65 054	25	9.69 900	32	0.30 100	9.95 154	6	26
35	9.65 079	25	9.69 932	32	0.30 068	9.95 148	6	**25**
36	9.65 104	25	9.69 963	31	0.30 037	9.95 141	7	24
37	9.65 130	26	9.69 995	32	0.30 005	9.95 135	6	23
38	9.65 155	25	9.70 026	31	0.29 974	9.95 129	6	22
39	9.65 180	25	9.70 058	32	0.29 942	9.95 122	7	21
40	9.65 205	25	9.70 089	31	0.29 911	9.95 116	6	**20**
41	9 65 230	25	9.70 121	32	0.29 879	9.95 110	6	19
42	9.65 255	25	9.70 152	31	0.29 848	9.95 103	7	18
43	9.65 281	26	9.70 184	32	0.29 816	9.95 097	6	17
44	9.65 306	25	9.70 215	31	0.29 785	9.95 090	7	16
45	9.65 331	25	9.70 247	32	0.29 753	9.95 084	6	**15**
46	9.65 356	25	9.70 278	31	0.29 722	9.95 078	6	14
47	9.65 381	25	9.70 309	31	0.29 691	9.95 071	7	13
48	9.65 406	25	9.70 341	32	0.29 659	9.95 065	6	12
49	9.65 431	25	9.70 372	31	0.29 628	9.95 059	6	11
50	9.65 456	25	9.70 404	32	0.29 596	9.95 052	7	**10**
51	9.65 481	25	9.70 435	31	0.29 565	9.95 046	6	9
52	9.65 506	25	9.70 466	31	0.29 534	9.95 039	7	8
53	9.65 531	25	9.70 498	32	0.29 502	9.95 033	6	7
54	9.65 556	25	9.70 529	31	0.29 471	9.95 027	6	6
55	9.65 580	24	9.70 560	31	0.29 440	9.95 020	7	**5**
56	9.65 605	25	9.70 592	32	0.29 408	9.95 014	6	4
57	9.65 630	25	9.70 623	31	0.29 377	9.95 007	7	3
58	9.65 655	25	9.70 654	31	0.29 346	9.95 001	6	2
59	9.65 680	25	9.70 685	31	0.29 315	9.94 995	6	1
60	9.65 705	25	9.70 717	32	0.29 283	9.94 988	7	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	32	31	26
2	6 4	6.2	5.2
3	9.6	9.3	7.8
4	12.8	12.4	10.4
5	16.0	15.5	13.0
6	19.2	18.6	15.6
7	22.4	21.7	18.2
8	25.6	24.8	20.8
9	28.8	27.9	23.4

	25	24
2	5.0	4.8
3	7.5	7.2
4	10.0	9.6
5	12.5	12.0
6	15.0	14.4
7	17.5	16.8
8	20.0	19.2
9	22.5	21.6

	7	6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

From the top:

For **26°**+ or **206°**+, read as printed; for **116°**+ or **296°**+, read co-function.

From the bottom:

For **63°**+ or **243°**+, read as printed; for **153°**+ or **333°**+, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.65 705	24	9.70 717	31	0.29 283	9.94 988	6	**60**
1	9.65 729	25	9.70 748	31	0.29 252	9.94 982	7	59
2	9.65 754	25	9.70 779	31	0.29 221	9.94 975	6	58
3	9.65 779	25	9.70 810	31	0.29 190	9.94 969	7	57
4	9.65 804	24	9.70 841	32	0.29 159	9.94 962	6	56
5	9.65 828	25	9.70 873	31	0.29 127	9.94 956	7	**55**
6	9.65 853	25	9.70 904	31	0.29 096	9.94 949	6	54
7	9.65 878	24	9.70 935	31	0.29 065	9.94 943	7	53
8	9.65 902	25	9.70 966	31	0.29 034	9.94 936	6	52
9	9.65 927	25	9.70 997	31	0.29 003	9.94 930	7	51
10	9.65 952	24	9.71 028	31	0.28 972	9.94 923	6	**50**
11	9.65 976	25	9.71 059	31	0.28 941	9.94 917	6	49
12	9.66 001	24	9.71 090	31	0.28 910	9.94 911	7	48
13	9.66 025	25	9.71 121	32	0.28 879	9.94 904	6	47
14	9.66 050	25	9.71 153	31	0.28 847	9.94 898	7	46
15	9.66 075	24	9.71 184	31	0.28 816	9.94 891	6	**45**
16	9.66 099	25	9.71 215	31	0.28 785	9.94 885	7	44
17	9.66 124	24	9.71 246	31	0.28 754	9.94 878	7	43
18	9.66 148	25	9.71 277	31	0.28 723	9.94 871	6	42
19	9.66 173	24	9.71 308	31	0.28 692	9.94 865	7	41
20	9.66 197	24	9.71 339	31	0.28 661	9.94 858	6	**40**
21	9.66 221	25	9.71 370	31	0.28 630	9.94 852	7	39
22	9.66 246	24	9.71 401	30	0.28 599	9.94 845	6	38
23	9.66 270	25	9.71 431	31	0.28 569	9.94 839	7	37
24	9.66 295	24	9.71 462	31	0.28 538	9.94 832	6	36
25	9.66 319	24	9.71 493	31	0.28 507	9.94 826	7	**35**
26	9.66 343	25	9.71 524	31	0.28 476	9.94 819	6	34
27	9.66 368	24	9.71 555	31	0.28 445	9.94 813	7	33
28	9.66 392	24	9.71 586	31	0.28 414	9.94 806	7	32
29	9.66 416	25	9.71 617	31	0.28 383	9.94 799	6	31
30	9.66 441	24	9.71 648	31	0.28 352	9.94 793	7	**30**
31	9.66 465	24	9.71 679	30	0.28 321	9.94 786	6	29
32	9.66 489	24	9.71 709	31	0.28 291	9.94 780	7	28
33	9.66 513	24	9.71 740	31	0.28 260	9.94 773	6	27
34	9.66 537	25	9.71 771	31	0.28 229	9.94 767	7	26
35	9.66 562	24	9.71 802	31	0.28 198	9.94 760	7	**25**
36	9.66 586	24	9.71 833	30	0.28 167	9.94 753	6	24
37	9.66 610	24	9.71 863	31	0.28 137	9.94 747	7	23
38	9.66 634	24	9.71 894	31	0.28 106	9.94 740	6	22
39	9.66 658	24	9.71 925	30	0.28 075	9.94 734	7	21
40	9.66 682	24	9.71 955	31	0.28 045	9.94 727	7	**20**
41	9.66 706	25	9.71 986	31	0.28 014	9.94 720	6	19
42	9.66 731	24	9.72 017	31	0.27 983	9.94 714	7	18
43	9.66 755	24	9.72 048	30	0.27 952	9.94 707	7	17
44	9.66 779	24	9.72 078	31	0.27 922	9.94 700	6	16
45	9.66 803	24	9.72 109	31	0.27 891	9.94 694	7	**15**
46	9.66 827	24	9.72 140	30	0.27 860	9.94 687	7	14
47	9.66 851	24	9.72 170	31	0.27 830	9.94 680	6	13
48	9.66 875	24	9.72 201	30	0.27 799	9.94 674	7	12
49	9.66 899	23	9.72 231	31	0.27 769	9.94 667	7	11
50	9.66 922	24	9.72 262	31	0.27 738	9.94 660	6	**10**
51	9.66 946	24	9.72 293	30	0.27 707	9.94 654	7	9
52	9.66 970	24	9.72 323	31	0.27 677	9.94 647	7	8
53	9.66 994	24	9.72 354	30	0.27 646	9.94 640	6	7
54	9.67 018	24	9.72 384	31	0.27 616	9.94 634	7	6
55	9.67 042	24	9.72 415	30	0.27 585	9.94 627	7	**5**
56	9.67 066	24	9.72 445	31	0.27 555	9.94 620	6	4
57	9.67 090	23	9.72 476	30	0.27 524	9.94 614	7	3
58	9.67 113	24	9.72 506	31	0.27 494	9.94 607	7	2
59	9.67 137	24	9.72 537	30	0.27 463	9.94 600	7	1
60	9.67 161		9.72 567		0.27 433	9.94 593		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	32	**31**	**30**
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

	25	**24**	**23**
2	5.0	4.8	4.6
3	7.5	7.2	6.9
4	10.0	9.6	9.2
5	12.5	12.0	11.5
6	15.0	14.4	13.8
7	17.5	16.8	16.1
8	20.0	19.2	18.4
9	22.5	21.6	20.7

	7	**6**
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

From the top:

For **27°+** or **207°+**, read as printed; for **117°+** or **297°+**, read co-function.

From the bottom:

For **62°+** or **242°+**, read as printed; for **152°+** or **332°+**, read co-function.

Prop. Pts.

28° — Logarithms of Trigonometric Functions

'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.67 161	24	9.72 567	31	0.27 433	9.94 593	6	**60**
1	9.67 185	23	9.72 598	30	0.27 402	9.94 587	7	59
2	9.67 208	24	9.72 628	31	0.27 372	9.94 580	7	58
3	9.67 232	24	9.72 659	30	0.27 341	9.94 573	6	57
4	9.67 256	24	9.72 689	31	0.27 311	9.94 567	7	56
5	9.67 280	23	9.72 720	30	0.27 280	9.94 560	7	**55**
6	9.67 303	24	9.72 750	30	0.27 250	9.94 553	7	54
7	9.67 327	23	9.72 780	31	0.27 220	9.94 546	6	53
8	9.67 350	24	9.72 811	30	0.27 189	9.94 540	7	52
9	9.67 374	24	9.72 841	31	0.27 159	9.94 533	7	51
10	9.67 398	23	9.72 872	30	0.27 128	9.94 526	7	**50**
11	9.67 421	24	9.72 902	30	0.27 098	9.94 519	6	49
12	9.67 445	23	9.72 932	31	0.27 068	9.94 513	7	48
13	9.67 468	24	9.72 963	30	0.27 037	9.94 506	7	47
14	9.67 492	23	9.72 993	30	0.27 007	9.94 499	7	46
15	9.67 515	24	9.73 023	31	0.26 977	9.94 492	7	**45**
16	9.67 539	23	9.73 054	30	0.26 946	9.94 485	6	44
17	9.67 562	24	9.73 084	30	0.26 916	9.94 479	7	43
18	9.67 586	23	9.73 114	30	0.26 886	9.94 472	7	42
19	9.67 609	24	9.73 144	31	0.26 856	9.94 465	7	41
20	9.67 633	23	9.73 175	30	0.26 825	9.94 458	7	**40**
21	9.67 656	24	9.73 205	30	0.26 795	9.94 451	6	39
22	9.67 680	23	9.73 235	30	0.26 765	9.94 445	7	38
23	9.67 703	23	9.73 265	30	0.26 735	9.94 438	7	37
24	9.67 726	24	9.73 295	31	0.26 705	9.94 431	7	36
25	9.67 750	23	9.73 326	30	0.26 674	9.94 424	7	**35**
26	9.67 773	23	9.73 356	30	0.26 644	9.94 417	7	34
27	9.67 796	24	9.73 386	30	0.26 614	9.94 410	6	33
28	9.67 820	23	9.73 416	30	0.26 584	9.94 404	7	32
29	9.67 843	23	9.73 446	30	0.26 554	9.94 397	7	31
30	9.67 866	24	9.73 476	31	0.26 524	9.94 390	7	**30**
31	9.67 890	23	9.73 507	30	0.26 493	9.94 383	7	29
32	9.67 913	23	9.73 537	30	0.26 463	9.94 376	7	28
33	9.67 936	23	9.73 567	30	0.26 433	9.94 369	7	27
34	9.67 959	23	9.73 597	30	0.26 403	9.94 362	7	26
35	9.67 982	24	9.73 627	30	0.26 373	9.94 355	6	**25**
36	9.68 006	23	9.73 657	30	0.26 343	9.94 349	7	24
37	9.68 029	23	9.73 687	30	0.26 313	9.94 342	7	23
38	9.68 052	23	9.73 717	30	0.26 283	9.94 335	7	22
39	9.68 075	23	9.73 747	30	0.26 253	9.94 328	7	21
40	9.68 098	23	9.73 777	30	0.26 223	9.94 321	7	**20**
41	9.68 121	23	9.73 807	30	0.26 193	9.94 314	7	19
42	9.68 144	23	9.73 837	30	0.26 163	9.94 307	7	18
43	9.68 167	23	9.73 867	30	0.26 133	9.94 300	7	17
44	9.68 190	23	9.73 897	30	0.26 103	9.94 293	7	16
45	9.68 213	24	9.73 927	30	0.26 073	9.94 286	7	**15**
46	9.68 237	23	9.73 957	30	0.26 043	9.94 279	6	14
47	9.68 260	23	9.73 987	30	0.26 013	9.94 273	7	13
48	9.68 283	22	9.74 017	30	0.25 983	9.94 266	7	12
49	9.68 305	23	9.74 047	30	0.25 953	9.94 259	7	11
50	9.68 328	23	9.74 077	30	0.25 923	9.94 252	7	**10**
51	9.68 351	23	9.74 107	30	0.25 893	9.94 245	7	9
52	9.68 374	23	9.74 137	29	0.25 863	9.94 238	7	8
53	9.68 397	23	9.74 166	30	0.25 834	9.94 231	7	7
54	9.68 420	23	9.74 196	30	0.25 804	9.94 224	7	6
55	9.68 443	23	9.74 226	30	0.25 774	9.94 217	7	**5**
56	9.68 466	23	9.74 256	30	0.25 744	9.94 210	7	4
57	9.68 489	23	9.74 286	30	0.25 714	9.94 203	7	3
58	9.68 512	22	9.74 316	29	0.25 684	9.94 196	7	2
59	9.68 534	23	9.74 345	30	0.25 655	9.94 189	7	1
60	9.68 557		9.74 375		0.25 625	9.94 182		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	'

Prop. Pts.

	31	**30**	**29**
2	6.2	6.0	5.8
3	9.3	9.0	8.7
4	12.4	12.0	11.6
5	15.5	15.0	14.5
6	18.6	18.0	17.4
7	21.7	21.0	20.3
8	24.8	24.0	23.2
9	27.9	27.0	26.1

	24	**23**	**22**
2	4.8	4.6	4.4
3	7.2	6.9	6.6
4	9.6	9.2	8.8
5	12.0	11.5	11.0
6	14.4	13.8	13.2
7	16.8	16.1	15.4
8	19.2	18.4	17.6
9	21.6	20.7	19.8

	7	**6**
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

From the top :

For **28°**+ or **208°**+, read as printed; for **118°**+ or **298°**+, read co-function.

From the bottom :

For **61°**+ or **241°**+, read as printed; for **151°**+ or **331°**+, read co-function.

Prop. Pts.

61° — Logarithms of Trigonometric Functions

'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.68 557	23	9.74 375	30	0.25 625	9.94 182	7	**60**
1	9.68 580	23	9.74 405	30	0.25 595	9.94 175	7	59
2	9.68 603	22	9.74 435	30	0.25 565	9.94 168	7	58
3	9.68 625	23	9.74 465	29	0.25 535	9.94 161	7	57
4	9.68 648	23	9.74 494	30	0.25 506	9.94 154	7	56
5	9.68 671	23	9.74 524	30	0.25 476	9.94 147	7	**55**
6	9.68 694	22	9.74 554	29	0.25 446	9.94 140	7	54
7	9.68 716	23	9.74 583	30	0.25 417	9.94 133	7	53
8	9.68 739	23	9.74 613	30	0.25 387	9.94 126	7	52
9	9.68 762	22	9.74 643	30	0.25 357	9.94 119	7	51
10	9.68 784	23	9.74 673	29	0.25 327	9.94 112	7	**50**
11	9.68 807	22	9.74 702	30	0.25 298	9.94 105	7	49
12	9.68 829	23	9.74 732	30	0.25 268	9.94 098	8	48
13	9.68 852	23	9.74 762	29	0.25 238	9.94 090	7	47
14	9.68 875	22	9.74 791	30	0.25 209	9.94 083	7	46
15	9.68 897	23	9.74 821	30	0.25 179	9.94 076	7	**45**
16	9.68 920	22	9.74 851	29	0.25 149	9.94 069	7	44
17	9.68 942	23	9.74 880	30	0.25 120	9.94 062	7	43
18	9.68 965	22	9.74 910	29	0.25 090	9.94 055	7	42
19	9.68 987	23	9.74 939	30	0.25 061	9.94 048	7	41
20	9.69 010	22	9.74 969	29	0.25 031	9.94 041	7	**40**
21	9.69 032	23	9.74 998	30	0.25 002	9.94 034	7	39
22	9.69 055	22	9.75 028	30	0.24 972	9.94 027	7	38
23	9.69 077	23	9.75 058	29	0.24 942	9.94 020	8	37
24	9.69 100	22	9.75 087	30	0.24 913	9.94 012	7	36
25	9.69 122	22	9.75 117	29	0.24 883	9.94 005	7	**35**
26	9.69 144	23	9.75 146	30	0.24 854	9.93 998	7	34
27	9.69 167	22	9.75 176	29	0 24 824	9.93 991	7	33
28	9.69 189	23	9.75 205	30	0.24 795	9.93 984	7	32
29	9.69 212	22	9.75 235	29	0.24 765	9.93 977	7	31
30	9.69 234	22	9.75 264	30	0.24 736	9.93 970	7	**30**
31	9.69 256	23	9.75 294	29	0.24 706	9.93 963	8	29
32	9.69 279	22	9.75 323	30	0.24 677	9.93 955	7	28
33	9.69 301	22	9.75 353	29	0.24 647	9.93 948	7	27
34	9.69 323	22	9.75 382	29	0.24 618	9.93 941	7	26
35	9.69 345	23	9.75 411	30	0.24 589	9.93 934	7	**25**
36	9.69 368	22	9.75 441	29	0.24 559	9.93 927	7	24
37	9.69 390	22	9.75 470	30	0.24 530	9.93 920	8	23
38	9.69 412	22	9.75 500	29	0.24 500	9.93 912	7	22
39	9.69 434	22	9.75 529	29	0.24 471	9.93 905	7	21
40	9.69 456	23	9.75 558	30	0.24 442	9.93 898	7	**20**
41	9.69 479	22	9.75 588	29	0.24 412	9.93 891	7	19
42	9.69 501	22	9.75 617	30	0.24 383	9.93 884	8	18
43	9.69 523	22	9.75 647	29	0.24 353	9.93 876	7	17
44	9.69 545	22	9.75 676	29	0.24 324	9.93 869	7	16
45	9.69 567	22	9.75 705	30	0.24 295	9.93 862	7	**15**
46	9.69 589	22	9.75 735	29	0.24 265	9.93 855	8	14
47	9.69 611	22	9.75 764	29	0.24 236	9.93 847	7	13
48	9.69 633	22	9.75 793	29	0.24 207	9.93 840	7	12
49	9.69 655	22	9.75 822	30	0.24 178	9.93 833	7	11
50	9.69 677	22	9.75 852	29	0.24 148	9.93 826	7	**10**
51	9.69 699	22	9.75 881	29	0.24 119	9.93 819	8	9
52	9.69 721	22	9.75 910	29	0.24 090	9.93 811	7	8
53	9.69 743	22	9.75 939	30	0.24 061	9.93 804	7	7
54	9.69 765	22	9.75 969	29.	0.24 031	9.93 797	8	6
55	9.69 787	22	9.75 998	29	0.24 002	9.93 789	7	**5**
56	9.69 809	22	9.76 027	29	0.23 973	9.93 782	7	4
57	9.69 831	22	9.76 056	30	0.23 944	9.93 775	7	3
58	9.69 853	22	9.76 086	29	0.23 914	9.93 768	8	2
59	9.69 875	22	9.76 115	29	0.23 885	9.93 760	7	1
60	9.69 897		9.76 144		0.23 856	9.93 753		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	'

Prop. Pts.

	30	29	23
2	6.0	5.8	4 6
3	9.0	8.7	6.9
4	12.0	11.6	9.2
5	15.0	14.5	11.5
6	18 0	17.4	13.8
7	21.0	20.3	16.1
8	24.0	23.2	18.4
9	27.0	26.1	20.7

	22	8	7
2	4.4	1.6	1.4
3	6.6	2.4	2.1
4	8.8	3.2	2.8
5	11.0	4.0	3.5
6	13.2	4.8	4.2
7	15.4	5.6	4.9
8	17.6	6.4	5.6
9	19.8	7.2	6.3

From the top:

For **29°+** or **209°+**, read as printed; for **119°+** or **299°+**, read co-function.

From the bottom:

For **60°+** or **240°+**, read as printed; for **150°+** or **330°+**, read co-function.

Prop. Pts.

60° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.69 897	22	9.76 144	29	0.23 856	9.93 753	7	**60**
1	9.69 919	22	9.76 173	29	0.23 827	9.93 746	8	59
2	9.69 941	22	9.76 202	29	0.23 798	9.93 738	7	58
3	9.69 963	21	9.76 231	30	0.23 769	9.93 731	7	57
4	9.69 984	22	9.76 261	29	0.23 739	9.93 724	7	56
5	9.70 006	22	9.76 290	29	0.23 710	9.93 717	8	**55**
6	9.70 028	22	9.76 319	29	0.23 681	9.93 709	7	54
7	9.70 050	22	9.76 348	29	0.23 652	9.93 702	7	53
8	9.70 072	21	9.76 377	29	0.23 623	9.93 695	8	52
9	9.70 093	22	9.76 406	29	0.23 594	9.93 687	7	51
10	9.70 115	22	9.76 435	29	0.23 565	9.93 680	7	**50**
11	9.70 137	22	9.76 464	29	9.23 536	9.93 673	8	49
12	9.70 159	21	9.76 493	29	0.23 507	9.93 665	7	48
13	9.70 180	22	9.76 522	29	0.23 478	9.93 658	8	47
14	9.70 202	22	9.76 551	29	0.23 449	9.93 650	7	46
15	9.70 224	21	9.76 580	29	0.23 420	9.93 643	7	**45**
16	9.70 245	22	9.76 609	30	0.23 391	9.93 636	8	44
17	9.70 267	21	9.76 639	29	0.23 361	9.93 628	7	43
18	9.70 288	22	9.76 668	29	0.23 332	9.93 621	7	42
19	9.70 310	22	9.76 697	28	0.23 303	9.93 614	8	41
20	9.70 332	21	9.76 725	29	0.23 275	9.93 606	7	**40**
21	9.70 353	22	9.76 754	29	0.23 246	9.93 599	8	39
22	9.70 375	21	9.76 783	29	0.23 217	9.93 591	7	38
23	9.70 396	22	9.76 812	29	0.23 188	9.93 584	7	37
24	9.70 418	21	9.76 841	29	0.23 159	9.93 577	8	36
25	9.70 439	22	9.76 870	29	0.23 130	9.93 569	7	**35**
26	9.70 461	21	9.76 899	29	0.23 101	9.93 562	8	34
27	9.70 482	22	9.76 928	29	0.23 072	9.93 554	7	33
28	9.70 504	21	9.76 957	29	0.23 043	9.93 547	8	32
29	9.70 525	22	9.76 986	29	0.23 014	9.93 539	7	31
30	9.70 547	21	9.77 015	29	0.22 985	9.93 532	7	**30**
31	9.70 568	22	9.77 044	29	0.22 956	9.93 525	8	29
32	9.70 590	21	9.77 073	28	0.22 927	9.93 517	7	28
33	9.70 611	22	9.77 101	29	0.22 899	9.93 510	8	27
34	9.70 633	21	9.77 130	29	0.22 870	9.93 502	7	26
35	9.70 654	21	9.77 159	29	0.22 841	9.93 495	8	**25**
36	9.70 675	22	9.77 188	29	0.22 812	9.93 487	7	24
37	9.70 697	21	9.77 217	29	0.22 783	9.93 480	8	23
38	9.70 718	21	9.77 246	28	0.22 754	9.93 472	7	22
39	9.70 739	22	9.77 274	29	0.22 726	9.93 465	8	21
40	9.70 761	21	9.77 303	29	0.22 697	9.93 457	7	**20**
41	9.70 782	21	9.77 332	29	0.22 668	9.93 450	8	19
42	9.70 803	21	9.77 361	29	0.22 639	9.93 442	7	18
43	9.70 824	22	9.77 390	28	0.22 610	9.93 435	8	17
44	9.70 846	21	9.77 418	29	0.22 582	9.93 427	7	16
45	9.70 867	21	9.77 447	29	0.22 553	9.93 420	8	**15**
46	9.70 888	21	9.77 476	29	0.22 524	9.93 412	7	14
47	9.70 909	22	9.77 505	28	0.22 495	9.93 405	8	13
48	9.70 931	21	9.77 533	29	0.22 467	9.93 397	7	12
49	9.70 952	21	9.77 562	29	0.22 438	9.93 390	8	11
50	9.70 973	21	9.77 591	28	0.22 409	9.93 382	7	**10**
51	9.70 994	21	9.77 619	29	0.22 381	9.93 375	8	9
52	9.71 015	21	9.77 648	29	0.22 352	9.93 367	7	8
53	9.71 036	22	9.77 677	29	0.22 323	9.93 360	8	7
54	9.71 058	21	9.77 706	28	0.22 294	9.93 352	8	6
55	9.71 079	21	9.77 734	29	0.22 266	9.93 344	7	**5**
56	9.71 100	21	9.77 763	28	0.22 237	9.93 337	8	4
57	9.71 121	21	9.77 791	29	0.22 209	9.93 329	7	3
58	9.71 142	21	9.77 820	29	0.22 180	9.93 322	8	2
59	9.71 163	21	9.77 849	28	0.22 151	9.93 314	7	1
60	9.71 184		9.77 877		0.22 123	9.93 307		**0**
	L Cos	**d**	**L Ctn**	**c d**	**L Tan**	**L Sin**	**d**	′

Prop. Pts.

	30	**29**	**28**
2	6.0	5.8	5.6
3	9.0	8.7	8.4
4	12.0	11.6	11.2
5	15.0	14.5	14.0
6	18.0	17.4	16.8
7	21.0	20.3	19.6
8	24.0	23.2	22.4
9	27.0	26.1	25.2

	22	**21**
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	8	**7**
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

From the top:

For **30°+** or **210°+**, read as printed; for **120°+** or **300°+**, read co-function.

From the bottom:

For **59°+** or **239°+**, read as printed; for **149°+** or **329°+**, read co-function.

Prop. Pts.

	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.71 184	21	9.77 877	29	0.22 123	9.93 307	8	**60**
1	9.71 205	21	9.77 906	29	0.22 094	9.93 299	8	59
2	9.71 226	21	9.77 935	28	0.22 065	9.93 291	7	58
3	9.71 247	21	9.77 963	29	0.22 037	9.93 284	8	57
4	9.71 268	21	9.77 992	28	0.22 008	9.93 276	7	56
5	9.71 289	21	9.78 020	29	0.21 980	9.93 269	8	**55**
6	9.71 310	21	9.78 049	28	0.21 951	9.93 261	8	54
7	9.71 331	21	9.78 077	29	0.21 923	9.93 253	7	53
8	9.71 352	21	9.78 106	29	0.21 894	9.93 246	8	52
9	9.71 373	20	9.78 135	28	0.21 865	9.93 238	8	51
10	9.71 393	21	9.78 163	29	0.21 837	9.93 230	7	**50**
11	9.71 414	21	9.78 192	28	0.21 808	9.93 223	8	49
12	9.71 435	21	9.78 220	29	0.21 780	9.93 215	8	48
13	9.71 456	21	9.78 249	28	0.21 751	9.93 207	7	47
14	9.71 477	21	9.78 277	29	0.21 723	9.93 200	8	46
15	9.71 498	21	9.78 306	28	0.21 694	9.93 192	8	**45**
16	9.71 519	20	9.78 334	29	0.21 666	9.93 184	7	44
17	9.71 539	21	9.78 363	28	0.21 637	9.93 177	8	43
18	9.71 560	21	9.78 391	28	0.21 609	9.93 169	8	42
19	9.71 581	21	9.78 419	29	0.21 581	9.93 161	7	41
20	9.71 602	20	9.78 448	28	0.21 552	9.93 154	8	**40**
21	9.71 622	21	9.78 476	29	0.21 524	9.93 146	8	39
22	9.71 643	21	9.78 505	28	0.21 495	9.93 138	7	38
23	9.71 664	21	9.78 533	29	0.21 467	9.93 131	8	37
24	9.71 685	20	9.78 562	28	0.21 438	9.93 123	8	36
25	9.71 705	21	9.78 590	28	0.21 410	9.93 115	7	**35**
26	9.71 726	21	9.78 618	29	0.21 382	9.93 108	8	34
27	9.71 747	20	9.78 647	28	0.21 353	9.93 100	8	33
28	9.71 767	21	9.78 675	29	0.21 325	9.93 092	8	32
29	9.71 788	21	9.78 704	28	0.21 296	9.93 084	7	31
30	9.71 809	20	9.78 732	28	0.21 268	9.93 077	8	**30**
31	9.71 829	21	9.78 760	29	0.21 240	9.93 069	8	29
32	9.71 850	20	9.78 789	28	0.21 211	9.93 061	8	28
33	9.71 870	21	9.78 817	28	0.21 183	9.93 053	7	27
34	9.71 891	20	9.78 845	29	0.21 155	9.93 046	8	26
35	9.71 911	21	9.78 874	28	0.21 126	9.93 038	8	**25**
36	9.71 932	20	9.78 902	28	0.21 098	9.93 030	8	24
37	9.71 952	21	9.78 930	29	0.21 070	9.93 022	8	23
38	9.71 973	21	9.78 959	28	0.21 041	9.93 014	7	22
39	9.71 994	20	9.78 987	28	0.21 013	9.93 007	8	21
40	9.72 014	20	9.79 015	28	0.20 985	9.92 999	8	**20**
41	9.72 034	21	9.79 043	29	0.20 957	9.92 991	8	19
42	9.72 055	20	9.79 072	28	0.20 928	9.92 983	7	18
43	9.72 075	21	9.79 100	28	0.20 900	9.92 976	8	17
44	9.72 096	20	9.79 128	28	0.20 872	9.92 968	8	16
45	9.72 116	21	9.79 156	29	0.20 844	9.92 960	8	**15**
46	9.72 137	20	9.79 185	28	0.20 815	9.92 952	8	14
47	9.72 157	20	9.79 213	28	0.20 787	9.92 944	8	13
48	9.72 177	21	9.79 241	28	0.20 759	9.92 936	7	12
49	9.72 198	20	9.79 269	28	0.20 731	9.92 929	8	11
50	9.72 218	20	9.79 297	29	0.20 703	9.92 921	8	**10**
51	9.72 238	21	9.79 326	28	0.20 674	9.92 913	8	9
52	9.72 259	20	9.79 354	28	0.20 646	9.92 905	8	8
53	9.72 279	20	9.79 382	28	0.20 618	9.92 897	8	7
54	9.72 299	21	9.79 410	28	0.20 590	9.92 889	8	6
55	9.72 320	20	9.79 438	28	0.20 562	9.92 881	7	**5**
56	9.72 340	20	9.79 466	29	0.20 534	9.92 874	8	4
57	9.72 360	21	9.79 495	28	0.20 505	9.92 866	8	3
58	9.72 381	20	9.79 523	28	0.20 477	9.92 858	8	2
59	9.72 401	20	9.79 551	28	0.20 449	9.92 850	8	1
60	9.72 421		9.79 579		0.20 421	9.92 842		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	29	**28**	**21**
2	5.8	5.6	4.2
3	8.7	8.4	6.3
4	11.6	11.2	8.4
5	14.5	14.0	10.5
6	17.4	16.8	12.6
7	20.3	19.6	14.7
8	23.2	22.4	16.8
9	26.1	25.2	18.9

	20	**8**	**7**
2	4.0	1.6	1.4
3	6.0	2.4	2.1
4	8.0	3.2	2.8
5	10.0	4.0	3.5
6	12.0	4.8	4.2
7	14.0	5.6	4.9
8	16.0	6.4	5.6
9	18.0	7.2	6.3

From the top:

For **31°+** or **211°+**, read as printed; for **121°+** or **301°+**, read co-function.

From the bottom:

For **58°+** or **238°+**, read as printed; for **148°+** or **328°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.72 421	20	9.79 579	28	0.20 421	9.92 842	8	**60**
1	9.72 441	20	9.79 607	28	0.20 393	9.92 834	8	59
2	9.72 461	21	9.79 635	28	0.20 365	9.92 826	8	58
3	9.72 482	20	9.79 663	28	0.20 337	9.92 818	8	57
4	9.72 502	20	9.79 691	28	0.20 309	9.92 810	7	56
5	9.72 522	20	9.79 719	28	0.20 281	9.92 803	8	**55**
6	9.72 542	20	9.79 747	29	0.20 253	9.92 795	8	54
7	9.72 562	20	9.79 776	28	0.20 224	9.92 787	8	53
8	9.72 582	20	9.79 804	28	0.20 196	9.92 779	8	52
9	9.72 602	20	9.79 832	28	0.20 168	9.92 771	8	51
10	9.72 622	21	9.79 860	28	0.20 140	9.92 763	8	**50**
11	9.72 643	20	9.79 888	28	0.20 112	9.92 755	8	49
12	9.72 663	20	9.79 916	28	0.20 084	9.92 747	8	48
13	9.72 683	20	9.79 944	28	0.20 056	9.92 739	8	47
14	9.72 703	20	9.79 972	28	0.20 028	9.92 731	8	46
15	9.72 723	20	9.80 000	28	0.20 000	9.92 723	8	**45**
16	9.72 743	20	9.80 028	28	0.19 972	9.92 715	8	44
17	9.72 763	20	9.80 056	28	0.19 944	9.92 707	8	43
18	9.72 783	20	9.80 084	28	0.19 916	9.92 699	8	42
19	9.72 803	20	9.80 112	28	0.19 888	9.92 691	8	41
20	9.72 823	20	9.80 140	28	0.19 860	9.92 683	8	**40**
21	9.72 843	20	9.80 168	27	0.19 832	9.92 675	8	39
22	9.72 863	20	9.80 195	28	0.19 805	9.92 667	8	38
23	9.72 883	19	9.80 223	28	0.19 777	9.92 659	8	37
24	9.72 902	20	9.80 251	28	0.19 749	9.92 651	8	36
25	9.72 922	20	9.80 279	28	0.19 721	9.92 643	8	**35**
26	9.72 942	20	9.80 307	28	0.19 693	9.92 635	8	34
27	9.72 962	20	9.80 335	28	0.19 665	9.92 627	8	33
28	9.72 982	20	9.80 363	28	0.19 637	9.92 619	8	32
29	9.73 002	20	9.80 391	28	0.19 609	9.92 611	8	31
30	9.73 022	19	9.80 419	28	0.19 581	9.92 603	8	**30**
31	9.73 041	20	9.80 447	27	0.19 553	9.92 595	8	29
32	9.73 061	20	9.80 474	28	0.19 526	9.92 587	8	28
33	9.73 081	20	9.80 502	28	0.19 498	9.92 579	8	27
34	9.73 101	20	9.80 530	28	0.19 470	9.92 571	8	26
35	9.73 121	19	9.80 558	28	0.19 442	9.92 563	8	**25**
36	9.73 140	20	9.80 586	28	0.19 414	9.92 555	9	24
37	9.73 160	20	9.80 614	28	0.19 386	9.92 546	8	23
38	9.73 180	20	9.80 642	27	0.19 358	9.92 538	8	22
39	9.73 200	19	9.80 669	28	0.19 331	9.92 530	8	21
40	9.73 219	20	9.80 697	28	0.19 303	9.92 522	8	**20**
41	9.73 239	20	9.80 725	28	0.19 275	9.92 514	8	19
42	9.73 259	19	9.80 753	28	0.19 247	9.92 506	8	18
43	9.73 278	20	9.80 781	27	0.19 219	9.92 498	8	17
44	9.73 298	20	9.80 808	28	0.19 192	9.92 490	8	16
45	9.73 318	19	9.80 836	28	0.19 164	9.92 482	9	**15**
46	9.73 337	20	9.80 864	28	0.19 136	9.92 473	8	14
47	9.73 357	20	9.80 892	27	0.19 108	9.92 465	8	13
48	9.73 377	19	9.80 919	28	0.19 081	9.92 457	8	12
49	9.73 396	20	9.80 947	28	0.19 053	9.92 449	8	11
50	9.73 416	19	9.80 975	28	0.19 025	9.92 441	8	**10**
51	9.73 435	20	9.81 003	27	0.18 997	9.92 433	8	9
52	9.73 455	19	9.81 030	28	0.18 970	9.92 425	9	8
53	9.73 474	20	9.81 058	28	0.18 942	9.92 416	8	7
54	9.73 494	19	9.81 086	27	0.18 914	9.92 408	8	6
55	9.73 513	20	9.81 113	28	0.18 887	9.92 400	8	**5**
56	9.73 533	19	9.81 141	28	0.18 859	9.92 392	8	4
57	9.73 552	20	9.81 169	27	0.18 831	9.92 384	8	3
58	9.73 572	19	9.81 196	28	0.18 804	9.92 376	9	2
59	9.73 591	20	9.81 224	28	0.18 776	9.92 367	8	1
60	9.73 611		9.81 252		0.18 748	9.92 359		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	29	**28**	**27**
2	5.8	5.6	5.4
3	8.7	8.4	8.1
4	11.6	11.2	10.8
5	14.5	14.0	13.5
6	17.4	16.8	16.2
7	20.3	19.6	18.9
8	23.2	22.4	21.6
9	26.1	25.2	24.3

	21	**20**	**19**
2	4.2	4.0	3.8
3	6.3	6.0	5.7
4	8.4	8.0	7.6
5	10.5	10.0	9.5
6	12.6	12.0	11.4
7	14.7	14.0	13.3
8	16.8	16.0	15.2
9	18.9	18.0	17.1

	9	**8**	**7**
2	1.8	1.6	1.4
3	2.7	2.4	2.1
4	3.6	3.2	2.8
5	4.5	4.0	3.5
6	5.4	4.8	4.2
7	6.3	5.6	4.9
8	7.2	6.4	5.6
9	8.1	7.2	6.3

From the top:

For **32°**+ or **212°**+, read as printed; for **122°**+ or **302°**+, read co-function.

From the bottom:

For **57°**+ or **237°**+, read as printed; for **147°**+ or **327°**+, read co-function.

Prop. Pts.

33°—Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.73 611	19	9.81 252	27	0.18 748	9.92 359	8	**60**
1	9.73 630	20	9.81 279	28	0.18 721	9.92 351	8	59
2	9.73 650	19	9.81 307	28	0.18 693	9.92 343	8	58
3	9.73 669	20	9.81 335	27	0.18 665	9.92 335	9	57
4	9.73 689	19	9.81 362	28	0.18 638	9.92 326	8	56
5	9.73 708	19	9.81 390	28	0.18 610	9.92 318	8	**55**
6	9.73 727	20	9.81 418	27	0.18 582	9.92 310	8	54
7	9.73 747	19	9.81 445	28	0.18 555	9.92 302	9	53
8	9.73 766	19	9.81 473	27	0.18 527	9.92 293	8	52
9	9.73 785	20	9.81 500	28	0.18 500	9.92 285	8	51
10	9.73 805	19	9.81 528	28	0.18 472	9.92 277	8	**50**
11	9.73 824	19	9.81 556	27	0.18 444	9.92 269	9	49
12	9.73 843	20	9.81 583	28	0.18 417	9.92 260	8	48
13	9.73 863	19	9.81 611	27	0.18 389	9.92 252	8	47
14	9.73 882	19	9.81 638	28	0.18 362	9.92 244	9	46
15	9.73 901	20	9.81 666	27	0.18 334	9.92 235	8	**45**
16	9.73 921	19	9.81 693	28	0.18 307	9.92 227	8	44
17	9.73 940	19	9.81 721	27	0.18 279	9.92 219	8	43
18	9.73 959	19	9.81 748	28	0.18 252	9.92 211	9	42
19	9.73 978	19	9.81 776	27	0.18 224	9.92 202	8	41
20	9.73 997	20	9.81 803	28	0.18 197	9.92 194	8	**40**
21	9.74 017	19	9.81 831	27	0.18 169	9.92 186	9	39
22	9.74 036	19	9.81 858	28	0.18 142	9.92 177	8	38
23	9.74 055	19	9.81 886	27	0.18 114	9.92 169	8	37
24	9.74 074	19	9.81 913	28	0.18 087	9.92 161	9	36
25	9.74 093	20	9.81 941	27	0.18 059	9.92 152	8	**35**
26	9.74 113	19	9.81 968	28	0.18 032	9.92 144	8	34
27	9.74 132	19	9.81 996	27	0.18 004	9.92 136	9	33
28	9.74 151	19	9.82 023	28	0.17 977	9.92 127	8	32
29	9.74 170	19	9.82 051	27	0.17 949	9.92 119	8	31
30	9.74 189	19	9.82 078	28	0.17 922	9.92 111	9	**30**
31	9.74 208	19	9.82 106	27	0.17 894	9.92 102	8	29
32	9.74 227	19	9.82 133	28	0.17 867	9.92 094	8	28
33	9.74 246	19	9.82 161	27	0.17 839	9.92 086	9	27
34	9.74 265	19	9.82 188	27	0.17 812	9.92 077	8	26
35	9.74 284	19	9.82 215	28	0.17 785	9.92 069	9	**25**
36	9.74 303	19	9.82 243	27	0.17 757	9.92 060	8	24
37	9.74 322	19	9.82 270	28	0.17 730	9.92 052	8	23
38	9.74 341	19	9.82 298	27	0.17 702	9.92 044	9	22
39	9.74 360	19	9.82 325	27	0.17 675	9.92 035	8	21
40	9.74 379	19	9.82 352	28	0.17 648	9.92 027	9	**20**
41	9.74 398	19	9.82 380	27	0.17 620	9.92 018	8	19
42	9.74 417	19	9.82 407	28	0.17 593	9.92 010	8	18
43	9.74 436	19	9.82 435	27	0.17 565	9.92 002	9	17
44	9.74 455	19	9.82 462	27	0.17 538	9.91 993	8	16
45	9.74 474	19	9.82 489	28	0.17 511	9.91 985	9	**15**
46	9.74 493	19	9.82 517	27	0.17 483	9.91 976	8	14
47	9.74 512	19	9.82 544	27	0.17 456	9.91 968	9	13
48	9.74 531	18	9.82 571	28	0.17 429	9.91 959	8	12
49	9.74 549	19	9.82 599	27	0.17 401	9.91 951	9	11
50	9.74 568	19	9.82 626	27	0.17 374	9.91 942	8	**10**
51	9.74 587	19	9.82 653	28	0.17 347	9.91 934	9	9
52	9.74 606	19	9.82 681	27	0.17 319	9.91 925	8	8
53	9.74 625	19	9.82 708	27	0.17 292	9.91 917	9	7
54	9.74 644	18	9.82 735	27	0.17 265	9.91 908	8	6
55	9.74 662	19	9.82 762	28	0.17 238	9.91 900	9	**5**
56	9.74 681	19	9.82 790	27	0.17 210	9.91 891	8	4
57	9.74 700	19	9.82 817	27	0.17 183	9.91 883	9	3
58	9.74 719	18	9.82 844	27	0.17 156	9.91 874	8	2
59	9.74 737	19	9.82 871	28	0.17 129	9.91 866	9	1
60	9.74 756		9.82 899		0.17 101	9.91 857		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	28	27	20
2	5.6	5.4	4.0
3	8.4	8.1	6.0
4	11.2	10.8	8.0
5	14.0	13.5	10.0
6	16.8	16.2	12.0
7	19.6	18.9	14.0
8	22.4	21.6	16.0
9	25.2	24.3	18.0

	19	18
2	3.8	3.6
3	5.7	5.4
4	7.6	7.2
5	9.5	9.0
6	11.4	10.8
7	13.3	12.6
8	15.2	14.4
9	17.1	16.2

	9	8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

From the top:

For **33°+** or **213°+**, read as printed; for **123°+** or **303°+**, read co-function.

From the bottom:

For **56°+** or **236°+**, read as printed; for **146°+** or **326°+**, read co-function.

Prop. Pts.

56°—Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.74 756	19	9.82 899	27	0.17 101	9.91 857	8	**60**
1	9.74 775	19	9.82 926	27	0.17 074	9.91 849	9	59
2	9.74 794	18	9.82 953	27	0.17 047	9.91 840	8	58
3	9.74 812	19	9.82 980	28	0.17 020	9.91 832	9	57
4	9.74 831	19	9.83 008	27	0.16 992	9.91 823	8	56
5	9.74 850	18	9.83 035	27	0.16 965	9.91 815	9	**55**
6	9.74 868	19	9.83 062	27	0.16 938	9.91 806	8	54
7	9.74 887	19	9.83 089	28	0.16 911	9.91 798	9	53
8	9.74 906	18	9.83 117	27	0.16 883	9.91 789	8	52
9	9.74 924	19	9.83 144	27	0.16 856	9.91 781	9	51
10	9.74 943	18	9.83 171	27	0.16 829	9.91 772	9	**50**
11	9.74 961	19	9.83 198	27	0.16 802	9.91 763	8	49
12	9.74 980	19	9.83 225	27	0.16 775	9.91 755	9	48
13	9.74 999	18	9.83 252	28	0.16 748	9.91 746	8	47
14	9.75 017	19	9.83 280	27	0.16 720	9.91 738	9	46
15	9.75 036	18	9.83 307	27	0.16 693	9.91 729	9	**45**
16	9.75 054	19	9.83 334	27	0.16 666	9.91 720	8	44
17	9.75 073	18	9.83 361	27	0.16 639	9.91 712	9	43
18	9.75 091	19	9.83 388	27	0.16 612	9.91 703	8	42
19	9.75 110	18	9.83 415	27	0.16 585	9.91 695	9	41
20	9.75 128	19	9.83 442	28	0.16 558	9.91 686	9	**40**
21	9.75 147	18	9.83 470	27	0.16 530	9.91 677	8	39
22	9.75 165	19	9.83 497	27	0.16 503	9.91 669	9	38
23	9.75 184	18	9.83 524	27	0.16 476	9.91 660	9	37
24	9.75 202	19	9.83 551	27	0.16 449	9.91 651	8	36
25	9.75 221	18	9.83 578	27	0.16 422	9.91 643	9	**35**
26	9.75 239	19	9.83 605	27	0.16 395	9.91 634	9	34
27	9.75 258	18	9.83 632	27	0.16 368	9.91 625	8	33
28	9.75 276	18	9.83 659	27	0.16 341	9.91 617	9	32
29	9.75 294	19	9.83 686	27	0.16 314	9.91 608	9	31
30	9.75 313	18	9.83 713	27	0.16 287	9.91 599	8	**30**
31	9.75 331	19	9.83 740	28	0.16 260	9.91 591	9	29
32	9.75 350	18	9.83 768	27	0.16 232	9.91 582	9	28
33	9.75 368	18	9.83 795	27	0.16 205	9.91 573	8	27
34	9.75 386	19	9.83 822	27	0.16 178	9.91 565	9	26
35	9.75 405	18	9.83 849	27	0.16 151	9.91 556	9	**25**
36	9.75 423	18	9.83 876	27	0.16 124	9.91 547	9	24
37	9.75 441	18	9.83 903	27	0.16 097	9.91 538	8	23
38	9.75 459	19	9.83 930	27	0.16 070	9.91 530	9	22
39	9.75 478	18	9.83 957	27	0.16 043	9.91 521	9	21
40	9.75 496	18	9.83 984	27	0.16 016	9.91 512	8	**20**
41	9.75 514	19	9.84 011	27	0.15 989	9.91 504	9	19
42	9.75 533	18	9.84 038	27	0.15 962	9.91 495	9	18
43	9.75 551	18	9.84 065	27	0.15 935	9.91 486	9	17
44	9.75 569	18	9.84 092	27	0.15 908	9.91 477	8	16
45	9.75 587	18	9.84 119	27	0.15 881	9.91 469	9	**15**
46	9.75 605	19	9.84 146	27	0.15 854	9.91 460	9	14
47	9.75 624	18	9.84 173	27	0.15 827	9.91 451	9	13
48	9.75 642	18	9.84 200	27	0.15 800	9.91 442	9	12
49	9.75 660	18	9.84 227	27	0.15 773	9.91 433	8	11
50	9.75 678	18	9.84 254	26	0.15 746	9.91 425	9	**10**
51	9.75 696	18	9.84 280	27	0.15 720	9.91 416	9	9
52	9.75 714	19	9.84 307	27	0.15 693	9.91 407	9	8
53	9.75 733	18	9.84 334	27	0.15 666	9.91 398	9	7
54	9.75 751	18	9.84 361	27	0.15 639	9.91 389	8	6
55	9.75 769	18	9.84 388	27	0.15 612	9.91 381	9	**5**
56	9.75 787	18	9.84 415	27	0.15 585	9.91 372	9	4
57	9.75 805	18	9.84 442	27	0.15 558	9.91 363	9	3
58	9.75 823	18	9 84 469	27	0.15 531	9.91 354	9	2
59	9.75 841	18	9.84 496	27	0 15 504	9.91 345	9	1
60	9.75 859		9.84 523		0.15 477	9.91 336		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	28	27	26
2	5.6	5.4	5.2
3	8.4	8.1	7.8
4	11.2	10.8	10.4
5	14.0	13.5	13.0
6	16.8	16.2	15.6
7	19.6	18.9	18.2
8	22.4	21.6	20.8
9	25.2	24.3	23.4

	19	18
2	3.8	3.6
3	5.7	5.4
4	7.6	7.2
5	9.5	9.0
6	11.4	10.8
7	13.3	12.6
8	15.2	14.4
9	17.1	16.2

	9	8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

From the top:

For **34°+** or **214°+**, read as printed; for **124°+** or **304°+**, read co-function.

From the bottom:

For **55°+** or **235°+**, read as printed; for **145°+** or **325°+**, read co-function.

Prop. Pts.

55° — Logarithms of Trigonometric Functions

35° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.75 859	18	9.84 523	27	0.15 477	9.91 336	8	**60**
1	9.75 877	18	9.84 550	26	0.15 450	9.91 328	9	59
2	9.75 895	18	9.84 576	27	0.15 424	9.91 319	9	58
3	9.75 913	18	9.84 603	27	0.15 397	9.91 310	9	57
4	9.75 931	18	9.84 630	27	0.15 370	9.91 301	9	56
5	9.75 949	18	9.84 657	27	0.15 343	9.91 292	9	**55**
6	9.75 967	18	9.84 684	27	0.15 316	9.91 283	9	54
7	9.75 985	18	9.84 711	27	0.15 289	9.91 274	8	53
8	9.76 003	18	9.84 738	26	0.15 262	9.91 266	9	52
9	9.76 021	18	9.84 764	27	0.15 236	9.91 257	9	51
10	9.76 039	18	9.84 791	27	0.15 209	9.91 248	9	**50**
11	9.76 057	18	9.84 818	27	0.15 182	9.91 239	9	49
12	9.76 075	18	9.84 845	27	0.15 155	9.91 230	9	48
13	9.76 093	18	9.84 872	27	0.15 128	9.91 221	9	47
14	9.76 111	18	9.84 899	26	0.15 101	9.91 212	9	46
15	9.76 129	17	9.84 925	27	0.15 075	9.91 203	9	**45**
16	9.76 146	18	9.84 952	27	0.15 048	9.91 194	9	44
17	9.76 164	18	9.84 979	27	0.15 021	9.91 185	9	43
18	9.76 182	18	9.85 006	27	0.14 994	9.91 176	9	42
19	9.76 200	18	9.85 033	26	0.14 967	9.91 167	9	41
20	9.76 218	18	9.85 059	27	0.14 941	9.91 158	9	**40**
21	9.76 236	17	9.85 086	27	0.14 914	9.91 149	8	39
22	9.76 253	18	9.85 113	27	0.14 887	9.91 141	9	38
23	9.76 271	18	9.85 140	26	0.14 860	9.91 132	9	37
24	9.76 289	18	9.85 166	27	0.14 834	9.91 123	9	36
25	9.76 307	17	9.85 193	27	0.14 807	9.91 114	9	**35**
26	9.76 324	18	9.85 220	27	0.14 780	9.91 105	9	34
27	9.76 342	18	9.85 247	26	0.14 753	9.91 096	9	33
28	9.76 360	18	9.85 273	27	0.14 727	9.91 087	9	32
29	9.76 378	17	9.85 300	27	0.14 700	9.91 078	9	31
30	9.76 395	18	9.85 327	27	0.14 673	9.91 069	9	**30**
31	9.76 413	18	9.85 354	26	0.14 646	9.91 060	9	29
32	9.76 431	17	9.85 380	27	0.14 620	9.91 051	9	28
33	9.76 448	18	9.85 407	27	0.14 593	9.91 042	9	27
34	9.76 466	18	9.85 434	26	0.14 566	9.91 033	10	26
35	9.76 484	17	9.85 460	27	0.14 540	9.91 023	9	**25**
36	9.76 501	18	9.85 487	27	0.14 513	9.91 014	9	24
37	9.76 519	18	9.85 514	26	0.14 486	9.91 005	9	23
38	9.76 537	17	9.85 540	27	0.14 460	9.90 996	9	22
39	9.76 554	18	9.85 567	27	0.14 433	9.90 987	9	21
40	9.76 572	18	9.85 594	26	0.14 406	9.90 978	9	**20**
41	9.76 590	17	9.85 620	27	0.14 380	9.90 969	9	19
42	9.76 607	18	9.85 647	27	0.14 353	9.90 960	9	18
43	9.76 625	17	9.85 674	26	0.14 326	9.90 951	9	17
44	9.76 642	18	9.85 700	27	0.14 300	9.90 942	9	16
45	9.76 660	17	9.85 727	27	0.14 273	9.90 933	9	**15**
46	9.76 677	18	9.85 754	26	0.14 246	9.90 924	9	14
47	9.76 695	17	9.85 780	27	0.14 220	9.90 915	9	13
48	9.76 712	18	9.85 807	27	0.14 193	9.90 906	10	12
49	9.76 730	17	9.85 834	26	0.14 166	9.90 896	9	11
50	9.76 747	18	9.85 860	27	0.14 140	9.90 887	9	**10**
51	9.76 765	17	9.85 887	26	0.14 113	9.90 878	9	9
52	9.76 782	18	9.85 913	27	0.14 087	9.90 869	9	8
53	9.76 800	17	9.85 940	27	0.14 060	9.90 860	9	7
54	9.76 817	18	9.85 967	26	0.14 033	9.90 851	9	6
55	9.76 835	17	9.85 993	27	0.14 007	9.90 842	10	**5**
56	9.76 852	18	9.86 020	26	0.13 980	9.90 832	9	4
57	9.76 870	17	9.86 046	27	0.13 954	9.90 823	9	3
58	9 76 887	17	9.86 073	27	0.13 927	9.90 814	9	2
59	9.76 904	18	9.86 100	26	0.13 900	9.90 805	9	1
60	9.76 922		9.86 126		0.13 874	9.90 796		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	27	26	18
2	5.4	5.2	3.6
3	8.1	7.8	5.4
4	10.8	10.4	7.2
5	13.5	13.0	9.0
6	16.2	15.6	10.8
7	18.9	18.2	12.6
8	21.6	20.8	14.4
9	24.3	23.4	16.2

	17	10
2	3.4	2.0
3	5.1	3.0
4	6.8	4.0
5	8.5	5.0
6	10.2	6.0
7	11.9	7.0
8	13.6	8.0
9	15.3	9.0

	9	8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

From the top:

For **35°+** or **215°+**, read as printed; for **125°+** or **305°+**, read co-function.

From the bottom:

For **54°+** or **234°+**, read as printed; for **144°+** or **324°+**, read co-function.

Prop. Pts.

54° — Logarithms of Trigonometric Functions

36° — Logarithms of Trigonometric Functions

'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.76 922	17	9.86 126	27	0.13 874	9.90 796	9	**60**
1	9.76 939	18	9.86 153	26	0.13 847	9.90 787	10	59
2	9.76 957	17	9.86 179	27	0.13 821	9.90 777	9	58
3	9.76 974	17	9.86 206	26	0.13 794	9.90 768	9	57
4	9.76 991	18	9.86 232	27	0.13 768	9.90 759	9	56
5	9.77 009	17	9.86 259	26	0.13 741	9.90 750	9	**55**
6	9.77 026	17	9.86 285	27	0.13 715	9.90 741	10	54
7	9.77 043	18	9.86 312	26	0.13 688	9.90 731	9	53
8	9.77 061	17	9.86 338	27	0.13 662	9.90 722	9	52
9	9.77 078	17	9.86 365	27	0.13 635	9.90 713	9	51
10	9.77 095	17	9.86 392	26	0.13 608	9.90 704	10	**50**
11	9.77 112	18	9.86 418	27	0.13 582	9.90 694	9	49
12	9.77 130	17	9.86 445	26	0.13 555	9.90 685	9	48
13	9.77 147	17	9.86 471	27	0.13 529	9.90 676	9	47
14	9.77 164	17	9.86 498	26	0.13 502	9.90 667	10	46
15	9.77 181	18	9.86 524	27	0.13 476	9.90 657	9	**45**
16	9.77 199	17	9.86 551	26	0.13 449	9.90 648	9	44
17	9.77 216	17	9.86 577	26	0.13 423	9.90 639	9	43
18	9.77 233	17	9.86 603	27	0.13 397	9.90 630	10	42
19	9.77 250	18	9.86 630	26	0.13 370	9.90 620	9	41
20	9.77 268	17	9.86 656	27	0.13 344	9.90 611	9	**40**
21	9.77 285	17	9.86 683	26	0.13 317	9.90 602	10	39
22	9.77 302	17	9.86 709	27	0.13 291	9.90 592	9	38
23	9.77 319	17	9.86 736	26	0.13 264	9.90 583	9	37
24	9.77 336	17	9.86 762	27	0.13 238	9.90 574	9	36
25	9.77 353	17	9.86 789	26	0.13 211	9.90 565	10	**35**
26	9.77 370	17	9.86 815	27	0.13 185	9.90 555	9	34
27	9.77 387	18	9.86 842	26	0.13 158	9.90 546	9	33
28	9.77 405	17	9.86 868	26	0.13 132	9.90 537	10	32
29	9.77 422	17	9.86 894	27	0.13 106	9.90 527	9	31
30	9.77 439	17	9.86 921	26	0.13 079	9.90 518	9	**30**
31	9.77 456	17	9.86 947	27	0.13 053	9.90 509	10	29
32	9.77 473	17	9.86 974	26	0.13 026	9.90 499	9	28
33	9.77 490	17	9.87 000	27	0.13 000	9.90 490	10	27
34	9.77 507	17	9.87 027	26	0.12 973	9.90 480	9	26
35	9.77 524	17	9.87 053	26	0.12 947	9.90 471	9	**25**
36	9.77 541	17	9.87 079	27	0.12 921	9.90 462	10	24
37	9.77 558	17	9.87 106	26	0.12 894	9.90 452	9	23
38	9.77 575	17	9.87 132	26	0.12 868	9.90 443	9	22
39	9.77 592	17	9.87 158	27	0.12 842	9.90 434	10	21
40	9.77 609	17	9.87 185	26	0.12 815	9.90 424	9	**20**
41	9.77 626	17	9.87 211	27	0.12 789	9.90 415	10	19
42	9.77 643	17	9.87 238	26	0.12 762	9.90 405	9	18
43	9.77 660	17	9.87 264	26	0.12 736	9.90 396	10	17
44	9.77 677	17	9.87 290	27	0.12 710	9.90 386	9	16
45	9.77 694	17	9.87 317	26	0.12 683	9.90 377	9	**15**
46	9.77 711	17	9.87 343	26	0.12 657	9.90 368	10	14
47	9.77 728	16	9.87 369	27	0.12 631	9.90 358	9	13
48	9.77 744	17	9.87 396	26	0.12 604	9.90 349	10	12
49	9.77 761	17	9.87 422	26	0.12 578	9.90 339	9	11
50	9.77 778	17	9.87 448	27	0.12 552	9.90 330	10	**10**
51	9.77 795	17	9.87 475	26	0.12 525	9.90 320	9	9
52	9.77 812	17	9.87 501	26	0.12 499	9.90 311	10	8
53	9.77 829	17	9.87 527	27	0.12 473	9.90 301	9	7
54	9.77 846	16	9.87 554	26	0.12 446	9.90 292	10	6
55	9.77 862	17	9.87 580	26	0.12 420	9.90 282	9	**5**
56	9.77 879	17	9.87 606	27	0.12 394	9.90 273	10	4
57	9.77 896	17	9.87 633	26	0.12 367	9.90 263	9	3
58	9.77 913	17	9.87 659	26	0.12 341	9.90 254	10	2
59	9.77 930	16	9.87 685	26	0.12 315	9.90 244	9	1
60	9.77 946		9.87 711		0.12 289	9.90 235		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	'

Prop. Pts.

	27	26	18
2	5.4	5.2	3.6
3	8.1	7.8	5.4
4	10.8	10.4	7.2
5	13.5	13.0	9.0
6	16.2	15.6	10.8
7	18.9	18.2	12.6
8	21.6	20.8	14.4
9	24.3	23.4	16.2

	17	16
2	3.4	3.2
3	5.1	4.8
4	6.8	6.4
5	8.5	8.0
6	10.2	9.6
7	11.9	11.2
8	13.6	12.8
9	15.3	14.4

	10	9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

From the top :

For **36°+** or **216°+**, read as printed; for **126°+** or **306°+**, read co-function.

From the bottom :

For **53°+** or **233°+**, read as printed; for **143°+** or **323°+**, read co-function.

Prop. Pts.

53° — Logarithms of Trigonometric Functions

37° — Logarithms of Trigonometric Functions

'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.77 946	17	9.87 711	27	0.12 289	9.90 235	10	**60**
1	9.77 963	17	9.87 738	26	0.12 262	9.90 225	9	59
2	9.77 980	17	9.87 764	26	0.12 236	9.90 216	10	58
3	9.77 997	16	9.87 790	27	0.12 210	9.90 206	9	57
4	9.78 013	17	9.87 817	26	0.12 183	9.90 197	10	56
5	9.78 030	17	9.87 843	26	0.12 157	9.90 187	9	**55**
6	9.78 047	16	9.87 869	26	0.12 131	9.90 178	10	54
7	9.78 063	17	9.87 895	27	0.12 105	9.90 168	9	53
8	9.78 080	17	9.87 922	26	0.12 078	9.90 159	10	52
9	9.78 097	16	9.87 948	26	0.12 052	9.90 149	10	51
10	9.78 113	17	9.87 974	26	0.12 026	9.90 139	9	**50**
11	9.78 130	17	9.88 000	27	0.12 000	9.90 130	10	49
12	9.78 147	16	9.88 027	26	0.11 973	9.90 120	9	48
13	9.78 163	17	9.88 053	26	0.11 947	9.90 111	10	47
14	9.78 180	17	9.88 079	26	0.11 921	9.90 101	10	46
15	9.78 197	16	9.88 105	26	0.11 895	9.90 091	9	**45**
16	9.78 213	17	9.88 131	27	0.11 869	9.90 082	10	44
17	9.78 230	16	9.88 158	26	0.11 842	9.90 072	9	43
18	9.78 246	17	9.88 184	26	0.11 816	9.90 063	10	42
19	9.78 263	17	9.88 210	26	0.11 790	9.90 053	10	41
20	9.78 280	16	9.88 236	26	0.11 764	9.90 043	9	**40**
21	9.78 296	17	9.88 262	27	0.11 738	9.90 034	10	39
22	9.78 313	16	9.88 289	26	0.11 711	9.90 024	10	38
23	9.78 329	17	9.88 315	26	0.11 685	9.90 014	9	37
24	9.78 346	16	9.88 341	26	0.11 659	9.90 005	10	36
25	9.78 362	17	9.88 367	26	0.11 633	9.89 995	10	**35**
26	9.78 379	16	9.88 393	27	0.11 607	9.89 985	9	34
27	9.78 395	17	9.88 420	26	0.11 580	9.89 976	10	33
28	9.78 412	16	9.88 446	26	0.11 554	9.89 966	10	32
29	9.78 428	17	9.88 472	26	0.11 528	9.89 956	9	31
30	9.78 445	16	9.88 498	26	0.11 502	9.89 947	10	**30**
31	9.78 461	17	9.88 524	26	0.11 476	9.89 937	10	29
32	9.78 478	16	9.88 550	27	0.11 450	9.89 927	9	28
33	9.78 494	16	9.88 577	26	0.11 423	9.89 918	10	27
34	9.78 510	17	9.88 603	26	0.11 397	9.89 908	10	26
35	9.78 527	16	9.88 629	26	0.11 371	9.89 898	10	**25**
36	9.78 543	17	9.88 655	26	0.11 345	9.89 888	9	24
37	9.78 560	16	9.88 681	26	0.11 319	9.89 879	10	23
38	9.78 576	16	9.88 707	26	0.11 293	9.89 869	10	22
39	9.78 592	17	9.88 733	26	0.11 267	9.89 859	10	21
40	9.78 609	16	9.88 759	27	0.11 241	9.89 849	9	**20**
41	9.78 625	17	9.88 786	26	0.11 214	9.89 840	10	19
42	9.78 642	16	9.88 812	26	0.11 188	9.89 830	10	18
43	9.78 658	16	9.88 838	26	0.11 162	9.89 820	10	17
44	9.78 674	17	9.88 864	26	0.11 136	9.89 810	9	16
45	9.78 691	16	9.88 890	26	0.11 110	9.89 801	10	**15**
46	9.78 707	16	9.88 916	26	0.11 084	9.89 791	10	14
47	9.78 723	16	9.88 942	26	0.11 058	9.89 781	10	13
48	9.78 739	17	9.88 968	26	0.11 032	9.89 771	10	12
49	9.78 756	16	9.88 994	26	0.11 006	9.89 761	9	11
50	9.78 772	16	9.89 020	26	0.10 980	9.89 752	10	**10**
51	9.78 788	17	9.89 046	27	0.10 954	9.89 742	10	9
52	9.78 805	16	9.89 073	26	0.10 927	9.89 732	10	8
53	9.78 821	16	9.89 099	26	0.10 901	9.89 722	10	7
54	9.78 837	16	9.89 125	26	0.10 875	9.89 712	10	6
55	9.78 853	16	9.89 151	26	0.10 849	9.89 702	9	**5**
56	9.78 869	17	9.89 177	26	0.10 823	9.89 693	10	4
57	9.78 886	16	9.89 203	26	0.10 797	9.89 683	10	3
58	9.78 902	16	9.89 229	26	0.10 771	9.89 673	10	2
59	9.78 918	16	9.89 255	26	0.10 745	9.89 663	10	1
60	9.78 934		9.89 281		0.10 719	9.89 653		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	'

Prop. Pts.

	27	**26**	**17**
2	5.4	5.2	3.4
3	8.1	7.8	5.1
4	10.8	10.4	6.8
5	13.5	13.0	8.5
6	16.2	15.6	10.2
7	18.9	18.2	11.9
8	21.6	20.8	13.6
9	24.3	23.4	15.3

	16	**10**	**9**
2	3.2	2.0	1.8
3	4.8	3.0	2.7
4	6.4	4.0	3.6
5	8.0	5.0	4.5
6	9.6	6.0	5.4
7	11.2	7.0	6.3
8	12.8	8.0	7.2
9	14.4	9.0	8.1

From the top:

For **37°**+ or **217°**+, read as printed; for **127°**+ or **307°**+, read co-function.

From the bottom:

For **52°**+ or **232°**+, read as printed; for **142°**+ or **322°**+, read co-function.

Prop. Pts.

52° — Logarithms of Trigonometric Functions

38° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.78 934	16	9.89 281	26	0.10 719	9.89 653	10	**60**
1	9.78 950	17	9.89 307	26	0.10 693	9.89 643	10	59
2	9.78 967	16	9.89 333	26	0.10 667	9.89 633	9	58
3	9.78 983	16	9.89 359	26	0.10 641	9.89 624	10	57
4	9.78 999	16	9.89 385	26	0.10 615	9.89 614	10	56
5	9.79 015	16	9.89 411	26	0.10 589	9.89 604	10	**55**
6	9.79 031	16	9.89 437	26	0.10 563	9.89 594	10	54
7	9.79 047	16	9.89 463	26	0.10 537	9.89 584	10	53
8	9.79 063	16	9.89 489	26	0.10 511	9.89 574	10	52
9	9.79 079	16	9.89 515	26	0.10 485	9.89 564	10	51
10	9.79 095	16	9.89 541	26	0.10 459	9.89 554	10	**50**
11	9.79 111	17	9.89 567	26	0.10 433	9.89 544	10	49
12	9.79 128	16	9.89 593	26	0.10 407	9.89 534	10	48
13	9.79 144	16	9.89 619	26	0.10 381	9.89 524	10	47
14	9.79 160	16	9.89 645	26	0.10 355	9.89 514	10	46
15	9.79 176	16	9.89 671	26	0.10 329	9.89 504	9	**45**
16	9.79 192	16	9.89 697	26	0.10 303	9.89 495	10	44
17	9.79 208	16	9.89 723	26	0.10 277	9.89 485	10	43
18	9.79 224	16	9.89 749	26	0.10 251	9.89 475	10	42
19	9.79 240	16	9.89 775	26	0.10 225	9.89 465	10	41
20	9.79 256	16	9.89 801	26	0.10 199	9.89 455	10	**40**
21	9.79 272	16	9.89 827	26	0.10 173	9.89 445	10	39
22	9.79 288	16	9.89 853	26	0.10 147	9.89 435	10	38
23	9.79 304	15	9.89 879	26	0.10 121	9.89 425	10	37
24	9.79 319	16	9.89 905	26	0.10 095	9.89 415	10	36
25	9.79 335	16	9.89 931	26	0.10 069	9.89 405	10	**35**
26	9.79 351	16	9.89 957	26	0.10 043	9.89 395	10	34
27	9.79 367	16	9.89 983	26	0.10 017	9.89 385	10	33
28	9.79 383	16	9.90 009	26	0.09 991	9.89 375	11	32
29	9.79 399	16	9.90 035	26	0.09 965	9.89 364	10	31
30	9.79 415	16	9.90 061	25	0.09 939	9.89 354	10	**30**
31	9.79 431	16	9.90 086	26	0.09 914	9.89 344	10	29
32	9.79 447	16	9.90 112	26	0.09 888	9.89 334	10	28
33	9.79 463	15	9.90 138	26	0.09 862	9.89 324	10	27
34	9.79 478	16	9.90 164	26	0.09 836	9.89 314	10	26
35	9.79 494	16	9.90 190	26	0.09 810	9.89 304	10	**25**
36	9.79 510	16	9.90 216	26	0.09 784	9.89 294	10	24
37	9.79 526	16	9.90 242	26	0.09 758	9.89 284	10	23
38	9.79 542	16	9.90 268	26	0.09 732	9.89 274	10	22
39	9.79 558	15	9.90 294	26	0.09 706	9.89 264	10	21
40	9.79 573	16	9.90 320	26	0.09 680	9.89 254	10	**20**
41	9.79 589	16	9.90 346	25	0.09 654	9.89 244	11	19
42	9.79 605	16	9.90 371	26	0.09 629	9.89 233	10	18
43	9.79 621	15	9.90 397	26	0.09 603	9.89 223	10	17
44	9.79 636	16	9.90 423	26	0.09 577	9.89 213	10	16
45	9.79 652	16	9.90 449	26	0.09 551	9.89 203	10	**15**
46	9.79 668	16	9.90 475	26	0.09 525	9.89 193	10	14
47	9.79 684	15	9.90 501	26	0.09 499	9.89 183	10	13
48	9.79 699	16	9.90 527	26	0.09 473	9.89 173	11	12
49	9.79 715	16	9.90 553	25	0.09 447	9.89 162	10	11
50	9.79 731	15	9.90 578	26	0.09 422	9.89 152	10	**10**
51	9.79 746	16	9.90 604	26	0.09 396	9.89 142	10	9
52	9.79 762	16	9.90 630	26	0.09 370	9.89 132	10	8
53	9.79 778	15	9.90 656	26	0.09 344	9.89 122	10	7
54	9.79 793	16	9.90 682	26	0.09 318	9.89 112	11	6
55	9.79 809	16	9.90 708	26	0.09 292	9.89 101	10	**5**
56	9.79 825	15	9.90 734	25	0.09 266	9.89 091	10	4
57	9.79 840	16	9.90 759	26	0.09 241	9.89 081	10	3
58	9.79 856	16	9.90 785	26	0.09 215	9.89 071	11	2
59	9.79 872	15	9.90 811	26	0.09 189	9.89 060	10	1
60	9.79 887		9.90 837		0.09 163	9.89 050		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	26	25	17
2	5.2	5.0	3.4
3	7.8	7.5	5.1
4	10.4	10.0	6.8
5	13.0	12.5	8.5
6	15.6	15.0	10.2
7	18.2	17.5	11.9
8	20.8	20.0	13.6
9	23.4	22.5	15.3

	16	15	11
2	3.2	3.0	2.2
3	4.8	4.5	3.3
4	6.4	6.0	4.4
5	8.0	7.5	5.5
6	9.6	9.0	6.6
7	11.2	10.5	7.7
8	12.8	12.0	8.8
9	14.4	13.5	9.9

	10	9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

From the top:

For **38°**+ or **218°**+, read as printed; for **128°**+ or **308°**+, read co-function.

From the bottom:

For **51°**+ or **231°**+, read as printed; for **141°**+ or **321°**+, read co-function.

Prop. Pts.

51° — Logarithms of Trigonometric Functions

39° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.79 887	16	9.90 837	26	0.09 163	9.89 050	10	**60**
1	9.79 903	15	9.90 863	26	0.09 137	9.89 040	10	59
2	9.79 918	16	9.90 889	25	0.09 111	9.89 030	10	58
3	9.79 934	16	9.90 914	26	0.09 086	9.89 020	11	57
4	9.79 950	15	9.90 940	26	0.09 060	9.89 009	10	56
5	9.79 965	16	9.90 966	26	0.09 034	9.88 999	10	**55**
6	9.79 981	15	9.90 992	26	0.09 008	9.88 989	11	54
7	9.79 996	16	9.91 018	25	0.08 982	9.88 978	10	53
8	9.80 012	15	9.91 043	26	0.08 957	9.88 968	10	52
9	9.80 027	16	9.91 069	26	0.08 931	9.88 958	10	51
10	9.80 043	15	9.91 095	26	0.08 905	9.88 948	11	**50**
11	9.80 058	16	9.91 121	26	0.08 879	9.88 937	10	49
12	9.80 074	15	9.91 147	25	0.08 853	9.88 927	10	48
13	9.80 089	16	9.91 172	26	0.08 828	9.88 917	11	47
14	9.80 105	15	9.91 198	26	0.08 802	9.88 906	10	46
15	9.80 120	16	9.91 224	26	0.08 776	9.88 896	10	**45**
16	9.80 136	15	9.91 250	26	0.08 750	9.88 886	11	44
17	9.80 151	15	9.91 276	25	0.08 724	9.88 875	10	43
18	9.80 166	16	9.91 301	26	0.08 699	9.88 865	10	42
19	9.80 182	15	9.91 327	26	0.08 673	9.88 855	11	41
20	9.80 197	16	9.91 353	26	0.08 647	9.88 844	10	**40**
21	9.80 213	15	9.91 379	25	0.08 621	9.88 834	10	39
22	9.80 228	16	9.91 404	26	0.08 596	9.88 824	11	38
23	9.80 244	15	9.91 430	26	0.08 570	9.88 813	10	37
24	9.80 259	15	9.91 456	26	0.08 544	9.88 803	10	36
25	9.80 274	16	9.91 482	25	0.08 518	9.88 793	11	**35**
26	9.80 290	15	9.91 507	26	0.08 493	9.88 782	10	34
27	9.80 305	15	9.91 533	26	0.08 467	9.88 772	11	33
28	9.80 320	16	9.91 559	26	0.08 441	9.88 761	10	32
29	9.80 336	15	9.91 585	25	0.08 415	9.88 751	10	31
30	9.80 351	15	9.91 610	26	0.08 390	9.88 741	11	**30**
31	9.80 366	16	9.91 636	26	0.08 364	9.88 730	10	29
32	9.80 382	15	9.91 662	26	0.08 338	9.88 720	11	28
33	9.80 397	15	9.91 688	25	0.08 312	9.88 709	10	27
34	9.80 412	16	9.91 713	26	0.08 287	9.88 699	11	26
35	9.80 428	15	9.91 739	26	0.08 261	9.88 688	10	**25**
36	9.80 443	15	9.91 765	26	0.08 235	9.88 678	10	24
37	9.80 458	15	9.91 791	25	0.08 209	9.88 668	11	23
38	9.80 473	16	9.91 816	26	0.08 184	9.88 657	10	22
39	9.80 489	15	9.91 842	26	0.08 158	9.88 647	11	21
40	9.80 504	15	9.91 868	25	0.08 132	9.88 636	10	**20**
41	9.80 519	15	9.91 893	26	0.08 107	9.88 626	11	19
42	9.80 534	16	9.91 919	26	0.08 081	9.88 615	10	18
43	9.80 550	15	9.91 945	26	0.08 055	9.88 605	11	17
44	9.80 565	15	9.91 971	25	0.08 029	9.88 594	10	16
45	9.80 580	15	9.91 996	26	0.08 004	9.88 584	11	**15**
46	9.80 595	15	9.92 022	26	0.07 978	9.88 573	10	14
47	9.80 610	15	9.92 048	25	0.07 952	9.88 563	11	13
48	9.80 625	16	9.92 073	26	0.07 927	9.88 552	10	12
49	9.80 641	15	9.92 099	26	0.07 901	9.88 542	11	11
50	9.80 656	15	9.92 125	25	0.07 875	9.88 531	10	**10**
51	9.80 671	15	9.92 150	26	0.07 850	9.88 521	11	9
52	9.80 686	15	9.92 176	26	0.07 824	9.88 510	11	8
53	9.80 701	15	9.92 202	25	0.07 798	9.88 499	10	7
54	9.80 716	15	9.92 227	26	0.07 773	9.88 489	11	6
55	9.80 731	15	9.92 253	26	0.07 747	9.88 478	10	**5**
56	9.80 746	16	9.92 279	25	0.07 721	9.88 468	11	4
57	9.80 762	15	9.92 304	26	0.07 696	9.88 457	10	3
58	9.80 777	15	9.92 330	26	0.07 670	9.88 447	11	2
59	9.80 792	15	9.92 356	25	0.07 644	9.88 436	11	1
60	9.80 807		9.92 381		0.07 619	9.88 425		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	26	25	16
2	5.2	5.0	3.2
3	7.8	7.5	4.8
4	10.4	10.0	6.4
5	13.0	12.5	8.0
6	15.6	15.0	9.6
7	18.2	17.5	11.2
8	20.8	20.0	12.8
9	23.4	22.5	14.4

	15	11	10
2	3.0	2.2	2.0
3	4.5	3.3	3.0
4	6.0	4.4	4.0
5	7.5	5.5	5.0
6	9.0	6.6	6.0
7	10.5	7.7	7.0
8	12.0	8.8	8.0
9	13.5	9.9	9.0

From the top:

For **39°+** or **219°+**, read as printed; for **129°+** or **309°+**, read co-function.

From the bottom:

For **50°+** or **230°+**, read as printed; for **140°+** or **320°+**, read co-function.

Prop. Pts.

50° — Logarithms of Trigonometric Functions

Google

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.80 807	15	9.92 381	26	0.07 619	9.88 425	10	**60**
1	9.80 822	15	9.92 407	26	0.07 593	9.88 415	11	59
2	9.80 837	15	9.92 433	25	0.07 567	9.88 404	10	58
3	9.80 852	15	9.92 458	26	0.07 542	9.88 394	11	57
4	9.80 867	15	9.92 484	26	0.07 516	9.88 383	11	56
5	9.80 882	15	9.92 510	25	0.07 490	9.88 372	10	**55**
6	9.80 897	15	9.92 535	26	0.07 465	9.88 362	11	54
7	9.80 912	15	9.92 561	26	0.07 439	9.88 351	11	53
8	9.80 927	15	9.92 587	25	0.07 413	9.88 340	10	52
9	9.80 942	15	9.92 612	26	0.07 388	9.88 330	11	51
10	9.80 957	15	9.92 638	25	0.07 362	9.88 319	11	**50**
11	9.80 972	15	9.92 663	26	0.07 337	9.88 308	10	49
12	9.80 987	15	9.92 689	26	0.07 311	9.88 298	11	48
13	9.81 002	15	9.92 715	25	0.07 285	9.88 287	11	47
14	9.81 017	15	9.92 740	26	0.07 260	9.88 276	10	46
15	9.81 032	15	9.92 766	26	0.07 234	9.88 266	11	**45**
16	9.81 047	14	9.92 792	25	0.07 208	9.88 255	11	44
17	9.81 061	15	9.92 817	26	0.07 183	9.88 244	10	43
18	9.81 076	15	9.92 843	25	0.07 157	9.88 234	11	42
19	9.81 091	15	9.92 868	26	0.07 132	9.88 223	11	41
20	9.81 106	15	9.92 894	26	0.07 106	9.88 212	11	**40**
21	9.81 121	15	9.92 920	25	0.07 080	9.88 201	10	39
22	9.81 136	15	9.92 945	26	0.07 055	9.88 191	11	38
23	9.81 151	15	9.92 971	25	0.07 029	9.88 180	11	37
24	9.81 166	14	9.92 996	26	0.07 004	9.88 169	11	36
25	9.81 180	15	9.93 022	26	0.06 978	9.88 158	10	**35**
26	9.81 195	15	9.93 048	25	0.06 952	9.88 148	11	34
27	9.81 210	15	9.93 073	26	0.06 927	9.88 137	11	33
28	9.81 225	15	9.93 099	25	0.06 901	9.88 126	11	32
29	9.81 240	14	9.93 124	26	0.06 876	9.88 115	10	31
30	9.81 254	15	9.93 150	25	0.06 850	9.88 105	11	**30**
31	9.81 269	15	9.93 175	26	0.06 825	9.88 094	11	29
32	9.81 284	15	9.93 201	26	0.06 799	9.88 083	11	28
33	9.81 299	15	9.93 227	25	0.06 773	9.88 072	11	27
34	9.81 314	14	9.93 252	26	0.06 748	9.88 061	10	26
35	9.81 328	15	9.93 278	25	0.06 722	9.88 051	11	**25**
36	9.81 343	15	9.93 303	26	0.06 697	9.88 040	11	24
37	9.81 358	14	9.93 329	25	0.06 671	9.88 029	11	23
38	9.81 372	15	9.93 354	26	0.06 646	9.88 018	11	22
39	9.81 387	15	9.93 380	26	0.06 620	9.88 007	11	21
40	9.81 402	15	9.93 406	25	0.06 594	9.87 996	11	**20**
41	9.81 417	14	9.93 431	26	0.06 569	9.87 985	10	19
42	9.81 431	15	9.93 457	25	0.06 543	9.87 975	11	18
43	9.81 446	15	9.93 482	26	0.06 518	9.87 964	11	17
44	9.81 461	14	9.93 508	25	0.06 492	9.87 953	11	16
45	9.81 475	15	9.93 533	26	0.06 467	9.87 942	11	**15**
46	9.81 490	15	9.93 559	25	0.06 441	9.87 931	11	14
47	9.81 505	14	9.93 584	26	0.06 416	9.87 920	11	13
48	9.81 519	15	9.93 610	26	0.06 390	9.87 909	11	12
49	9.81 534	15	9.93 636	25	0.06 364	9.87 898	11	11
50	9.81 549	14	9.93 661	26	0.06 339	9.87 887	10	**10**
51	9.81 563	15	9.93 687	25	0.06 313	9.87 877	11	9
52	9.81 578	14	9.93 712	26	0.06 288	9.87 866	11	8
53	9.81 592	15	9.93 738	25	0.06 262	9.87 855	11	7
54	9.81 607	15	9.93 763	26	0.06 237	9.87 844	11	6
55	9.81 622	14	9.93 789	25	0.06 211	9.87 833	11	**5**
56	9.81 636	15	9.93 814	26	0.06 186	9.87 822	11	4
57	9.81 651	14	9.93 840	25	0.06 160	9.87 811	11	3
58	9.81 665	15	9.93 865	26	0.06 135	9.87 800	11	2
59	9.81 680	14	9.93 891	25	0.06 109	9.87 789	11	1
60	9.81 694		9.93 916		0.06 084	9.87 778		**0**
	L Cos	**d**	**L Ctn**	**c d**	**L Tan**	**L Sin**	**d**	′

Prop. Pts.

	26	**25**	**15**
2	5.2	5.0	3.0
3	7.8	7.5	4.5
4	10.4	10.0	6.0
5	13.0	12.5	7.5
6	15.6	15.0	9.0
7	18.2	17.5	10.5
8	20.8	20.0	12.0
9	23.4	22.5	13.5

	14	**11**	**10**
2	2.8	2.2	2.0
3	4.2	3.3	3.0
4	5.6	4.4	4.0
5	7.0	5.5	5.0
6	8.4	6.6	6.0
7	9.8	7.7	7.0
8	11.2	8.8	8.0
9	12.6	9.9	9.0

From the top:

For **40°+** or **220°+**, read as printed; for **130°+** or **310°+**, read co-function.

From the bottom:

For **49°+** or **229°+**, read as printed; for **139°+** or **319°+**, read co-function.

Prop. Pts.

41° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.81 694	15	9.93 916	26	0.06 084	9.87 778	11	**60**
1	9.81 709	14	9.93 942	25	0.06 058	9.87 767	11	59
2	9.81 723	15	9.93 967	26	0.06 033	9.87 756	11	58
3	9.81 738	14	9.93 993	25	0.06 007	9.87 745	11	57
4	9.81 752	15	9.94 018	26	0.05 982	9.87 734	11	56
5	9.81 767	14	9.94 044	25	0.05 956	9.87 723	11	**55**
6	9.81 781	15	9.94 069	26	0.05 931	9.87 712	11	54
7	9.81 796	14	9.94 095	25	0.05 905	9.87 701	11	53
8	9.81 810	15	9.94 120	26	0.05 880	9.87 690	11	52
9	9.81 825	14	9.94 146	25	0.05 854	9.87 679	11	51
10	9.81 839	15	9.94 171	26	0.05 829	9.87 668	11	**50**
11	9.81 854	14	9.94 197	25	0.05 803	9.87 657	11	49
12	9.81 868	14	9.94 222	26	0.05 778	9.87 646	11	48
13	9.81 882	15	9.94 248	25	0.05 752	9.87 635	11	47
14	9.81 897	14	9.94 273	26	0.05 727	9.87 624	11	46
15	9.81 911	15	9.94 299	25	0.05 701	9.87 613	12	**45**
16	9.81 926	14	9.94 324	26	0.05 676	9.87 601	11	44
17	9.81 940	15	9.94 350	25	0.05 650	9.87 590	11	43
18	9.81 955	14	9.94 375	26	0.05 625	9.87 579	11	42
19	9.81 969	14	9.94 401	25	0.05 599	9.87 568	11	41
20	9.81 983	15	9.94 426	26	0.05 574	9.87 557	11	**40**
21	9.81 998	14	9.94 452	25	0.05 548	9.87 546	11	39
22	9.82 012	14	9.94 477	26	0.05 523	9.87 535	11	38
23	9.82 026	15	9.94 503	25	0.05 497	9.87 524	11	37
24	9.82 041	14	9.94 528	26	0.05 472	9.87 513	12	36
25	9.82 055	14	9.94 554	25	0.05 446	9.87 501	11	**35**
26	9.82 069	15	9.94 579	25	0.05 421	9.87 490	11	34
27	9.82 084	14	9.94 604	26	0.05 396	9.87 479	11	33
28	9.82 098	14	9.94 630	25	0.05 370	9.87 468	11	32
29	9.82 112	14	9.94 655	26	0.05 345	9.87 457	11	31
30	9.82 126	15	9.94 681	25	0.05 319	9.87 446	12	**30**
31	9.82 141	14	9.94 706	26	0.05 294	9.87 434	11	29
32	9.82 155	14	9.94 732	25	0.05 268	9.87 423	11	28
33	9.82 169	15	9.94 757	26	0.05 243	9.87 412	11	27
34	9.82 184	14	9.94 783	25	0.05 217	9.87 401	11	26
35	9.82 198	14	9.94 808	26	0.05 192	9.87 390	12	**25**
36	9.82 212	14	9.94 834	25	0.05 166	9.87 378	11	24
37	9.82 226	14	9.94 859	25	0.05 141	9.87 367	11	23
38	9.82 240	15	9.94 884	26	0.05 116	9.87 356	11	22
39	9.82 255	14	9.94 910	25	0.05 090	9.87 345	11	21
40	9.82 269	14	9.94 935	26	0.05 065	9.87 334	12	**20**
41	9.82 283	14	9.94 961	25	0.05 039	9.87 322	11	19
42	9.82 297	14	9.94 986	26	0.05 014	9.87 311	11	18
43	9.82 311	15	9.95 012	25	0.04 988	9.87 300	12	17
44	9.82 326	14	9.95 037	25	0.04 963	9.87 288	11	16
45	9.82 340	14	9.95 062	26	0.04 938	9.87 277	11	**15**
46	9.82 354	14	9.95 088	25	0.04 912	9.87 266	11	14
47	9.82 368	14	9.95 113	26	0.04 887	9.87 255	12	13
48	9.82 382	14	9.95 139	25	0.04 861	9.87 243	11	12
49	9.82 396	14	9.95 164	26	0.04 836	9.87 232	11	11
50	9.82 410	14	9.95 190	25	0.04 810	9.87 221	12	**10**
51	9.82 424	15	9.95 215	25	0.04 785	9.87 209	11	9
52	9.82 439	14	9.95 240	26	0.04 760	9.87 198	11	8
53	9.82 453	14	9.95 266	25	0.04 734	9.87 187	12	7
54	9.82 467	14	9.95 291	26	0.04 709	9.87 175	11	6
55	9.82 481	14	9.95 317	25	0.04 683	9.87 164	11	**5**
56	9.82 495	14	9.95 342	26	0.04 658	9.87 153	12	4
57	9.82 509	14	9.95 368	25	0.04 632	9.87 141	11	3
58	9.82 523	14	9.95 393	25	0.04 607	9.87 130	11	2
59	9.82 537	14	9.95 418	26	0.04 582	9.87 119	12	1
60	9.82 551		9.95 444		0.04 556	9.87 107		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	26	**25**	**15**
2	5.2	5.0	3.0
3	7.8	7.5	4.5
4	10.4	10.0	6.0
5	13.0	12.5	7.5
6	15.6	15.0	9.0
7	18.2	17.5	10.5
8	20.8	20.0	12.0
9	23.4	22.5	13.5

	14	**12**	**11**
2	2.8	2.4	2.2
3	4.2	3.6	3.3
4	5.6	4.8	4.4
5	7.0	6.0	5.5
6	8.4	7.2	6.6
7	9.8	8.4	7.7
8	11.2	9.6	8.8
9	12.6	10.8	9.9

From the top:

For **41°**+ or **221°**+, read as printed; for **131°**+ or **311°**+, read co-function.

From the bottom:

For **48°**+ or **228°**+, read as printed; for **138°**+ or **318°**+, read co-function.

Prop. Pts.

48° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.82 551	14	9.95 444	25	0.04 556	9.87 107	11	**60**
1	9.82 565	14	9.95 469	26	0.04 531	9.87 096	11	59
2	9.82 579	14	9.95 495	25	0.04 505	9.87 085	12	58
3	9.82 593	14	9.95 520	25	0.04 480	9.87 073	11	57
4	9.82 607	14	9.95 545	26	0.04 455	9.87 062	12	56
5	9.82 621	14	9.95 571	25	0.04 429	9.87 050	11	**55**
6	9.82 635	14	9.95 596	26	0.04 404	9.87 039	11	54
7	9.82 649	14	9.95 622	25	0.04 378	9.87 028	12	53
8	9.82 663	14	9.95 647	25	0.04 353	9.87 016	11	52
9	9.82 677	14	9.95 672	26	0.04 328	9.87 005	12	51
10	9.82 691	14	9.95 698	25	0.04 302	9.86 993	11	**50**
11	9.82 705	14	9.95 723	25	0.04 277	9.86 982	12	49
12	9.82 719	14	9.95 748	26	0.04 252	9.86 970	11	48
13	9.82 733	14	9.95 774	25	0.04 226	9.86 959	12	47
14	9.82 747	14	9.95 799	26	0.04 201	9.86 947	11	46
15	9.82 761	14	9.95 825	25	0.04 175	9.86 936	12	**45**
16	9.82 775	13	9.95 850	25	0.04 150	9.86 924	11	44
17	9.82 788	14	9.95 875	26	0.04 125	9.86 913	11	43
18	9.82 802	14	9.95 901	25	0.04 099	9.86 902	12	42
19	9.82 816	14	9.95 926	26	0.04 074	9.86 890	11	41
20	9.82 830	14	9.95 952	25	0.04 048	9.86 879	12	**40**
21	9.82 844	14	9.95 977	25	0.04 023	9.86 867	12	39
22	9.82 858	14	9.96 002	26	0.03 998	9.86 855	11	38
23	9.82 872	13	9.96 028	25	0.03 972	9.86 844	12	37
24	9.82 885	14	9.96 053	25	0.03 947	9.86 832	11	36
25	9.82 899	14	9.96 078	26	0.03 922	9.86 821	12	**35**
26	9.82 913	14	9.96 104	25	0.03 896	9.86 809	11	34
27	9.82 927	14	9.96 129	26	0.03 871	9.86 798	12	33
28	9.82 941	14	9.96 155	25	0.03 845	9.86 786	11	32
29	9.82 955	13	9.96 180	25	0.03 820	9.86 775	12	31
30	9.82 968	14	9.96 205	26	0.03 795	9.86 763	11	**30**
31	9.82 982	14	9.96 231	25	0.03 769	9.86 752	12	29
32	9.82 996	14	9.96 256	25	0.03 744	9.86 740	12	28
33	9.83 010	13	9.96 281	26	0.03 719	9.86 728	11	27
34	9.83 023	14	9.96 307	25	0.03 693	9.86 717	12	26
35	9.83 037	14	9.96 332	25	0.03 668	9.86 705	11	**25**
36	9.83 051	14	9.96 357	26	0.03 643	9.86 694	12	24
37	9.83 065	13	9.96 383	25	0.03 617	9.86 682	12	23
38	9.83 078	14	9.96 408	25	0.03 592	9.86 670	11	22
39	9.83 092	14	9.96 433	26	0.03 567	9.86 659	12	21
40	9.83 106	14	9.96 459	25	0 03 541	9.86 647	12	**20**
41	9.83 120	13	9.96 484	26	0.03 516	9.86 635	11	19
42	9.83 133	14	9.96 510	25	0.03 490	9.86 624	12	18
43	9.83 147	14	9.96 535	25	0.03 465	9.86 612	12	17
44	9.83 161	13	9.96 560	26	0.03 440	9.86 600	11	16
45	9.83 174	14	9.96 586	25	0.03 414	9.86 589	12	**15**
46	9.83 188	14	9.96 611	25	0.03 389	9.86 577	12	14
47	9.83 202	13	9.96 636	26	0.03 364	9.86 565	11	13
48	9.83 215	14	9.96 662	25	0.03 338	9.86 554	12	12
49	9.83 229	13	9.96 687	25	0.03 313	9.86 542	12	11
50	9.83 242	14	9.96 712	26	0.03 288	9.86 530	12	**10**
51	9.83 256	14	9.96 738	25	0.03 262	9.86 518	11	9
52	9.83 270	13	9.96 763	25	0.03 237	9.86 507	12	8
53	9.83 283	14	9.96 788	26	0.03 212	9.86 495	12	7
54	9.83 297	13	9.96 814	25	0.03 186	9.86 483	11	6
55	9.83 310	14	9.96 839	25	0.03 161	9.86 472	12	**5**
56	9.83 324	14	9.96 864	26	0.03 136	9.86 460	12	4
57	9.83 338	13	9.96 890	25	0.03 110	9.86 448	12	3
58	9.83 351	14	9.96 915	25	0.03 085	9.86 436	11	2
59	9.83 365	13	9.96 940	26	0.03 060	9.86 425	12	1
60	9.83 378		9.96 966		0.03 034	9.86 413		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	26	**25**	**14**
2	5.2	5.0	2.8
3	7.8	7.5	4.2
4	10.4	10.0	5.6
5	13.0	12.5	7.0
6	15.6	15.0	8.4
7	18.2	17.5	9.8
8	20.8	20.0	11.2
9	23.4	22.5	12.6

	13	**12**	**11**
2	2.6	2.4	2.2
3	3.9	3.6	3.3
4	5.2	4.8	4.4
5	6.5	6.0	5.5
6	7.8	7.2	6.6
7	9.1	8.4	7.7
8	10.4	9.6	8.8
9	11.7	10.8	9.9

From the top:

For **42°**+ or **222°**+, read as printed; for **132°**+ or **312°**+, read co-function.

From the bottom:

For **47°**+ or **227°**+, read as printed; for **137°**+ or **317°**+, read co-function.

Prop. Pts.

47° — Logarithms of Trigonometric Functions

43° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.83 378	14	9.96 966	25	0.03 034	9.86 413	12	**60**
1	9.83 392	13	9.96 991	25	0.03 009	9.86 401	12	59
2	9.83 405	14	9.97 016	26	0.02 984	9.86 389	12	58
3	9.83 419	13	9.97 042	25	0.02 958	9.86 377	11	57
4	9.83 432	14	9.97 067	25	0.02 933	9.86 366	12	56
5	9.83 446	13	9.97 092	26	0.02 908	9.86 354	12	**55**
6	9.83 459	14	9.97 118	25	0.02 882	9.86 342	12	54
7	9.83 473	13	9.97 143	25	0.02 857	9.86 330	12	53
8	9.83 486	14	9.97 168	25	0.02 832	9.86 318	12	52
9	9.83 500	13	9.97 193	26	0.02 807	9.86 306	11	51
10	9.83 513	14	9.97 219	25	0.02 781	9.86 295	12	**50**
11	9.83 527	13	9.97 244	25	0.02 756	9.86 283	12	49
12	9.83 540	14	9.97 269	26	0.02 731	9.86 271	12	48
13	9.83 554	13	9.97 295	25	0.02 705	9.86 259	12	47
14	9.83 567	14	9.97 320	25	0.02 680	9.86 247	12	46
15	9.83 581	13	9.97 345	26	0.02 655	9.86 235	12	**45**
16	9.83 594	14	9.97 371	25	0.02 629	9.86 223	12	44
17	9.83 608	13	9.97 396	25	0.02 604	9.86 211	11	43
18	9.83 621	13	9.97 421	26	0.02 579	9.86 200	12	42
19	9.83 634	14	9.97 447	25	0.02 553	9.86 188	12	41
20	9.83 648	13	9.97 472	25	0.02 528	9.86 176	12	**40**
21	9.83 661	13	9.97 497	26	0.02 503	9.86 164	12	39
22	9.83 674	14	9.97 523	25	0.02 477	9.86 152	12	38
23	9.83 688	13	9.97 548	25	0.02 452	9.86 140	12	37
24	9.83 701	14	9.97 573	25	0.02 427	9.86 128	12	36
25	9.83 715	13	9.97 598	26	0.02 402	9.86 116	12	**35**
26	9.83 728	13	9.97 624	25	0.02 376	9.86 104	12	34
27	9.83 741	14	9.97 649	25	0.02 351	9.86 092	12	33
28	9.83 755	13	9.97 674	26	0.02 326	9.86 080	12	32
29	9.83 768	13	9.97 700	25	0.02 300	9.86 068	12	31
30	9.83 781	14	9.97 725	25	0.02 275	9.86 056	12	**30**
31	9.83 795	13	9.97 750	26	0.02 250	9.86 044	12	29
32	9.83 808	13	9.97 776	25	0.02 224	9.86 032	12	28
33	9.83 821	13	9.97 801	25	0.02 199	9.86 020	12	27
34	9.83 834	14	9.97 826	25	0.02 174	9.86 008	12	26
35	9.83 848	13	9.97 851	26	0.02 149	9.85 996	12	**25**
36	9.83 861	13	9.97 877	25	0.02 123	9.85 984	12	24
37	9.83 874	13	9.97 902	25	0.02 098	9.85 972	12	23
38	9.83 887	14	9.97 927	26	0.02 073	9.85 960	12	22
39	9.83 901	13	9.97 953	25	0.02 047	9.85 948	12	21
40	9.83 914	13	9.97 978	25	0.02 022	9.85 936	12	**20**
41	9.83 927	13	9.98 003	26	0.01 997	9.85 924	12	19
42	9.83 940	14	9.98 029	25	0.01 971	9.85 912	12	18
43	9.83 954	13	9.98 054	25	0.01 946	9.85 900	12	17
44	9.83 967	13	9.98 079	25	0.01 921	9.85 888	12	16
45	9.83 980	13	9.98 104	26	0.01 896	9.85 876	12	**15**
46	9.83 993	13	9.98 130	25	0.01 870	9.85 864	13	14
47	9.84 006	14	9.98 155	25	0.01 845	9.85 851	12	13
48	9.84 020	13	9.98 180	26	0.01 820	9.85 839	12	12
49	9.84 033	13	9.98 206	25	0.01 794	8.85 827	12	11
50	9.84 046	13	9.98 231	25	0.01 769	9.85 815	12	**10**
51	9.84 059	13	9.98 256	25	0.01 744	9.85 803	12	9
52	9.84 072	13	9.98 281	26	0.01 719	9.85 791	12	8
53	9.84 085	13	9.98 307	25	0.01 693	9.85 779	13	7
54	9.84 098	14	9.98 332	25	0.01 668	9.85 766	12	6
55	9.84 112	13	9.98 357	26	0.01 643	9.85 754	12	**5**
56	9.84 125	13	9.98 383	25	0.01 617	9.85 742	12	4
57	9.84 138	13	9.98 408	25	0.01 592	9.85 730	12	3
58	9.84 151	13	9.98 433	25	0.01 567	9.85 718	12	2
59	9.84 164	13	9.98 458	26	0.01 542	9.85 706	13	1
60	9.84 177		9.98 484		0.01 516	9.85 693		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	26	**25**	**14**
2	5.2	5.0	2.8
3	7.8	7.5	4.2
4	10.4	10.0	5.6
5	13.0	12.5	7.0
6	15.6	15.0	8.4
7	18.2	17.5	9.8
8	20.8	20.0	11.2
9	23.4	22.5	12.6

	13	**12**	**11**
2	2.6	2.4	2.2
3	3.9	3.6	3.3
4	5.2	4.8	4.4
5	6.5	6.0	5.5
6	7.8	7.2	6.6
7	9.1	8.4	7.7
8	10.4	9.6	8.8
9	11.7	10.8	9.9

From the top :

For **43°**+ or **223°**+, read as printed; for **133°**+ or **313°**+, read co-function.

From the bottom :

For **46°**+ or **226°**+, read as printed; for **136°**+ or **316°**+, read co-function.

Prop. Pts.

46° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.84 177	13	9.98 484	25	0.01 516	9.85 693	12	**60**
1	9.84 190	13	9.98 509	25	0.01 491	9.85 681	12	59
2	9.84 203	13	9.98 534	26	0.01 466	9.85 669	12	58
3	9.84 216	13	9.98 560	25	0.01 440	9.85 657	12	57
4	9.84 229	13	9.98 585	25	0.01 415	9.85 645	13	56
5	9.84 242	13	9.98 610	25	0.01 390	9.85 632	12	**55**
6	9.84 255	14	9.98 635	26	0.01 365	9.85 620	12	54
7	9.84 269	13	9.98 661	25	0.01 339	9.85 608	12	53
8	9.84 282	13	9.98 686	25	0.01 314	9.85 596	13	52
9	9.84 295	13	9.98 711	26	0.01 289	9.85 583	12	51
10	9.84 308	13	9.98 737	25	0.01 263	9.85 571	12	**50**
11	9.84 321	13	9.98 762	25	0.01 238	9.85 559	12	49
12	9.84 334	13	9.98 787	25	0.01 213	9.85 547	13	48
13	9.84 347	13	9.98 812	26	0.01 188	9.85 534	12	47
14	9.84 360	13	9.98 838	25	0.01 162	9.85 522	12	46
15	9.84 373	12	9.98 863	25	0.01 137	9.85 510	13	**45**
16	9.84 385	13	9.98 888	25	0.01 112	9.85 497	12	44
17	9.84 398	13	9.98 913	26	0.01 087	9.85 485	12	43
18	9.84 411	13	9.98 939	25	0.01 061	9.85 473	13	42
19	9.84 424	13	9.98 964	25	0.01 036	9.85 460	12	41
20	9.84 437	13	9.98 989	26	0.01 011	9.85 448	12	**40**
21	9.84 450	13	9.99 015	25	0.00 985	9.85 436	13	39
22	9.84 463	13	9.99 040	25	0.00 960	9.85 423	12	38
23	9.84 476	13	9.99 065	25	0.00 935	9.85 411	12	37
24	9.84 489	13	9.99 090	26	0.00 910	9.85 399	13	36
25	9.84 502	13	9.99 116	25	0.00 884	9.85 386	12	**35**
26	9.84 515	13	9.99 141	25	0.00 859	9.85 374	13	34
27	9.84 528	12	9.99 166	25	0.00 834	9.85 361	12	33
28	9.84 540	13	9.99 191	26	0.00 809	9.85 349	12	32
29	9.84 553	13	9.99 217	25	0.00 783	9.85 337	13	31
30	9.84 566	13	9.99 242	25	0.00 758	9.85 324	12	**30**
31	9.84 579	13	9.99 267	26	0.00 733	9.85 312	13	29
32	9.84 592	13	9.99 293	25	0.00 707	9.85 299	12	28
33	9.84 605	13	9.99 318	25	0.00 682	9.85 287	13	27
34	9.84 618	12	9.99 343	25	0.00 657	9.85 274	12	26
35	9.84 630	13	9.99 368	26	0.00 632	9.85 262	12	**25**
36	9.84 643	13	9.99 394	25	0.00 606	9.85 250	13	24
37	9.84 656	13	9.99 419	25	0.00 581	9.85 237	12	23
38	9.84 669	13	9.99 444	25	0.00 556	9.85 225	13	22
39	9.84 682	12	9.99 469	26	0.00 531	9.85 212	12	21
40	9.84 694	13	9.99 495	25	0.00 505	9.85 200	13	**20**
41	9.84 707	13	9.99 520	25	0.00 480	9.85 187	12	19
42	9.84 720	13	9.99 545	25	0.00 455	9.85 175	13	18
43	9.84 733	12	9.99 570	26	0.00 430	9.85 162	12	17
44	9.84 745	13	9.99 596	25	0.00 404	9.85 150	13	16
45	9.84 758	13	9.99 621	25	0.00 379	9.85 137	12	**15**
46	9.84 771	13	9.99 646	26	0.00 354	9.85 125	13	14
47	9.84 784	12	9.99 672	25	0.00 328	9.85 112	12	13
48	9.84 796	13	9.99 697	25	0.00 303	9.85 100	13	12
49	9.84 809	13	9.99 722	25	0.00 278	9.85 087	13	11
50	9.84 822	13	9.99 747	26	0.00 253	9.85 074	12	**10**
51	9.84 835	12	9.99 773	25	0.00 227	9.85 062	13	9
52	9.84 847	13	9.99 798	25	0.00 202	9.85 049	12	8
53	9.84 860	13	9.99 823	25	0.00 177	9.85 037	13	7
54	9.84 873	12	9.99 848	26	0.00 152	9.85 024	12	6
55	9.84 885	13	9.99 874	25	0.00 126	9.85 012	13	**5**
56	9.84 898	13	9.99 899	25	0.00 101	9.84 999	13	4
57	9.84 911	12	9.99 924	25	0.00 076	9.84 986	12	3
58	9.84 923	13	9.99 949	26	0.00 051	9.84 974	13	2
59	9.84 936	13	9.99 975	25	0.00 025	9.84 961	12	1
60	9.84 949		0.00 000		0.00 000	9 84 949		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	26	25	14
2	5.2	5.0	2.8
3	7.8	7.5	4.2
4	10.4	10.0	5.6
5	13.0	12.5	7.0
6	15.6	15.0	8.4
7	18.2	17.5	9.8
8	20.8	20.0	11.2
9	23.4	22.5	12.6

	13	12
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

From the top :

For **44°+** or **224°+**, read as printed; for **134°+** or **314°+**, read co-function.

From the bottom :

For **45°+** or **225°+**, read as printed; for **135°+** or **315°+**, read co-function.

Prop. Pts.

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Degrees						Minutes		Seconds	
0°	0.00000 00	**60°**	1.04719 76	**120°**	2.09439 51	**0′**	0.00000 00	**0″**	0.00000 00
1	0.01745 33	61	1.06465 08	121	2.11184 84	1	0.00029 09	1	0.00000 48
2	0.03490 66	62	1.08210 41	122	2.12930 17	2	0.00058 18	2	0.00000 97
3	0.05235 99	63	1.09955 74	123	2.14675 50	3	0.00087 27	3	0.00001 45
4	0.06981 32	64	1.11701 07	124	2.16420 83	4	0.00116 36	4	0.00001 94
5	0.08726 65	**65**	1.13446 40	**125**	2.18166 16	**5**	0.00145 44	**5**	0.00002 42
6	0.10471 98	66	1.15191 73	126	2.19911 49	6	0.00174 53	6	0.00002 91
7	0.12217 30	67	1.16937 06	127	2.21656 82	7	0.00203 62	7	0.00003 39
8	0.13962 63	68	1.18682 39	128	2.23402 14	8	0.00232 71	8	0.00003 88
9	0.15707 96	69	1.20427 72	129	2.25147 47	9	0.00261 80	9	0.00004 36
10	0.17453 29	**70**	1.22173 05	**130**	2.26892 80	**10**	0.00290 89	**10**	0.00004 85
11	0.19198 62	71	1.23918 38	131	2.28638 13	11	0.00319 98	11	0.00005 33
12	0.20943 95	72	1.25663 71	132	2.30383 46	12	0.00349 07	12	0.00005 82
13	0.22689 28	73	1.27409 04	133	2.32128 79	13	0.00378 15	13	0.00006 30
14	0.24434 61	74	1.29154 36	134	2.33874 12	14	0.00407 24	14	0.00006 79
15	0.26179 94	**75**	1.30899 69	**135**	2.35619 45	**15**	0.00436 33	**15**	0.00007 27
16	0.27925 27	76	1.32645 02	136	2.37364 78	16	0.00465 42	16	0.00007 76
17	0.29670 60	77	1.34390 35	137	2.39110 11	17	0.00494 51	17	0.00008 24
18	0.31415 93	78	1.36135 68	138	2.40855 44	18	0.00523 60	18	0.00008 73
19	0.33161 26	79	1.37881 01	139	2.42600 77	19	0.00552 69	19	0.00009 21
20	0.34906 59	**80**	1.39626 34	**140**	2.44346 10	**20**	0.00581 78	**20**	0.00009 70
21	0.36651 91	81	1.41371 67	141	2.46091 42	21	0.00610 87	21	0.00010 18
22	0.38397 24	82	1.43117 00	142	2.47836 75	22	0.00639 95	22	0.00010 67
23	0.40142 57	83	1.44862 33	143	2.49582 08	23	0.00669 04	23	0.00011 15
24	0.41887 90	84	1.46607 66	144	2.51327 41	24	0.00698 13	24	0.00011 64
25	0.43633 23	**85**	1.48352 99	**145**	2.53072 74	**25**	0.00727 22	**25**	0.00012 12
26	0.45378 56	86	1.50098 32	146	2.54818 07	26	0.00756 31	26	0.00012 61
27	0.47123 89	87	1.51843 64	147	2.56563 40	27	0.00785 40	27	0.00013 09
28	0.48869 22	88	1.53588 97	148	2.58308 73	28	0.00814 49	28	0.00013 57
29	0.50614 55	89	1.55334 30	149	2.60054 06	29	0.00843 58	29	0.00014 06
30	0.52359 88	**90**	1.57079 63	**150**	2.61799 39	**30**	0.00872 66	**30**	0.00014 54
31	0.54105 21	91	1.58824 96	151	2.63544 72	31	0.00901 75	31	0.00015 03
32	0.55850 54	92	1.60570 29	152	2.65290 05	32	0.00930 84	32	0.00015 51
33	0.57595 87	93	1.62315 62	153	2.67035 38	33	0.00959 93	33	0.00016 00
34	0.59341 19	94	1.64060 95	154	2.68780 70	34	0.00989 02	34	0.00016 48
35	0.61086 52	**95**	1.65806 28	**155**	2.70526 03	**35**	0.01018 11	**35**	0.00016 97
36	0.62831 85	96	1.67551 61	156	2.72271 36	36	0.01047 20	36	0.00017 45
37	0.64577 18	97	1.69296 94	157	2.74016 69	37	0.01076 29	37	0.00017 94
38	0.66322 51	98	1.71042 27	158	2.75762 02	38	0.01105 38	38	0.00018 42
39	0.68067 84	99	1.72787 60	159	2.77507 35	39	0.01134 46	39	0.00018 91
40	0.69813 17	**100**	1.74532 93	**160**	2.79252 68	**40**	0.01163 55	**40**	0.00019 39
41	0.71558 50	101	1.76278 25	161	2.80998 01	41	0.01192 64	41	0.00019 88
42	0.73303 83	102	1.78023 58	162	2.82743 34	42	0.01221 73	42	0.00020 36
43	0.75049 16	103	1.79768 91	163	2.84488 67	43	0.01250 82	43	0.00020 85
44	0.76794 49	104	1.81514 24	164	2.86234 00	44	0.01279 91	44	0.00021 33
45	0.78539 82	**105**	1.83259 57	**165**	2.87979 33	**45**	0.01309 00	**45**	0.00021 82
46	0.80285 15	106	1.85004 90	166	2.89724 66	46	0.01338 09	46	0.00022 30
47	0.82030 47	107	1.86750 23	167	2.91469 99	47	0.01367 17	47	0.00022 79
48	0.83775 80	108	1.88495 56	168	2.93215 31	48	0.01396 26	48	0.00023 27
49	0.85521 13	109	1.90240 89	169	2.94960 64	49	0.01425 35	49	0.00023 76
50	0.87266 46	**110**	1.91986 22	**170**	2.96705 97	**50**	0.01454 44	**50**	0.00024 24
51	0.89011 79	111	1.93731 55	171	2.98451 30	51	0.01483 53	51	0.00024 73
52	0.90757 12	112	1.95476 88	172	3.00196 63	52	0.01512 62	52	0.00025 21
53	0.92502 45	113	1.97222 21	173	3.01941 96	53	0.01541 71	53	0.00025 70
54	0.94247 78	114	1.98967 53	174	3.03687 29	54	0.01570 80	54	0.00026 18
55	0.95993 11	**115**	2.00712 86	**175**	3.05432 62	**55**	0.01599 89	**55**	0.00026 66
56	0.97738 44	116	2.02458 19	176	3.07177 95	56	0.01628 97	56	0.00027 15
57	0.99483 77	117	2.04203 52	177	3.08923 28	57	0.01658 06	57	0.00027 63
58	1.01229 10	118	2.05948 85	178	3.10668 61	58	0.01687 15	58	0.00028 12
59	1.02974 43	119	2.07694 18	179	3.12413 94	59	0.01716 24	59	0.00028 60
60	1.04719 76	**120**	2.09439 51	**180**	3.14159 27	**60**	0.01745 33	**60**	0.00029 09

x Radians	Sin x	Cos x	Tan x	Equivalent of x
.00	.00000	1.0000	.00000	0° 00′.0
.01	.01000	.99995	.01000	0° 34′.4
.02	.02000	.99980	.02000	1° 08′.8
.03	.03000	.99955	.03001	1° 43′.1
.04	.03999	.99920	.04002	2° 17′.5
.05	.04998	.99875	.05004	2° 51′.9
.06	.05996	.99820	.06007	3° 26′.3
.07	.06994	.99755	.07011	4° 00′.6
.08	.07991	.99680	.08017	4° 35′.0
.09	.08988	.99595	.09024	5° 09′.4
.10	.09983	.99500	.10033	5° 43′.8
.11	.10978	.99396	.11045	6° 18′.2
.12	.11971	.99281	.12058	6° 52′.5
.13	.12963	.99156	.13074	7° 26′.9
.14	.13954	.99022	.14092	8° 01′.3
.15	.14944	.98877	.15114	8° 35′.7
.16	.15932	.98723	.16138	9° 10′.0
.17	.16918	.98558	.17166	9° 44′.4
.18	.17903	.98384	.18197	10° 18′.8
.19	.18886	.98200	.19232	10° 53′.2
.20	.19867	.98007	.20271	11° 27′.5
.21	.20846	.97803	.21314	12° 01′.9
.22	.21823	.97590	.22362	12° 36′.3
.23	.22798	.97367	.23414	13° 10′.7
.24	.23770	.97134	.24472	13° 45′.1
.25	.24740	.96891	.25534	14° 19′.4
.26	.25708	.96639	.26602	14° 53′.8
.27	.26673	.96377	.27676	15° 28′.2
.28	.27636	.96106	.28755	16° 02′.6
.29	.28595	.95824	.29841	16° 36′.9
.30	.29552	.95534	.30934	17° 11′.3
.31	.30506	.95233	.32033	17° 45′.7
.32	.31457	.94924	.33139	18° 20′.1
.33	.32404	.94604	.34252	18° 54′.5
.34	.33349	.94275	.35374	19° 28′.8
.35	.34290	.93937	.36503	20° 03′.2
.36	.35227	.93590	.37640	20° 37′.6
.37	.36162	.93233	.38786	21° 12′.0
.38	.37092	.92866	.39941	21° 46′.3
.39	.38019	.92491	.41106	22° 20′.7
.40	.38942	.92106	.42279	22° 55′.1
.41	.39861	.91712	.43463	23° 29′.5
.42	.40776	.91309	.44657	24° 03′.9
.43	.41687	.90897	.45862	24° 38′.2
.44	.42594	.90475	.47078	25° 12′.6
.45	.43497	.90045	.48305	25° 47′.0
.46	.44395	.89605	.49545	26° 21′.4
.47	.45289	.89157	.50795	26° 55′.7
.48	.46178	.88699	.52061	27° 30′.1
.49	.47063	.88233	.53339	28° 04′.5
.50	.47943	.87758	.54630	28° 38′.9

x Radians	Sin x	Cos x	Tan x	Equivalent of x
.50	.47943	.87758	.54630	28° 38′.9
.51	.48818	.87274	.55936	29° 13′.3
.52	.49688	.86782	.57256	29° 47′.6
.53	.50553	.86281	.58592	30° 22′.0
.54	.51414	.85771	.59943	30° 56′.4
.55	.52269	.85252	.61311	31° 30′.8
.56	.53119	.84726	.62695	32° 05′.1
.57	.53963	.84190	.64097	32° 39′.5
.58	.54802	.83646	.65517	33° 13′.9
.59	.55636	.83094	.66956	33° 48′.3
.60	.56464	.82534	.68414	34° 22′.6
.61	.57287	.81965	.69892	34° 57′.0
.62	.58104	.81388	.71391	35° 31′.4
.63	.58914	.80803	.72911	36° 05′.8
.64	.59720	.80210	.74454	36° 40′.2
.65	.60519	.79608	.76020	37° 14′.5
.66	.61312	.78999	.77610	37° 48′.9
.67	.62099	.78382	.79225	38° 23′.3
.68	.62879	.77757	.80866	38° 57′.7
.69	.63654	.77125	.82533	39° 32′.0
.70	.64422	.76484	.84229	40° 06′.4
.71	.65183	.75836	.85953	40° 40′.8
.72	.65938	.75181	.87707	41° 15′.2
.73	.66687	.74517	.89492	41° 49′.6
.74	.67429	.73847	.91309	42° 23′.9
.75	.68164	.73169	.93160	42° 58′.3
.76	.68892	.72484	.95055	43° 32′.7
.77	.69614	.71791	.96967	44° 07′.1
.78	.70328	.71091	.98926	44° 41′.4
.79	.71035	.70385	1.0092	45° 15′.8
.80	.71736	.69671	1.0296	45° 50′.2
.81	.72429	.68950	1.0505	46° 24′.6
.82	.73115	.68222	1.0717	46° 59′.0
.83	.73793	.67488	1.0934	47° 33′.3
.84	.74464	.66746	1.1156	48° 07′.7
.85	.75128	.65998	1.1383	48° 42′.1
.86	.75784	.65244	1.1616	49° 16′.5
.87	.76433	.64483	1.1853	49° 50′.8
.88	.77074	.63715	1.2097	50° 25′.2
.89	.77707	.62941	1.2346	50° 59′.6
.90	.78333	.62161	1.2602	51° 34′.0
.91	.78950	.61375	1.2864	52° 08′.3
.92	.79560	.60582	1.3133	52° 42′.7
.93	.80162	.59783	1.3409	53° 17′.1
.94	.80756	.58979	1.3692	53° 51′.5
.95	.81342	.58168	1.3984	54° 25′.9
.96	.81919	.57352	1.4284	55° 00′.2
.97	.82489	.56530	1.4592	55° 34′.6
.98	.83050	.55702	1.4910	56° 09′.0
.99	.83603	.54869	1.5237	56° 43′.4
1.00	.84147	.54030	1.5574	57° 17′.7

x Radians	Sin x	Cos x	Tan x	Equivalent of x
1.00	.84147	.54030	1.5574	57° 17′.7
1.01	.84683	.53186	1.5922	57° 52′.1
1.02	.85211	.52337	1.6281	58° 26′.5
1.03	.85730	.51482	1.6652	59° 00′.9
1.04	.86240	.50622	1.7036	59° 35′.3
1.05	.86742	.49757	1.7433	60° 09′.6
1.06	.87236	.48887	1.7844	60° 44′.0
1.07	.87720	.48012	1.8270	61° 18′.4
1.08	.88196	.47133	1.8712	61° 52′.8
1.09	.88663	.46249	1.9171	62° 27′.1
1.10	.89121	.45360	1.9648	63° 01′.5
1.11	.89570	.44466	2.0143	63° 35′.9
1.12	.90010	.43568	2.0660	64° 10′.3
1.13	.90441	.42666	2.1198	64° 44′.7
1.14	.90863	.41759	2.1759	65° 19′.0
1.15	.91276	.40849	2.2345	65° 53′.4
1.16	.91680	.39934	2.2958	66° 27′.8
1.17	.92075	.39015	2.3600	67° 02′.2
1.18	.92461	.38092	2.4273	67° 36′.5
1.19	.92837	.37166	2.4979	68° 10′.9
1.20	.93204	.36236	2.5722	68° 45′.3
1.21	.93562	.35302	2.6503	69° 19′.7
1.22	.93910	.34365	2.7328	69° 54′.1
1.23	.94249	.33424	2.8198	70° 28′.4
1.24	.94578	.32480	2.9119	71° 02′.8
1.25	.94898	.31532	3.0096	71° 37′.2
1.26	.95209	.30582	3.1133	72° 11′.6
1.27	.95510	.29628	3.2236	72° 45′.9
1.28	.95802	.28672	3.3413	73° 20′.3
1.29	.96084	.27712	3.4672	73° 54′.7
1.30	.96356	.26750	3.6021	74° 29′.1

x Radians	Sin x	Cos x	Tan x	Equivalent of x
1.30	.96356	.26750	3.6021	74° 29′.1
1.31	.96618	.25785	3.7470	75° 03′.4
1.32	.96872	.24818	3.9033	75° 37′.8
1.33	.97115	.23848	4.0723	76° 12′.2
1.34	.97348	.22875	4.2556	76° 46′.6
1.35	.97572	.21901	4.4552	77° 21′.0
1.36	.97786	.20924	4.6734	77° 55′.3
1.37	.97991	.19945	4.9131	78° 29′.7
1.38	.98185	.18964	5.1774	79° 04′.1
1.39	.98370	.17981	5.4707	79° 38′.5
1.40	.98545	.16997	5.7979	80° 12′.8
1.41	.98710	.16010	6.1654	80° 47′.2
1.42	.98865	.15023	6.5811	81° 21′.6
1.43	.99010	.14033	7.0555	81° 56′.0
1.44	.99146	.13042	7.6018	82° 30′.4
1.45	.99271	.12050	8.2381	83° 04′.7
1.46	.99387	.11057	8.9886	83° 39′.1
1.47	.99492	.10063	9.8874	84° 13′.5
1.48	.99588	.09067	10.983	84° 47′.9
1.49	.99674	.08071	12.350	85° 22′.2
1.50	.99749	.07074	14.101	85° 56′.6
1.51	.99815	.06076	16.428	86° 31′.0
1.52	.99871	.05077	19.670	87° 05′.4
1.53	.99917	.04079	24.498	87° 39′.8
1.54	.99953	.03079	32.461	88° 14′.1
1.55	.99978	.02079	48.078	88° 48′.5
1.56	.99994	.01080	92.621	89° 22′.9
1.57	1.0000	.00080	1255.8	89° 57′.3
1.58	.99996	-.00920	-108.65	90° 31′.6
1.59	.99982	-.01920	-52.067	91° 06′.0
1.60	.99957	-.02920	-34.233	91° 40′.4

π radians = 180° $\pi = 3.14159265$

1 radian = 57° 17′ 44″.806 = 57.° 2957795

3600″ = 60′ = 1° = .01745329 radian

TABLE Va — RADIANS TO DEGREES

	RADIANS	TENTHS	HUNDREDTHS	THOUSANDTHS	TEN-THOUSANDTHS
1	57°17′44″.8	5°43′46″.5	0°34′22″.6	0° 3′26″.3	0° 0′20″.6
2	114°35′29″.6	11°27′33″.0	1° 8′45″.3	0° 6′52″.5	0° 0′41″.3
3	171°53′14″.4	17°11′19″.4	1°43′07″.9	0°10′18″.8	0° 1′01″.9
4	229°10′59″.2	22°55′05″.9	2°17′30″.6	0°13′45″.1	0° 1′22″.5
5	286°28′44″.0	28°38′52″.4	2°51′53″.2	0°17′11″.3	0° 1′43″.1
6	343°46′28″.8	34°22′38″.9	3°26′15″.9	0°20′37″.6	0° 2′03″.8
7	401° 4′13″.6	40° 6′25″.4	4° 0′38″.5	0°24′03″.9	0° 2′24″.4
8	458°21′58″.4	45°50′11″.8	4°35′01″.2	0°27′30″.1	0° 2′45″.0
9	515°39′43″.3	51°33′58″.3	5° 9′23″.8	0°30′56″.4	0° 3′05″.6

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
1.00	1.0000	1.00000	3.16228	1.00000	1.00000	2.15443	4.64159	1.00000
1.01	1.0201	1.00499	3.17805	1.03030	1.00332	2.16159	4.65701	.990099
1.02	1.0404	1.00995	3.19374	1.06121	1.00662	2.16870	4.67233	.980392
1.03	1.0609	1.01489	3.20936	1.09273	1.00990	2.17577	4.68755	.970874
1.04	1.0816	1.01980	3.22490	1.12486	1.01316	2.18279	4.70267	.961538
1.05	1.1025	1.02470	3.24037	1.15762	1.01640	2.18976	4.71769	.952381
1.06	1.1236	1.02956	3.25576	1.19102	1.01961	2.19669	4.73262	.943396
1.07	1.1449	1.03441	3.27109	1.22504	1.02281	2.20358	4.74746	.934579
1.08	1.1664	1.03923	3.28634	1.25971	1.02599	2.21042	4.76220	.925926
1.09	1.1881	1.04403	3.30151	1.29503	1.02914	2.21722	4.77686	.917431
1.10	1.2100	1.04881	3.31662	1.33100	1.03228	2.22398	4.79142	.909091
1.11	1.2321	1.05357	3.33167	1.36763	1.03540	2.23070	4.80590	.900901
1.12	1.2544	1.05830	3.34664	1.40493	1.03850	2.23738	4.82028	.892857
1.13	1.2769	1.06301	3.36155	1.44290	1.04158	2.24402	4.83459	.884956
1.14	1.2996	1.06771	3.37639	1.48154	1.04464	2.25062	4.84881	.877193
1.15	1.3225	1.07238	3.39116	1.52088	1.04769	2.25718	4.86294	.869565
1.16	1.3456	1.07703	3.40588	1.56090	1.05072	2.26370	4.87700	.862069
1.17	1.3689	1.08167	3.42053	1.60161	1.05373	2.27019	4.89097	.854701
1.18	1.3924	1.08628	3.43511	1.64303	1.05672	2.27664	4.90487	.847458
1.19	1.4161	1.09087	3.44964	1.68516	1.05970	2.28305	4.91868	.840336
1.20	1.4400	1.09545	3.46410	1.72800	1.06266	2.28943	4.93242	.833333
1.21	1.4641	1.10000	3.47851	1.77156	1.06560	2.29577	4.94609	.826446
1.22	1.4884	1.10454	3.49285	1.81585	1.06853	2.30208	4.95968	.819672
1.23	1.5129	1.10905	3.50714	1.86087	1.07144	2.30835	4.97319	.813008
1.24	1.5376	1.11355	3.52136	1.90662	1.07434	2.31459	4.98663	.806452
1.25	1.5625	1.11803	3.53553	1.95312	1.07722	2.32079	5.00000	.800000
1.26	1.5876	1.12250	3.54965	2.00038	1.08008	2.32697	5.01330	.793651
1.27	1.6129	1.12694	3.56371	2.04838	1.08293	2.33311	5.02653	.787402
1.28	1.6384	1.13137	3.57771	2.09715	1.08577	2.33921	5.03968	.781250
1.29	1.6641	1.13578	3.59166	2.14669	1.08859	2.34529	5.05277	.775194
1.30	1.6900	1.14018	3.60555	2.19700	1.09139	2.35133	5.06580	.769231
1.31	1.7161	1.14455	3.61939	2.24809	1.09418	2.35735	5.07875	.763359
1.32	1.7424	1.14891	3.63318	2.29997	1.09696	2.36333	5.09164	.757576
1.33	1.7689	1.15326	3.64692	2.35264	1.09972	2.36928	5.10447	.751880
1.34	1.7956	1.15758	3.66060	2.40610	1.10247	2.37521	5.11723	.746269
1.35	1.8225	1.16190	3.67423	2.46038	1.10521	2.38110	5.12993	.740741
1.36	1.8496	1.16619	3.68782	2.51546	1.10793	2.38697	5.14256	.735294
1.37	1.8769	1.17047	3.70135	2.57135	1.11064	2.39280	5.15514	.729927
1.38	1.9044	1.17473	3.71484	2.62807	1.11334	2.39861	5.16765	.724638
1.39	1.9321	1.17898	3.72827	2.68562	1.11602	2.40439	5.18010	.719424
1.40	1.9600	1.18322	3.74166	2.74400	1.11869	2.41014	5.19249	.714286
1.41	1.9881	1.18743	3.75500	2.80322	1.12135	2.41587	5.20483	.709220
1.42	2.0164	1.19164	3.76829	2.86329	1.12399	2.42156	5.21710	.704225
1.43	2.0449	1.19583	3.78153	2.92421	1.12662	2.42724	5.22932	.699301
1.44	2.0736	1.20000	3.79473	2.98598	1.12924	2.43288	5.24148	.694444
1.45	2.1025	1.20416	3.80789	3.04862	1.13185	2.43850	5.25359	.689655
1.46	2.1316	1.20830	3.82099	3.11214	1.13445	2.44409	5.26564	.684932
1.47	2.1609	1.21244	3.83406	3.17652	1.13703	2.44966	5.27763	.680272
1.48	2.1904	1.21655	3.84708	3.24179	1.13960	2.45520	5.28957	.675676
1.49	2.2201	1.22066	3.86005	3.30795	1.14216	2.46072	5.30146	.671141
1.50	2.2500	1.22474	3.87298	3.37500	1.14471	2.46621	5.31329	.666667
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
1.50	2.2500	1.22474	3.87298	3.37500	1.14471	2.46621	5.31329	.666667
1.51	2.2801	1.22882	3.88587	3.44295	1.14725	2.47168	5.32507	.662252
1.52	2.3104	1.23288	3.89872	3.51181	1.14978	2.47712	5.33680	.657895
1.53	2.3409	1.23693	3.91152	3.58158	1.15230	2.48255	5.34848	.653595
1.54	2.3716	1.24097	3.92428	3.65226	1.15480	2.48794	5.36011	.649351
1.55	2.4025	1.24499	3.93700	3.72388	1.15729	2.49332	5.37169	.645161
1.56	2.4336	1.24900	3.94968	3.79642	1.15978	2.49867	5.38321	.641026
1.57	2.4649	1.25300	3.96232	3.86989	1.16225	2.50399	5.39469	.636943
1.58	2.4964	1.25698	3.97492	3.94431	1.16471	2.50930	5.40612	.632911
1.59	2.5281	1.26095	3.98748	4.01968	1.16717	2.51458	5.41750	.628931
1.60	2.5600	1.26491	4.00000	4.09600	1.16961	2.51984	5.42884	.625000
1.61	2.5921	1.26886	4.01248	4.17328	1.17204	2.52508	5.44012	.621118
1.62	2.6244	1.27279	4.02492	4.25153	1.17446	2.53030	5.45136	.617284
1.63	2.6569	1.27671	4.03733	4.33075	1.17687	2.53549	5.46256	.613497
1.64	2.6896	1.28062	4.04969	4.41094	1.17927	2.54067	5.47370	.609756
1.65	2.7225	1.28452	4.06202	4.49212	1.18167	2.54582	5.48481	.606061
1.66	2.7556	1.28841	4.07431	4.57430	1.18405	2.55095	5.49586	.602410
1.67	2.7889	1.29228	4.08656	4.65746	1.18642	2.55607	5.50688	.598802
1.68	2.8224	1.29615	4.09878	4.74163	1.18878	2.56116	5.51785	.595238
1.69	2.8561	1.30000	4.11096	4.82681	1.19114	2.56623	5.52877	.591716
1.70	2.8900	1.30384	4.12311	4.91300	1.19348	2.57128	5.53966	.588235
1.71	2.9241	1.30767	4.13521	5.00021	1.19582	2.57631	5.55050	.584795
1.72	2.9584	1.31149	4.14729	5.08845	1.19815	2.58133	5.56130	.581395
1.73	2.9929	1.31529	4.15933	5.17772	1.20046	2.58632	5.57205	.578035
1.74	3.0276	1.31909	4.17133	5.26802	1.20277	2.59129	5.58277	.574713
1.75	3.0625	1.32288	4.18330	5.35938	1.20507	2.59625	5.59344	.571429
1.76	3.0976	1.32665	4.19524	5.45178	1.20736	2.60118	5.60408	.568182
1.77	3.1329	1.33041	4.20714	5.54523	1.20964	2.60610	5.61467	.564972
1.78	3.1684	1.33417	4.21900	5.63975	1.21192	2.61100	5.62523	.561798
1.79	3.2041	1.33791	4.23084	5.73534	1.21418	2.61588	5.63574	.558659
1.80	3.2400	1.34164	4.24264	5.83200	1.21644	2.62074	5.64622	.555556
1.81	3.2761	1.34536	4.25441	5.92974	1.21869	2.62559	5.65665	.552486
1.82	3.3124	1.34907	4.26615	6.02857	1.22093	2.63041	5.66705	.549451
1.83	3.3489	1.35277	4.27785	6.12849	1.22316	2.63522	5.67741	.546448
1.84	3.3856	1.35647	4.28952	6.22950	1.22539	2.64001	5.68773	.543478
1.85	3.4225	1.36015	4.30116	6.33162	1.22760	2.64479	5.69802	.540541
1.86	3.4596	1.36382	4.31277	6.43486	1.22981	2.64954	5.70827	.537634
1.87	3.4969	1.36748	4.32435	6.53920	1.23201	2.65428	5.71848	.534759
1.88	3.5344	1.37113	4.33590	6.64467	1.23420	2.65901	5.72865	.531915
1.89	3.5721	1.37477	4.34741	6.75127	1.23639	2.66371	5.73879	.529101
1.90	3.6100	1.37840	4.35890	6.85900	1.23856	2.66840	5.74890	.526316
1.91	3.6481	1.38203	4.37035	6.96787	1.24073	2.67307	5.75897	.523560
1.92	3.6864	1.38564	4.38178	7.07789	1.24289	2.67773	5.76900	.520833
1.93	3.7249	1.38924	4.39318	7.18906	1.24505	2.68237	5.77900	.518135
1.94	3.7636	1.39284	4.40454	7.30138	1.24719	2.68700	5.78896	.515464
1.95	3.8025	1.39642	4.41588	7.41488	1.24933	2.69161	5.79889	.512821
1.96	3.8416	1.40000	4.42719	7.52954	1.25146	2.69620	5.80879	.510204
1.97	3.8809	1.40357	4.43847	7.64537	1.25359	2.70078	5.81865	.507614
1.98	3.9204	1.40712	4.44972	7.76239	1.25571	2.70534	5.82848	.505051
1.99	3.9601	1.41067	4.46094	7.88060	1.25782	2.70989	5.83827	.502513
2.00	4.0000	1.41421	4.47214	8.00000	1.25992	2.71442	5.84804	.500000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
2.00	4.0000	1.41421	4.47214	8.00000	1.25992	2.71442	5.84804	.500000
2.01	4.0401	1.41774	4.48330	8.12060	1.26202	2.71893	5.85777	.497512
2.02	4.0804	1.42127	4.49444	8.24241	1.26411	2.72344	5.86746	.495050
2.03	4.1209	1.42478	4.50555	8.36543	1.26619	2.72792	5.87713	.492611
2.04	4.1616	1.42829	4.51664	8.48966	1.26827	2.73239	5.88677	.490196
2.05	4.2025	1.43178	4.52769	8.61512	1.27033	2.73685	5.89637	.487805
2.06	4.2436	1.43527	4.53872	8.74182	1.27240	2.74129	5.90594	.485437
2.07	4.2849	1.43875	4.54973	8.86974	1.27445	2.74572	5.91548	.483092
2.08	4.3264	1.44222	4.56070	8.99891	1.27650	2.75014	5.92499	.480769
2.09	4.3681	1.44568	4.57165	9.12933	1.27854	2.75454	5.93447	.478469
2.10	4.4100	1.44914	4.58258	9.26100	1.28058	2.75892	5.94392	.476190
2.11	4.4521	1.45258	4.59347	9.39393	1.28261	2.76330	5.95334	.473934
2.12	4.4944	1.45602	4.60435	9.52813	1.28463	2.76766	5.96273	.471698
2.13	4.5369	1.45945	4.61519	9.66360	1.28665	2.77200	5.97209	.469434
2.14	4.5796	1.46287	4.62601	9.80034	1.28866	2.77633	5.98142	.467290
2.15	4.6225	1.46629	4.63681	9.93838	1.29066	2.78065	5.99073	.465116
2.16	4.6656	1.46969	4.64758	10.0777	1.29266	2.78495	6.00000	.462963
2.17	4.7089	1.47309	4.65833	10.2183	1.29465	2.78924	6.00925	.460829
2.18	4.7524	1.47648	4.66905	10.3602	1.29664	2.79352	6.01846	.458716
2.19	4.7961	1.47986	4.67974	10.5035	1.29862	2.79779	6.02765	.456621
2.20	4.8400	1.48324	4.69042	10.6480	1.30059	2.80204	6.03681	.454545
2.21	4.8841	1.48661	4.70106	10.7939	1.30256	2.80628	6.04594	.452489
2.22	4.9284	1.48997	4.71169	10.9410	1.30452	2.81050	6.05505	.450450
2.23	4.9729	1.49332	4.72229	11.0896	1.30648	2.81472	6.06413	.448430
2.24	5.0176	1.49666	4.73286	11.2394	1.30843	2.81892	6.07318	.446429
2.25	5.0625	1.50000	4.74342	11.3906	1.31037	2.82311	6.08220	.444444
2.26	5.1076	1.50333	4.75395	11.5432	1.31231	2.82728	6.09120	.442478
2.27	5.1529	1.50665	4.76445	11.6971	1.31424	2.83145	6.10017	.440529
2.28	5.1984	1.50997	4.77493	11.8524	1.31617	2.83560	6.10911	.438596
2.29	5.2441	1.51327	4.78539	12.0090	1.31809	2.83974	6.11803	.436681
2.30	5.2900	1.51658	4.79583	12.1670	1.32001	2.84387	6.12693	.434783
2.31	5.3361	1.51987	4.80625	12.3264	1.32192	2.84798	6.13579	.432900
2.32	5.3824	1.52315	4.81664	12.4872	1.32382	2.85209	6.14463	.431034
2.33	5.4289	1.52643	4.82701	12.6493	1.32572	2.85618	6.15345	.429185
2.34	5.4756	1.52971	4.83735	12.8129	1.32761	2.86026	6.16224	.427350
2.35	5.5225	1.53297	4.84768	12.9779	1.32950	2.86433	6.17101	.425532
2.36	5.5696	1.53623	4.85798	13.1443	1.33139	2.86838	6.17975	.423729
2.37	5.6169	1.53948	4.86826	13.3121	1.33326	2.87243	6.18846	.421941
2.38	5.6644	1.54272	4.87852	13.4813	1.33514	2.87646	6.19715	.420168
2.39	5.7121	1.54596	4.88876	13.6519	1.33700	2.88049	6.20582	.418410
2.40	5.7600	1.54919	4.89898	13.8240	1.33887	2.88450	6.21447	.416667
2.41	5.8081	1.55242	4.90918	13.9975	1.34072	2.88850	6.22308	.414938
2.42	5.8564	1.55563	4.91935	14.1725	1.34257	2.89249	6.23168	.413223
2.43	5.9049	1.55885	4.92950	14.3489	1.34442	2.89647	6.24025	.411523
2.44	5.9536	1.56205	4.93964	14.5268	1.34626	2.90044	6.24880	.409836
2.45	6.0025	1.56525	4.94975	14.7061	1.34810	2.90439	6.25732	.408163
2.46	6.0516	1.56844	4.95984	14.8869	1.34993	2.90834	6.26583	.406504
2.47	6.1009	1.57162	4.96991	15.0692	1.35176	2.91227	6.27431	.404858
2.48	6.1504	1.57480	4.97996	15.2530	1.35358	2.91620	6.28276	.403226
2.49	6.2001	1.57797	4.98999	15.4382	1.35540	2.92011	6.29119	.401606
2.50	6.2500	1.58114	5.00000	15.6250	1.35721	2.92402	6.29961	.400000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
2.50	6.2500	1.58114	5.00000	15.6250	1.35721	2.92402	6.29961	.400000
2.51	6.3001	1.58430	5.00999	15.8133	1.35902	2.92791	6.30799	.398406
2.52	6.3504	1.58745	5.01996	16.0030	1.36082	2.93179	6.31636	.396825
2.53	6.4009	1.59060	5.02991	16.1943	1.36262	2.93567	6.32470	.395257
2.54	6.4516	1.59374	5.03984	16.3871	1.36441	2.93953	6.33303	.393701
2.55	6.5025	1.59687	5.04975	16.5814	1.36620	2.94338	6.34133	.392157
2.56	6.5536	1.60000	5.05964	16.7772	1.36798	2.94723	6.34960	.390625
2.57	6.6049	1.60312	5.06952	16.9746	1.36976	2.95106	6.35786	.389105
2.58	6.6564	1.60624	5.07937	17.1735	1.37153	2.95488	6.36610	.387597
2.59	6.7081	1.60935	5.08920	17.3740	1.37330	2.95869	6.37431	.386100
2.60	6.7600	1.61245	5.09902	17.5760	1.37507	2.96250	6.38250	.384615
2.61	6.8121	1.61555	5.10882	17.7796	1.37683	2.96629	6.39068	.383142
2.62	6.8644	1.61864	5.11859	17.9847	1.37859	2.97007	6.39883	.381679
2.63	6.9169	1.62173	5.12835	18.1914	1.38034	2.97385	6.40696	.380228
2.64	6.9696	1.62481	5.13809	18.3997	1.38208	2.97761	6.41507	.378788
2.65	7.0225	1.62788	5.14782	18.6096	1.38383	2.98137	6.42316	.377358
2.66	7.0756	1.63095	5.15752	18.8211	1.38557	2.98511	6.43123	.375940
2.67	7.1289	1.63401	5.16720	19.0342	1.38730	2.98885	6.43928	.374532
2.68	7.1824	1.63707	5.17687	19.2488	1.38903	2.99257	6.44731	.373134
2.69	7.2361	1.64012	5.18652	19.4651	1.39076	2.99629	6.45531	.371747
2.70	7.2900	1.64317	5.19615	19.6830	1.39248	3.00000	6.46330	.370370
2.71	7.3441	1.64621	5.20577	19.9025	1.39419	3.00370	6.47127	.369004
2.72	7.3984	1.64924	5.21536	20.1236	1.39591	3.00739	6.47922	.367647
2.73	7.4529	1.65227	5.22494	20.3464	1.39761	3.01107	6.48715	.366300
2.74	7.5076	1.65529	5.23450	20.5708	1.39932	3.01474	6.49507	.364964
2.75	7.5625	1.65831	5.24404	20.7969	1.40102	3.01841	6.50296	.363636
2.76	7.6176	1.66132	5.25357	21.0246	1.40272	3.02206	6.51083	.362319
2.77	7.6729	1.66433	5.26308	21.2539	1.40441	3.02570	6.51868	.361011
2.78	7.7284	1.66733	5.27257	21.4850	1.40610	3.02934	6.52652	.359712
2.79	7.7841	1.67033	5.28205	21.7176	1.40778	3.03297	6.53434	.358423
2.80	7.8400	1.67332	5.29150	21.9520	1.40946	3.03659	6.54213	.357143
2.81	7.8961	1.67631	5.30094	22.1880	1.41114	3.04020	6.54991	.355872
2.82	7.9524	1.67929	5.31037	22.4258	1.41281	3.04380	6.55767	.354610
2.83	8.0089	1.68226	5.31977	22.6652	1.41448	3.04740	6.56541	.353357
2.84	8.0656	1.68523	5.32917	22.9063	1.41614	3.05098	6.57314	.352113
2.85	8.1225	1.68819	5.33854	23.1491	1.41780	3.05456	6.58084	.350877
2.86	8.1796	1.69115	5.34790	23.3937	1.41946	3.05813	6.58853	.349650
2.87	8.2369	1.69411	5.35724	23.6399	1.42111	3.06169	6.59620	.348432
2.88	8.2944	1.69706	5.36656	23.8879	1.42276	3.06524	6.60385	.347222
2.89	8.3521	1.70000	5.37587	24.1376	1.42440	3.06878	6.61149	.346021
2.90	8.4100	1.70294	5.38516	24.3890	1.42604	3.07232	6.61911	.344828
2.91	8.4681	1.70587	5.39444	24.6422	1.42768	3.07584	6.62671	.343643
2.92	8.5264	1.70880	5.40370	24.8971	1.42931	3.07936	6.63429	.342466
2.93	8.5849	1.71172	5.41295	25.1538	1.43094	3.08287	6.64185	.341297
2.94	8.6436	1.71464	5.42218	25.4122	1.43257	3.08638	6.64940	.340136
2.95	8.7025	1.71756	5.43139	25.6724	1.43419	3.08987	6.65693	.338983
2.96	8.7616	1.72047	5.44059	25.9343	1.43581	3.09336	6.66444	.337838
2.97	8.8209	1.72337	5.44977	26.1981	1.43743	3.09684	6.67194	.336700
2.98	8.8804	1.72627	5.45894	26.4636	1.43904	3.10031	6.67942	.335570
2.99	8.9401	1.72916	5.46809	26.7309	1.44065	3.10378	6.68688	.334448
3.00	9.0000	1.73205	5.47723	27.0000	1.44225	3.10723	6.69433	.333333
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
3.00	9.0000	1.73205	5.47723	27.0000	1.44225	3.10723	6.69433	.333333
3.01	9.0601	1.73494	5.48635	27.2709	1.44385	3.11068	6.70176	.332226
3.02	9.1204	1.73781	5.49545	27.5436	1.44545	3.11412	6.70917	.331126
3.03	9.1809	1.74069	5.50454	27.8181	1.44704	3.11756	6.71657	.330033
3.04	9.2416	1.74356	5.51362	28.0945	1.44863	3.12098	6.72395	.328947
3.05	9.3025	1.74642	5.52268	28.3726	1.45022	3.12440	6.73132	.327869
3.06	9.3636	1.74929	5.53173	28.6526	1.45180	3.12781	6.73866	.326797
3.07	9.4249	1.75214	5.54076	28.9344	1.45338	3.13121	6.74600	.325733
3.08	9.4864	1.75499	5.54977	29.2181	1.45496	3.13461	6.75331	.324675
3.09	9.5481	1.75784	5.55878	29.5036	1.45653	3.13800	6.76061	.323625
3.10	9.6100	1.76068	5.56776	29.7910	1.45810	3.14138	6.76790	.322581
3.11	9.6721	1.76352	5.57674	30.0802	1.45967	3.14475	6.77517	.321543
3.12	9.7344	1.76635	5.58570	30.3713	1.46123	3.14812	6.78242	.320513
3.13	9.7969	1.76918	5.59464	30.6643	1.46279	3.15148	6.78966	.319489
3.14	9.8596	1.77200	5.60357	30.9591	1.46434	3.15483	6.79688	.318471
3.15	9.9225	1.77482	5.61249	31.2559	1.46590	3.15818	6.80409	.317460
3.16	9.9856	1.77764	5.62139	31.5545	1.46745	3.16152	6.81128	.316456
3.17	10.0489	1.78045	5.63028	31.8550	1.46899	3.16485	6.81846	.315457
3.18	10.1124	1.78326	5.63915	32.1574	1.47054	3.16817	6.82562	.314465
3.19	10.1761	1.78606	5.64801	32.4618	1.47208	3.17149	6.83277	.313480
3.20	10.2400	1.78885	5.65685	32.7680	1.47361	3.17480	6.83990	.312500
3.21	10.3041	1.79165	5.66569	33.0762	1.47515	3.17811	6.84702	.311526
3.22	10.3684	1.79444	5.67450	33.3862	1.47668	3.18140	6.85412	.310559
3.23	10.4329	1.79722	5.68331	33.6983	1.47820	3.18469	6.86121	.309598
3.24	10.4976	1.80000	5.69210	34.0122	1.47973	3.18798	6.86829	.308642
3.25	10.5625	1.80278	5.70088	34.3281	1.48125	3.19125	6.87534	.307692
3.26	10.6276	1.80555	5.70964	34.6460	1.48277	3.19452	6.88239	.306748
3.27	10.6929	1.80831	5.71839	34.9658	1.48428	3.19778	6.88942	.305810
3.28	10.7584	1.81108	5.72713	35.2876	1.48579	3.20104	6.89643	.304878
3.29	10.8241	1.81384	5.73585	35.6113	1.48730	3.20429	6.90344	.303951
3.30	10.8900	1.81659	5.74456	35.9370	1.48881	3.20753	6.91042	.303030
3.31	10.9561	1.81934	5.75326	36.2647	1.49031	3.21077	6.91740	.302115
3.32	11.0224	1.82209	5.76194	36.5944	1.49181	3.21400	6.92436	.301205
3.33	11.0889	1.82483	5.77062	36.9260	1.49330	3.21722	6.93130	.300300
3.34	11.1556	1.82757	5.77927	37.2597	1.49480	3.22044	6.93823	.299401
3.35	11.2225	1.83030	5.78792	37.5954	1.49629	3.22365	6.94515	.298507
3.36	11.2896	1.83303	5.79655	37.9331	1.49777	3.22686	6.95205	.297619
3.37	11.3569	1.83576	5.80517	38.2728	1.49926	3.23006	6.95894	.296736
3.38	11.4244	1.83848	5.81378	38.6145	1.50074	3.23325	6.96582	.295858
3.39	11.4921	1.84120	5.82237	38.9582	1.50222	3.23643	6.97268	.294985
3.40	11.5600	1.84391	5.83095	39.3040	1.50369	3.23961	6.97953	.294118
3.41	11.6281	1.84662	5.83952	39.6518	1.50517	3.24278	6.98637	.293255
3.42	11.6964	1.84932	5.84808	40.0017	1.50664	3.24595	6.99319	.292398
3.43	11.7649	1.85203	5.85662	40.3536	1.50810	3.24911	7.00000	.291545
3.44	11.8336	1.85472	5.86515	40.7076	1.50957	3.25227	7.00680	.290698
3.45	11.9025	1.85742	5.87367	41.0636	1.51103	3.25542	7.01358	.289855
3.46	11.9716	1.86011	5.88218	41.4217	1.51249	3.25856	7.02035	.289017
3.47	12.0409	1.86279	5.89067	41.7819	1.51394	3.26169	7.02711	.288184
3.48	12.1104	1.86548	5.89915	42.1442	1.51540	3.26482	7.03385	.287356
3.49	12.1801	1.86815	5.90762	42.5085	1.51685	3.26795	7.04058	.286533
3.50	12.2500	1.87083	5.91608	42.8750	1.51829	3.27107	7.04730	.285714
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

Powers — Roots — Reciprocals

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
3.50	12.2500	1.87083	5.91608	42.8750	1.51829	3.27107	7.04730	.285714
3.51	12.3201	1.87350	5.92453	43.2436	1.51974	3.27418	7.05400	.284900
3.52	12.3904	1.87617	5.93296	43.6142	1.52118	3.27729	7.06070	.284091
3.53	12.4609	1.87883	5.94138	43.9870	1.52262	3.28039	7.06738	.283286
3.54	12.5316	1.88149	5.94979	44.3619	1.52406	3.28348	7.07404	.282486
3.55	12.6025	1.88414	5.95819	44.7389	1.52549	3.28657	7.08070	.281690
3.56	12.6736	1.88680	5.96657	45.1180	1.52692	3.28965	7.08734	.280899
3.57	12.7449	1.88944	5.97495	45.4993	1.52835	3.29273	7.09397	.280112
3.58	12.8164	1.89209	5.98331	45.8827	1.52978	3.29580	7.10059	.279330
3.59	12.8881	1.89473	5.99166	46.2683	1.53120	3.29887	7.10719	.278552
3.60	12.9600	1.89737	6.00000	46.6560	1.53262	3.30193	7.11379	.277778
3.61	13.0321	1.90000	6.00833	47.0459	1.53404	3.30498	7.12037	.277008
3.62	13.1044	1.90263	6.01664	47.4379	1.53545	3.30803	7.12694	.276243
3.63	13.1769	1.90526	6.02495	47.8321	1.53686	3.31107	7.13349	.275482
3.64	13.2496	1.90788	6.03324	48.2285	1.53827	3.31411	7.14004	.274725
3.65	13.3225	1.91050	6.04152	48.6271	1.53968	3.31714	7.14657	.273973
3.66	13.3956	1.91311	6.04979	49.0279	1.54109	3.32017	7.15309	.273224
3.67	13.4689	1.91572	6.05805	49.4309	1.54249	3.32319	7.15960	.272480
3.68	13.5424	1.91833	6.06630	49.8360	1.54389	3.32621	7.16610	.271739
3.69	13.6161	1.92094	6.07454	50.2434	1.54529	3.32922	7.17258	.271003
3.70	13.6900	1.92354	6.08276	50.6530	1.54668	3.33222	7.17905	.270270
3.71	13.7641	1.92614	6.09098	51.0648	1.54807	3.33522	7.18552	.269542
3.72	13.8384	1.92873	6.09918	51.4788	1.54946	3.33822	7.19197	.268817
3.73	13.9129	1.93132	6.10737	51.8951	1.55085	3.34120	7.19840	.268097
3.74	13.9876	1.93391	6.11555	52.3136	1.55223	3.34419	7.20483	.267380
3.75	14.0625	1.93649	6.12372	52.7344	1.55362	3.34716	7.21125	.266667
3.76	14.1376	1.93907	6.13188	53.1574	1.55500	3.35014	7.21765	.265957
3.77	14.2129	1.94165	6.14003	53.5826	1.55637	3.35310	7.22405	.265252
3.78	14.2884	1.94422	6.14817	54.0102	1.55775	3.35607	7.23043	.264550
3.79	14.3641	1.94679	6.15630	54.4399	1.55912	3.35902	7.23680	.263852
3.80	14.4400	1.94936	6.16441	54.8720	1.56049	3.36198	7.24316	.263158
3.81	14.5161	1.95192	6.17252	55.3063	1.56186	3.36492	7.24950	.262467
3.82	14.5924	1.95448	6.18061	55.7430	1.56322	3.36786	7.25584	.261780
3.83	14.6689	1.95704	6.18870	56.1819	1.56459	3.37080	7.26217	.261097
3.84	14.7456	1.95959	6.19677	56.6231	1.56595	3.37373	7.26848	.260417
3.85	14.8225	1.96214	6.20484	57.0666	1.56731	3.37666	7.27479	.259740
3.86	14.8996	1.96469	6.21289	57.5125	1.56866	3.37958	7.28108	.259067
3.87	14.9769	1.96723	6.22093	57.9606	1.57001	3.38249	7.28736	.258398
3.88	15.0544	1.96977	6.22896	58.4111	1.57137	3.38540	7.29363	.257732
3.89	15.1321	1.97231	6.23699	58.8639	1.57271	3.38831	7.29989	.257069
3.90	15.2100	1.97484	6.24500	59.3190	1.57406	3.39121	7.30614	.256410
3.91	15.2881	1.97737	6.25300	59.7765	1.57541	3.39411	7.31238	.255754
3.92	15.3664	1.97990	6.26099	60.2363	1.57675	3.39700	7.31861	.255102
3.93	15.4449	1.98242	6.26897	60.6985	1.57809	3.39988	7.32483	.254453
3.94	15.5236	1.98494	6.27694	61.1630	1.57942	3.40277	7.33104	.253807
3.95	15.6025	1.98746	6.28490	61.6299	1.58076	3.40564	7.33723	.253165
3.96	15.6816	1.98997	6.29285	62.0991	1.58209	3.40851	7.34342	.252525
3.97	15.7609	1.99249	6.30079	62.5708	1.58342	3.41138	7.34960	.251889
3.98	15.8404	1.99499	6.30872	63.0448	1.58475	3.41424	7.35576	.251256
3.99	15.9201	1.99750	6.31664	63.5212	1.58608	3.41710	7.36192	.250627
4.00	16.0000	2.00000	6.32456	64.0000	1.58740	3.41995	7.36806	.250000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
4.00	16.0000	2.00000	6.32456	64.0000	1.58740	3.41995	7.36806	.250000
4.01	16.0801	2.00250	6.33246	64.4812	1.58872	3.42280	7.37420	.249377
4.02	16.1604	2.00499	6.34035	64.9648	1.59004	3.42564	7.38032	.248756
4.03	16.2409	2.00749	6.34823	65.4508	1.59136	3.42848	7.38644	.248139
4.04	16.3216	2.00998	6.35610	65.9393	1.59267	3.43131	7.39254	.247525
4.05	16.4025	2.01246	6.36396	66.4301	1.59399	3.43414	7.39864	.246914
4.06	16.4836	2.01494	6.37181	66.9234	1.59530	3.43697	7.40472	.246305
4.07	16.5649	2.01742	6.37966	67.4191	1.59661	3.43979	7.41080	.245700
4.08	16.6464	2.01990	6.38749	67.9173	1.59791	3.44260	7.41686	.245098
4.09	16.7281	2.02237	6.39531	68.4179	1.59922	3.44541	7.42291	.244499
4.10	16.8100	2.02485	6.40312	68.9210	1.60052	3.44822	7.42896	.243902
4.11	16.8921	2.02731	6.41093	69.4265	1.60182	3.45102	7.43499	.243309
4.12	16.9744	2.02978	6.41872	69.9345	1.60312	3.45382	7.44102	.242718
4.13	17.0569	2.03224	6.42651	70.4450	1.60441	3.45661	7.44703	.242131
4.14	17.1396	2.03470	6.43428	70.9579	1.60571	3.45939	7.45304	.241546
4.15	17.2225	2.03715	6.44205	71.4734	1.60700	3.46218	7.45904	.240964
4.16	17.3056	2.03961	6.44981	71.9913	1.60829	3.46496	7.46502	.240385
4.17	17.3889	2.04206	6.45755	72.5117	1.60958	3.46773	7.47100	.239808
4.18	17.4724	2.04450	6.46529	73.0346	1.61086	3.47050	7.47697	.239234
4.19	17.5561	2.04695	6.47302	73.5601	1.61215	3.47327	7.48292	.238663
4.20	17.6400	2.04939	6.48074	74.0880	1.61343	3.47603	7.48887	.238095
4.21	17.7241	2.05183	6.48845	74.6185	1.61471	3.47878	7.49481	.237530
4.22	17.8084	2.05426	6.49615	75.1514	1.61599	3.48154	7.50074	.236967
4.23	17.8929	2.05670	6.50384	75.6870	1.61726	3.48428	7.50666	.236407
4.24	17.9776	2.05913	6.51153	76.2250	1.61853	3.48703	7.51257	.235849
4.25	18.0625	2.06155	6.51920	76.7656	1.61981	3.48977	7.51847	.235294
4.26	18.1476	2.06398	6.52687	77.3088	1.62108	3.49250	7.52437	.234742
4.27	18.2329	2.06640	6.53452	77.8545	1.62234	3.49523	7.53025	.234192
4.28	18.3184	2.06882	6.54217	78.4028	1.62361	3.49796	7.53612	.233645
4.29	18.4041	2.07123	6.54981	78.9536	1.62487	3.50068	7.54199	.233100
4.30	18.4900	2.07364	6.55744	79.5070	1.62613	3.50340	7.54784	.232558
4.31	18.5761	2.07605	6.56506	80.0630	1.62739	3.50611	7.55369	.232019
4.32	18.6624	2.07846	6.57267	80.6216	1.62865	3.50882	7.55953	.231481
4.33	18.7489	2.08087	6.58027	81.1827	1.62991	3.51153	7.56535	.230947
4.34	18.8356	2.08327	6.58787	81.7465	1.63116	3.51423	7.57117	.230415
4.35	18.9225	2.08567	6.59545	82.3129	1.63241	3.51692	7.57698	.229885
4.36	19.0096	2.08806	6.60303	82.8819	1.63366	3.51962	7.58279	.229358
4.37	19.0969	2.09045	6.61060	83.4535	1.63491	3.52231	7.58858	.228833
4.38	19.1844	2.09284	6.61816	84.0277	1.63619	3.52499	7.59436	.228311
4.39	19.2721	2.09523	6.62571	84.6045	1.63740	3.52767	7.60014	.227790
4.40	19.3600	2.09762	6.63325	85.1840	1.63864	3.53035	7.60590	.227273
4.41	19.4481	2.10000	6.64078	85.7661	1.63988	3.53302	7.61166	.226757
4.42	19.5364	2.10238	6.64831	86.3509	1.64112	3.53569	7.61741	.226244
4.43	19.6249	2.10476	6.65582	86.9383	1.64236	3.53835	7.62315	.225734
4.44	19.7136	2.10713	6.66333	87.5284	1.64359	3.54101	7.62888	.225225
4.45	19.8025	2.10950	6.67083	88.1211	1.64483	3.54367	7.63461	.224719
4.46	19.8916	2.11187	6.67832	88.7165	1.64606	3.54632	7.64032	.224215
4.47	19.9809	2.11424	6.68581	89.3146	1.64729	3.54897	7.64603	.223714
4.48	20.0704	2.11660	6.69328	89.9154	1.64851	3.55162	7.65172	.223214
4.49	20.1601	2.11896	6.70075	90.5188	1.64974	3.55426	7.65741	.222717
4.50	20.2500	2.12132	6.70820	91.1250	1.65096	3.55689	7.66309	.222222
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
4.50	20.2500	2.12132	6.70820	91.1250	1.65096	3.55689	7.66309	.222222
4.51	20.3401	2.12368	6.71565	91.7339	1.65219	3.55953	7.66877	.221729
4.52	20.4304	2.12603	6.72309	92.3454	1.65341	3.56215	7.67443	.221239
4.53	20.5209	2.12838	6.73053	92.9597	1.65462	3.56478	7.68009	.220751
4.54	20.6116	2.13073	6.73795	93.5767	1.65584	3.56740	7.68573	.220264
4.55	20.7025	2.13307	6.74537	94.1964	1.65706	3.57002	7.69137	.219780
4.56	20.7936	2.13542	6.75278	94.8188	1.65827	3.57263	7.69700	.219298
4.57	20.8849	2.13776	6.76018	95.4440	1.65948	3.57524	7.70262	.218818
4.58	20.9764	2.14009	6.76757	96.0719	1.66069	3.57785	7.70824	.218341
4.59	21.0681	2.14243	6.77495	96.7026	1.66190	3.58045	7.71384	.217865
4.60	21.1600	2.14476	6.78233	97.3360	1.66310	3.58305	7.71944	.217391
4.61	21.2521	2.14709	6.78970	97.9722	1.66431	3.58564	7.72503	.216920
4.62	21.3444	2.14942	6.79706	98.6111	1.66551	3.58823	7.73061	.216450
4.63	21.4369	2.15174	6.80441	99.2528	1.66671	3.59082	7.73619	.215983
4.64	21.5296	2.15407	6.81175	99.8973	1.66791	3.59340	7.74175	.215517
4.65	21.6225	2.15639	6.81909	100.545	1.66911	3.59598	7.74731	.215054
4.66	21.7156	2.15870	6.82642	101.195	1.67030	3.59856	7.75286	.214592
4.67	21.8089	2.16102	6.83374	101.848	1.67150	3.60113	7.75840	.214133
4.68	21.9024	2.16333	6.84105	102.503	1.67269	3.60370	7.76394	.213675
4.69	21.9961	2.16564	6.84836	103.162	1.67388	3.60626	7.76946	.213220
4.70	22.0900	2.16795	6.85565	103.823	1.67507	3.60883	7.77498	.212766
4.71	22.1841	2.17025	6.86294	104.487	1.67626	3.61138	7.78049	.212314
4.72	22.2784	2.17256	6.87023	105.154	1.67744	3.61394	7.78599	.211864
4.73	22.3729	2.17486	6.87750	105.824	1.67863	3.61649	7.79149	.211416
4.74	22.4676	2.17715	6.88477	106.496	1.67981	3.61903	7.79697	.210970
4.75	22.5625	2.17945	6.89202	107.172	1.68099	3.62158	7.80245	.210526
4.76	22.6576	2.18174	6.89928	107.850	1.68217	3.62412	7.80793	.210084
4.77	22.7529	2.18403	6.90652	108.531	1.68334	3.62665	7.81339	.209644
4.78	22.8484	2.18632	6.91375	109.215	1.68452	3.62919	7.81885	.209205
4.79	22.9441	2.18861	6.92098	109.902	1.68569	3.63172	7.82429	.208768
4.80	23.0400	2.19089	6.92820	110.592	1.68687	3.63424	7.82974	.208333
4.81	23.1361	2.19317	6.93542	111.285	1.68804	3.63676	7.83517	.207900
4.82	23.2324	2.19545	6.94262	111.980	1.68920	3.63928	7.84059	.207469
4.83	23.3289	2.19773	6.94982	112.679	1.69037	3.64180	7.84601	.207039
4.84	23.4256	2.20000	6.95701	113.380	1.69154	3.64431	7.85142	.206612
4.85	23.5225	2.20227	6.96419	114.084	1.69270	3.64682	7.85683	.206186
4.86	23.6196	2.20454	6.97137	114.791	1.69386	3.64932	7.86222	.205761
4.87	23.7169	2.20681	6.97854	115.501	1.69503	3.65182	7.86761	.205339
4.88	23.8144	2.20907	6.98570	116.214	1.69619	3.65432	7.87299	.204918
4.89	23.9121	2.21133	6.99285	116.930	1.69734	3.65681	7.87837	.204499
4.90	24.0100	2.21359	7.00000	117.649	1.69850	3.65931	7.88374	.204082
4.91	24.1081	2.21585	7.00714	118.371	1.69965	3.66179	7.88909	.203666
4.92	24.2064	2.21811	7.01427	119.095	1.70081	3.66428	7.89445	.203252
4.93	24.3049	2.22036	7.02140	119.823	1.70196	3.66676	7.89979	.202840
4.94	24.4036	2.22261	7.02851	120.554	1.70311	3.66924	7.90513	.202429
4.95	24.5025	2.22486	7.03562	121.287	1.70426	3.67171	7.91046	.202020
4.96	24.6016	2.22711	7.04273	122.024	1.70540	3.67418	7.91578	.201613
4.97	24.7009	2.22935	7.04982	122.763	1.70655	3.67665	7.92110	.201207
4.98	24.8004	2.23159	7.05691	123.506	1.70769	3.67911	7.92641	.200803
4.99	24.9001	2.23383	7.06399	124.251	1.70884	3.68157	7.93171	.200401
5.00	25.0000	2.23607	7.07107	125.000	1.70998	3.68403	7.93701	.200000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
5.00	25.0000	2.23607	7.07107	125.000	1.70998	3.68403	7.93701	.200000
5.01	25.1001	2.23830	7.07814	125.752	1.71112	3.68649	7.94229	.199601
5.02	25.2004	2.24054	7.08520	126.506	1.71225	3.68894	7.94757	.199203
5.03	25.3009	2.24277	7.09225	127.264	1.71339	3.69138	7.95285	.198807
5.04	25.4016	2.24499	7.09930	128.024	1.71452	3.69383	7.95811	.198413
5.05	25.5025	2.24722	7.10634	128.788	1.71566	3.69627	7.96337	.198020
5.06	25.6036	2.24944	7.11337	129.554	1.71679	3.69871	7.96863	.197628
5.07	25.7049	2.25167	7.12039	130.324	1.71792	3.70114	7.97387	.197239
5.08	25.8064	2.25389	7.12741	131.097	1.71905	3.70357	7.97911	.196850
5.09	25.9081	2.25610	7.13442	131.872	1.72017	3.70600	7.98434	.196464
5.10	26.0100	2.25832	7.14143	132.651	1.72130	3.70843	7.98957	.196078
5.11	26.1121	2.26053	7.14843	133.433	1.72242	3.71085	7.99479	.195695
5.12	26.2144	2.26274	7.15542	134.218	1.72355	3.71327	8.00000	.195312
5.13	26.3169	2.26495	7.16240	135.006	1.72467	3.71569	8.00520	.194932
5.14	26.4196	2.26716	7.16938	135.797	1.72579	3.71810	8.01040	.194553
5.15	26.5225	2.26936	7.17635	136.591	1.72691	3.72051	8.01559	.194175
5.16	26.6256	2.27156	7.18331	137.388	1.72802	3.72292	8.02078	.193798
5.17	26.7289	2.27376	7.19027	138.188	1.72914	3.72532	8.02596	.193424
5.18	26.8324	2.27596	7.19722	138.992	1.73025	3.72772	8.03113	.193050
5.19	26.9361	2.27816	7.20417	139.798	1.73137	3.73012	8.03629	.192678
5.20	27.0400	2.28035	7.21110	140.608	1.73248	3.73251	8.04145	.192308
5.21	27.1441	2.28254	7.21803	141.421	1.73359	3.73490	8.04660	.191939
5.22	27.2484	2.28473	7.22496	142.237	1.73470	3.73729	8.05175	.191571
5.23	27.3529	2.28692	7.23187	143.056	1.73580	3.73968	8.05689	.191205
5.24	27.4576	2.28910	7.23878	143.878	1.73691	3.74206	8.06202	.190840
5.25	27.5625	2.29129	7.24569	144.703	1.73801	3.74443	8.06714	.190476
5.26	27.6676	2.29347	7.25259	145.532	1.73912	3.74681	8.07226	.190114
5.27	27.7729	2.29565	7.25948	146.363	1.74022	3.74918	8.07737	.189753
5.28	27.8784	2.29783	7.26636	147.198	1.74132	3.75155	8.08248	.189394
5.29	27.9841	2.30000	7.27324	148.036	1.74242	3.75392	8.08758	.189036
5.30	28.0900	2.30217	7.28011	148.877	1.74351	3.75629	8.09267	.188679
5.31	28.1961	2.30434	7.28697	149.721	1.74461	3.75865	8.09776	.188324
5.32	28.3024	2.30651	7.29383	150.569	1.74570	3.76101	8.10284	.187970
5.33	28.4089	2.30868	7.30068	151.419	1.74680	3.76336	8.10791	.187617
5.34	28.5156	2.31084	7.30753	152.273	1.74789	3.76571	8.11298	.187266
5.35	28.6225	2.31301	7.31437	153.130	1.74898	3.76806	8.11804	.186916
5.36	28.7296	2.31517	7.32120	153.991	1.75007	3.77041	8.12310	.186567
5.37	28.8369	2.31733	7.32803	154.854	1.75116	3.77275	8.12814	.186220
5.38	28.9444	2.31948	7.33485	155.721	1.75224	3.77509	8.13319	.185874
5.39	29.0521	2.32164	7.34166	156.591	1.75333	3.77743	8.13822	.185529
5.40	29.1600	2.32379	7.34847	157.464	1.75441	3.77976	8.14325	.185185
5.41	29.2681	2.32594	7.35527	158.340	1.75549	3.78209	8.14828	.184843
5.42	29.3764	2.32809	7.36206	159.220	1.75657	3.78442	8.15329	.184502
5.43	29.4849	2.33024	7.36885	160.103	1.75765	3.78675	8.15831	.184162
5.44	29.5936	2.33238	7.37564	160.989	1.75873	3.78907	8.16331	.183824
5.45	29.7025	2.33452	7.38241	161.879	1.75981	3.79139	8.16831	.183486
5.46	29.8116	2.33666	7.38918	162.771	1.76088	3.79371	8.17330	.183150
5.47	29.9209	2.33880	7.39594	163.667	1.76196	3.79603	8.17829	.182815
5.48	30.0304	2.34094	7.40270	164.567	1.76303	3.79834	8.18327	.182482
5.49	30.1401	2.34307	7.40945	165.469	1.76410	3.80065	8.18824	.182149
5.50	30.2500	2.34521	7.41620	166.375	1.76517	3.80295	8.19321	.181818
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

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n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
5.50	30.2500	2.34521	7.41620	166.375	1.76517	3.80295	8.19321	.181818
5.51	30.3601	2.34734	7.42294	167.284	1.76624	3.80526	8.19818	.181488
5.52	30.4704	2.34947	7.42967	168.197	1.76731	3.80756	8.20313	.181159
5.53	30.5809	2.35160	7.43640	169.112	1.76838	3.80985	8.20808	.180832
5.54	30.6916	2.35372	7.44312	170.031	1.76944	3.81215	8.21303	.180505
5.55	30.8025	2.35584	7.44983	170.954	1.77051	3.81444	8.21797	.180180
5.56	30.9136	2.35797	7.45654	171.880	1.77157	3.81673	8.22290	.179856
5.57	31.0249	2.36008	7.46324	172.809	1.77263	3.81902	8.22783	.179533
5.58	31.1364	2.36220	7.46994	173.741	1.77369	3.82130	8.23275	.179211
5.59	31.2481	2.36432	7.47663	174.677	1.77475	3.82358	8.23766	.178891
5.60	31.3600	2.36643	7.48331	175.616	1.77581	3.82586	8.24257	.178571
5.61	31.4721	2.36854	7.48999	176.558	1.77686	3.82814	8.24747	.178253
5.62	31.5844	2.37065	7.49667	177.504	1.77792	3.83041	8.25237	.177936
5.63	31.6969	2.37276	7.50333	178.454	1.77897	3.83268	8.25726	.177620
5.64	31.8096	2.37487	7.50999	179.406	1.78003	3.83495	8.26215	.177305
5.65	31.9225	2.37697	7.51665	180.362	1.78108	3.83722	8.26703	.176991
5.66	32.0356	2.37908	7.52330	181.321	1.78213	3.83948	8.27190	.176678
5.67	32.1489	2.38118	7.52994	182.284	1.78318	3.84174	8.27677	.176367
5.68	32.2624	2.38328	7.53658	183.250	1.78422	3.84399	8.28164	.176056
5.69	32.3761	2.38537	7.54321	184.220	1.78527	3.84625	8.28649	.175747
5.70	32.4900	2.38747	7.54983	185.193	1.78632	3.84850	8.29134	.175439
5.71	32.6041	2.38956	7.55645	186.169	1.78736	3.85075	8.29619	.175131
5.72	32.7184	2.39165	7.56307	187.149	1.78840	3.85300	8.30103	.174825
5.73	32.8329	2.39374	7.56968	188.133	1.78944	3.85524	8.30587	.174520
5.74	32.9476	2.39583	7.57628	189.119	1.79048	3.85748	8.31069	.174216
5.75	33.0625	2.39792	7.58288	190.109	1.79152	3.85972	8.31552	.173913
5.76	33.1776	2.40000	7.58947	191.103	1.79256	3.86196	8.32034	.173611
5.77	33.2929	2.40208	7.59605	192.100	1.79360	3.86419	8.32515	.173310
5.78	33.4084	2.40416	7.60263	193.101	1.79463	3.86642	8.32995	.173010
5.79	33.5241	2.40624	7.60920	194.105	1.79567	3.86865	8.33476	.172712
5.80	33.6400	2.40832	7.61577	195.112	1.79670	3.87088	8.33955	.172414
5.81	33.7561	2.41039	7.62234	196.123	1.79773	3.87310	8.34434	.172117
5.82	33.8724	2.41247	7.62889	197.137	1.79876	3.87532	8.34913	.171821
5.83	33.9889	2.41454	7.63544	198.155	1.79979	3.87754	8.35390	.171527
5.84	34.1056	2.41661	7.64199	199.177	1.80082	3.87975	8.35868	.171233
5.85	34.2225	2.41868	7.64853	200.202	1.80185	3.88197	8.36345	.170940
5.86	34.3396	2.42074	7.65506	201.230	1.80288	3.88418	8.36821	.170649
5.87	34.4569	2.42281	7.66159	202.262	1.80390	3.88639	8.37297	.170358
5.88	34.5744	2.42487	7.66812	203.297	1.80492	3.88859	8.37772	.170068
5.89	34.6921	2.42693	7.67463	204.336	1.80595	3.89080	8.38247	.169779
5.90	34.8100	2.42899	7.68115	205.379	1.80697	3.89300	8.38721	.169492
5.91	34.9281	2.43105	7.68765	206.425	1.80799	3.89519	8.39194	.169205
5.92	35.0464	2.43311	7.69415	207.475	1.80901	3.89739	8.39667	.168919
5.93	35.1649	2.43516	7.70065	208.528	1.81003	3.89958	8.40140	.168634
5.94	35.2836	2.43721	7.70714	209.585	1.81104	3.90177	8.40612	.168350
5.95	35.4025	2.43926	7.71362	210.645	1.81206	3.90396	8 41083	.168067
5.96	35.5216	2.44131	7.72010	211.709	1.81307	3.90615	8.41554	.167785
5.97	35.6409	2.44336	7.72658	212.776	1.81409	3.90833	8.42025	.167504
5.98	35.7604	2.44540	7.73305	213.847	1.81510	3.91051	8.42494	.167224
5.99	35.8801	2.44745	7.73951	214.922	1.81611	3.91269	8.42964	.166945
6.00	36.0000	2.44949	7.74597	216.000	1.81712	3.91487	8.43433	.166667
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
6.00	36.0000	2.44949	7.74597	216.000	1.81712	3.91487	8.43433	.166667
6.01	36.1201	2.45153	7.75242	217.082	1.81813	3.91704	8.43901	.166389
6.02	36.2404	2.45357	7.75887	218.167	1.81914	3.91921	8.44369	.166113
6.03	36.3609	2.45561	7.76531	219.256	1.82014	3.92138	8.44836	.165837
6.04	36.4816	2.45764	7.77174	220.349	1.82115	3.92355	8.45303	.165563
6.05	36.6025	2.45967	7.77817	221.445	1.82215	3.92571	8.45769	.165289
6.06	36.7236	2.46171	7.78460	222.545	1.82316	3.92787	8.46235	.165017
6.07	36.8449	2.46374	7.79102	223.649	1.82416	3.93003	8.46700	.164745
6.08	36.9664	2.46577	7.79744	224.756	1.82516	3.93219	8.47165	.164474
6.09	37.0881	2.46779	7.80385	225.867	1.82616	3.93434	8.47629	.164204
6.10	37.2100	2.46982	7.81025	226.981	1.82716	3.93650	8.48093	.163934
6.11	37.3321	2.47184	7.81665	228.099	1.82816	3.93865	8.48556	.163666
6.12	37.4544	2.47386	7.82304	229.221	1.82915	3.94079	8.49018	.163399
6.13	37.5769	2.47588	7.82943	230.346	1.83015	3.94294	8.49481	.163132
6.14	37.6996	2.47790	7.83582	231.476	1.83115	3.94508	8.49942	.162866
6.15	37.8225	2.47992	7.84219	232.608	1.83214	3.94722	8.50403	.162602
6.16	37.9456	2.48193	7.84857	233.745	1.83313	3.94936	8.50864	.162338
6.17	38.0689	2.48395	7.85493	234.885	1.83412	3.95150	8.51324	.162075
6.18	38.1924	2.48596	7.86130	236.029	1.83511	3.95363	8.51784	.161812
6.19	38.3161	2.48797	7.86766	237.177	1.83610	3.95576	8.52243	.161551
6.20	38.4400	2.48998	7.87401	238.328	1.83709	3.95789	8.52702	.161290
6.21	38.5641	2.49199	7.88036	239.483	1.83808	3.96002	8.53160	.161031
6.22	38.6884	2.49399	7.88670	240.642	1.83906	3.96214	8.53618	.160772
6.23	38.8129	2.49600	7.89303	241.804	1.84005	3.96427	8.54075	.160514
6.24	38.9376	2.49800	7.89937	242.971	1.84103	3.96638	8.54532	.160256
6.25	39.0625	2.50000	7.90569	244.141	1.84202	3.96850	8.54988	.160000
6.26	39.1876	2.50200	7.91202	245.314	1.84300	3.97062	8.55444	.159744
6.27	39.3129	2.50400	7.91833	246.492	1.84398	3.97273	8.55899	.159490
6.28	39.4384	2.50599	7.92465	247.673	1.84496	3.97484	8.56354	.159236
6.29	39.5641	2.50799	7.93095	248.858	1.84594	3.97695	8.56808	.158983
6.30	39.6900	2.50998	7.93725	250.047	1.84691	3.97906	8.57262	.158730
6.31	39.8161	2.51197	7.94355	251.240	1.84789	3.98116	8.57715	.158479
6.32	39.9424	2.51396	7.94984	252.436	1.84887	3.98326	8.58168	.158228
6.33	40.0689	2.51595	7.95613	253.636	1.84984	3.98536	8.58620	.157978
6.34	40.1956	2.51794	7.96241	254.840	1.85082	3.98746	8.59072	.157729
6.35	40.3225	2.51992	7.96869	256.048	1.85179	3.98956	8.59524	.157480
6.36	40.4496	2.52190	7.97496	257.259	1.85276	3.99165	8.59975	.157233
6.37	40.5769	2.52389	7.98123	258.475	1.85373	3.99374	8.60425	.156986
6.38	40.7044	2.52587	7.98749	259.694	1.85470	3.99583	8.60875	.156740
6.39	40.8321	2.52784	7.99375	260.917	1.85567	3.99792	8.61325	.156495
6.40	40.9600	2.52982	8.00000	262.144	1.85664	4.00000	8.61774	.156250
6.41	41.0881	2.53180	8.00625	263.375	1.85760	4.00208	8.62222	.156006
6.42	41.2164	2.53377	8.01249	264.609	1.85857	4.00416	8.62671	.155763
6.43	41.3449	2.53574	8.01873	265.848	1.85953	4.00624	8.63118	.155521
6.44	41.4736	2.53772	8.02496	267.090	1.86050	4.00832	8.63566	.155280
6.45	41.6025	2.53969	8.03119	268.336	1.86146	4.01039	8.64012	.155039
6.46	41.7316	2.54165	8.03741	269.586	1.86242	4.01246	8.64459	.154799
6.47	41.8609	2.54362	8.04363	270.840	1.86338	4.01453	8.64904	.154560
6.48	41.9904	2.54558	8.04984	272.098	1.86434	4.01660	8.65350	.154321
6.49	42.1201	2.54755	8.05605	273.359	1.86530	4.01866	8.65795	.154083
6.50	42.2500	2.54951	8.06226	274.625	1.86626	4.02073	8.66239	.153846
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
6.50	42.2500	2.54951	8.06226	274.625	1.86626	4.02073	8.66239	.153846
6.51	42.3801	2.55147	8.06846	275.894	1.86721	4.02279	8.66683	.153610
6.52	42.5104	2.55343	8.07465	277.168	1.86817	4.02485	8.67127	.153374
6.53	42.6409	2.55539	8.08084	278.445	1.86912	4.02690	8.67570	.153139
6.54	42.7716	2.55734	8.08703	279.726	1.87008	4.02896	8.68012	.152905
6.55	42.9025	2.55930	8.09321	281.011	1.87103	4.03101	8.68455	.152672
6.56	43.0336	2.56125	8.09938	282.300	1.87198	4.03306	8.68896	.152439
6.57	43.1649	2.56320	8.10555	283.593	1.87293	4.03511	8.69338	.152207
6.58	43.2964	2.56515	8.11172	284.890	1.87388	4.03715	8.69778	.151976
6.59	43.4281	2.56710	8.11788	286.191	1.87483	4.03920	8.70219	.151745
6.60	43.5600	2.56905	8.12404	287.496	1.87578	4.04124	8.70659	.151515
6.61	43.6921	2.57099	8.13019	288.805	1.87672	4.04328	8.71098	.151286
6.62	43.8244	2.57294	8.13634	290.118	1.87767	4.04532	8.71537	.151057
6.63	43.9569	2.57488	8.14248	291.434	1.87862	4.04735	8.71976	.150830
6.64	44.0896	2.57682	8.14862	292.755	1.87956	4.04939	8.72414	.150602
6.65	44.2225	2.57876	8.15475	294.080	1.88050	4.05142	8.72852	.150376
6.66	44.3556	2.58070	8.16088	295.408	1.88144	4.05345	8.73289	.150150
6.67	44.4889	2.58263	8.16701	296.741	1.88239	4.05548	8.73726	.149925
6.68	44.6224	2.58457	8.17313	298.078	1.88333	4.05750	8.74162	.149701
6.69	44.7561	2.58650	8.17924	299.418	1.88427	4.05953	8.74598	.149477
6.70	44.8900	2.58844	8.18535	300.763	1.88520	4.06155	8.75034	.149254
6.71	45.0241	2.59037	8.19146	302.112	1.88614	4.06357	8.75469	.149031
6.72	45.1584	2.59230	8.19756	303.464	1.88708	4.06559	8.75904	.148810
6.73	45.2929	2.59422	8.20366	304.821	1.88801	4.06760	8.76338	.148588
6.74	45.4276	2.59615	8.20975	306.182	1.88895	4.06961	8.76772	.148368
6.75	45.5625	2.59808	8.21584	307.547	1.88988	4.07163	8.77205	.148148
6.76	45.6976	2.60000	8.22192	308.916	1.89081	4.07364	8.77638	.147929
6.77	45.8329	2.60192	8.22800	310.289	1.89175	4.07564	8.78071	.147710
6.78	45.9684	2.60384	8.23408	311.666	1.89268	4.07765	8.78503	.147493
6.79	46.1041	2.60576	8.24015	313.047	1.89361	4.07965	8.78935	.147275
6.80	46.2400	2.60768	8.24621	314.432	1.89454	4.08166	8.79366	.147059
6.81	46.3761	2.60960	8.25227	315.821	1.89546	4.08365	8.79797	.146843
6.82	46.5124	2.61151	8.25833	317.215	1.89639	4.08565	8.80227	.146628
6.83	46.6489	2.61343	8.26438	318.612	1.89732	4.08765	8.80657	.146413
6.84	46.7856	2.61534	8.27043	320.014	1.89824	4.08964	8.81087	.146199
6.85	46.9225	2.61725	8.27647	321.419	1.89917	4.09163	8.81516	.145985
6.86	47.0596	2.61916	8.28251	322.829	1.90009	4.09362	8.81945	.145773
6.87	47.1969	2.62107	8.28855	324.243	1.90102	4.09561	8.82373	.145560
6.88	47.3344	2.62298	8.29458	325.661	1.90194	4.09760	8.82801	.145349
6.89	47.4721	2.62488	8.30060	327.083	1.90286	4.09958	8.83228	.145138
6.90	47.6100	2.62679	8.30662	328.509	1.90378	4.10157	8.83656	.144928
6.91	47.7481	2.62869	8.31264	329.939	1.90470	4.10355	8.84082	.144718
6.92	47.8864	2.63059	8.31865	331.374	1.90562	4.10552	8.84509	.144509
6.93	48.0249	2.63249	8.32466	332.813	1.90653	4.10750	8.84934	.144300
6.94	48.1636	2.63439	8.33067	334.255	1.90745	4.10948	8.85360	.144092
6.95	48.3025	2.63629	8.33667	335.702	1.90837	4.11145	8.85785	.143885
6.96	48.4416	2.63818	8.34266	337.154	1.90928	4.11342	8.86210	.143678
6.97	48.5809	2.64008	8.34865	338.609	1.91019	4.11539	8.86634	.143472
6.98	48.7204	2.64197	8.35464	340.068	1.91111	4.11736	8.87058	.143266
6.99	48.8601	2.64386	8.36062	341.532	1.91202	4.11932	8.87481	.143062
7.00	49.0000	2.64575	8.36660	343.000	1.91293	4.12129	8.87904	.142857
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
7.00	49.0000	2.64575	8.36660	343.000	1.91293	4.12129	8.87904	.142857
7.01	49.1401	2.64764	8.37257	344.472	1.91384	4.12325	8.88327	.142653
7.02	49.2804	2.64953	8.37854	345.948	1.91475	4.12521	8.88749	.142450
7.03	49.4209	2.65141	8.38451	347.429	1.91566	4.12716	8.89171	.142248
7.04	49.5616	2.65330	8.39047	348.914	1.91657	4.12912	8.89592	.142045
7.05	49.7025	2.65518	8.39643	350.403	1.91747	4.13107	8.90013	.141844
7.06	49.8436	2.65707	8.40238	351.896	1.91838	4.13303	8.90434	.141643
7.07	49.9849	2.65895	8.40833	353.393	1.91929	4.13498	8.90854	.141443
7.08	50.1264	2.66083	8.41427	354.895	1.92019	4.13693	8.91274	.141243
7.09	50.2681	2.66271	8.42021	356.401	1.92109	4.13887	8.91693	.141044
7.10	50.4100	2.66458	8.42615	357.911	1.92200	4.14082	8.92112	.140845
7.11	50.5521	2.66646	8.43208	359.425	1.92290	4.14276	8.92531	.140647
7.12	50.6944	2.66833	8.43801	360.944	1.92380	4.14470	8.92949	.140449
7.13	50.8369	2.67021	8.44393	362.467	1.92470	4.14664	8.93367	.140252
7.14	50.9796	2.67208	8.44985	363.994	1.92560	4.14858	8.93784	.140056
7.15	51.1225	2.67395	8.45577	365.526	1.92650	4.15052	8.94201	.139860
7.16	51.2656	2.67582	8.46168	367.062	1.92740	4.15245	8.94618	.139665
7.17	51.4089	2.67769	8.46759	368.602	1.92829	4.15438	8.95034	.139470
7.18	51.5524	2.67955	8.47349	370.146	1.92919	4.15631	8.95450	.139276
7.19	51.6961	2.68142	8.47939	371.695	1.93008	4.15824	8.95866	.139082
7.20	51.8400	2.68328	8.48528	373.248	1.93098	4.16017	8.96281	.138889
7.21	51.9841	2.68514	8.49117	374.805	1.93187	4.16209	8.96696	.138696
7.22	52.1284	2.68701	8.49706	376.367	1.93277	4.16402	8.97110	.138504
7.23	52.2729	2.68887	8.50294	377.933	1.93366	4.16594	8.97524	.138313
7.24	52.4176	2.69072	8.50882	379.503	1.93455	4.16786	8.97938	.138122
7.25	52.5625	2.69258	8.51469	381.078	1.93544	4.16978	8.98351	.137931
7.26	52.7076	2.69444	8.52056	382.657	1.93633	4.17169	8.98764	.137741
7.27	52.8529	2.69629	8.52643	384.241	1.93722	4.17361	8.99176	.137552
7.28	52.9984	2.69815	8.53229	385.828	1.93810	4.17552	8.99588	.137363
7.29	53.1441	2.70000	8.53815	387.420	1.93899	4.17743	9.00000	.137174
7.30	53.2900	2.70185	8.54400	389.017	1.93988	4.17934	9.00411	.136986
7.31	53.4361	2.70370	8.54985	390.618	1.94076	4.18125	9.00822	.136799
7.32	53.5824	2.70555	8.55570	392.223	1.94165	4.18315	9.01233	.136612
7.33	53.7289	2.70740	8.56154	393.833	1.94253	4.18506	9.01643	.136426
7.34	53.8756	2.70924	8.56738	395.447	1.94341	4.18696	9.02053	.136240
7.35	54.0225	2.71109	8.57321	397.065	1.94430	4.18886	9.02462	.136054
7.36	54.1696	2.71293	8.57904	398.688	1.94518	4.19076	9.02871	.135870
7.37	54.3169	2.71477	8.58487	400.316	1.94606	4.19266	9.03280	.135685
7.38	54.4644	2.71662	8.59069	401.947	1.94694	4.19455	9.03689	.135501
7.39	54.6121	2.71846	8.59651	403.583	1.94782	4.19644	9.04097	.135318
7.40	54.7600	2.72029	8.60233	405.224	1.94870	4.19834	9.04504	.135135
7.41	54.9081	2.72213	8.60814	406.869	1.94957	4.20023	9.04911	.134953
7.42	55.0564	2.72397	8.61394	408.518	1.95045	4.20212	9.05318	.134771
7.43	55.2049	2.72580	8.61974	410.172	1.95132	4.20400	9.05725	.134590
7.44	55.3536	2.72764	8.62554	411.831	1.95220	4.20589	9.06131	.134409
7.45	55.5025	2.72947	8.63134	413.494	1.95307	4.20777	9.06537	.134228
7.46	55.6516	2.73130	8.63713	415.161	1.95395	4.20965	9.06942	.134048
7.47	55.8009	2.73313	8.64292	416.833	1.95482	4.21153	9.07347	.133869
7.48	55.9504	2.73496	8.64870	418.509	1.95569	4.21341	9.07752	.133690
7.49	56.1001	2.73679	8.65448	420.190	1.95656	4.21529	9.08156	.133511
7.50	56.2500	2.73861	8.66025	421.875	1.95743	4.21716	9.08560	.133333
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
7.50	56.2500	2.73861	8.66025	421.875	1.95743	4.21716	9.08560	.133333
7.51	56.4001	2.74044	8.66603	423.565	1.95830	4.21904	9.08964	.133156
7.52	56.5504	2.74226	8.67179	425.259	1.95917	4.22091	9.09367	.132979
7.53	56.7009	2.74408	8.67756	426.958	1.96004	4.22278	9.09770	.132802
7.54	56.8516	2.74591	8.68332	428.661	1.96091	4.22465	9.10173	.132626
7.55	57.0025	2.74773	8.68907	430.369	1.96177	4.22651	9.10575	.132450
7.56	57.1536	2.74955	8.69483	432.081	1.96264	4.22838	9.10977	.132275
7.57	57.3049	2.75136	8.70057	433.798	1.96350	4.23024	9.11378	.132100
7.58	57.4564	2.75318	8.70632	435.520	1.96437	4.23210	9.11779	.131926
7.59	57.6081	2.75500	8.71206	437.245	1.96523	4.23396	9.12180	.131752
7.60	57.7600	2.75681	8.71780	438.976	1.96610	4.23582	9.12581	.131579
7.61	57.9121	2.75862	8.72353	440.711	1.96696	4.23768	9.12981	.131406
7.62	58.0644	2.76043	8.72926	442.451	1.96782	4.23954	9.13380	.131234
7.63	58.2169	2.76225	8.73499	444.195	1.96868	4.24139	9.13780	.131062
7.64	58.3696	2.76405	8.74071	445.944	1.96954	4.24324	9.14179	.130890
7.65	58.5225	2.76586	8.74643	447.697	1.97040	4.24509	9.14577	.130719
7.66	58.6756	2.76767	8.75214	449.455	1.97126	4.24694	9.14976	.130548
7.67	58.8289	2.76948	8.75785	451.218	1.97211	4.24879	9.15374	.130378
7.68	58.9824	2.77128	8.76356	452.985	1.97297	4.25063	9.15771	.130208
7.69	59.1361	2.77308	8.76926	454.757	1.97383	4.25248	9.16169	.130039
7.70	59.2900	2.77489	8.77496	456.533	1.97468	4.25432	9.16566	.129870
7.71	59.4441	2.77669	8.78066	458.314	1.97554	4.25616	9.16962	.129702
7.72	59.5984	2.77849	8.78635	460.100	1.97639	4.25800	9.17359	.129534
7.73	59.7529	2.78029	8.79204	461.890	1.97724	4.25984	9.17754	.129366
7.74	59.9076	2.78209	8.79773	463.685	1.97809	4.26167	9.18150	.129199
7.75	60.0625	2.78388	8.80341	465.484	1.97895	4.26351	9.18545	.129032
7.76	60.2176	2.78568	8.80909	467.289	1.97980	4.26534	9.18940	.128866
7.77	60.3729	2.78747	8.81476	469.097	1.98065	4.26717	9.19335	.128700
7.78	60.5284	2.78927	8.82043	470.911	1.98150	4.26900	9.19729	.128535
7.79	60.6841	2.79106	8.82610	472.729	1.98234	4.27083	9.20123	.128370
7.80	60.8400	2.79285	8.83176	474.552	1.98319	4.27266	9.20516	.128205
7.81	60.9961	2.79464	8.83742	476.380	1.98404	4.27448	9.20910	.128041
7.82	61.1524	2.79643	8.84308	478.212	1.98489	4.27631	9.21302	.127877
7.83	61.3089	2.79821	8.84873	480.049	1.98573	4.27813	9.21695	.127714
7.84	61.4656	2.80000	8.85438	481.890	1.98658	4.27995	9.22087	.127551
7.85	61.6225	2.80179	8.86002	483.737	1.98742	4.28177	9.22479	.127389
7.86	61.7796	2.80357	8.86566	485.588	1.98826	4.28359	9.22871	.127226
7.87	61.9369	2.80535	8.87130	487.443	1.98911	4.28540	9.23262	.127065
7.88	62.0944	2.80713	8.87694	489.304	1.98995	4.28722	9.23653	.126904
7.89	62.2521	2.80891	8.88257	491.169	1.99079	4.28903	9.24043	.126743
7.90	62.4100	2.81069	8.88819	493.039	1.99163	4.29084	9.24434	.126582
7.91	62.5681	2.81247	8.89382	494.914	1.99247	4.29265	9.24823	.126422
7.92	62.7264	2.81425	8.89944	496.793	1.99331	4.29446	9.25213	.126263
7.93	62.8849	2.81603	8.90505	498.677	1.99415	4.29627	9.25602	.126103
7.94	63.0436	2.81780	8.91067	500.566	1.99499	4.29807	9.25991	.125945
7.95	63.2025	2.81957	8.91628	502.460	1.99582	4.29987	9.26380	.125786
7.96	63.3616	2.82135	8.92188	504.358	1.99666	4.30168	9.26768	.125628
7.97	63.5209	2.82312	8.92749	506.262	1.99750	4.30348	9.27156	.125471
7.98	63.6804	2.82489	8.93308	508.170	1.99833	4.30528	9.27544	.125313
7.99	63.8401	2.82666	8.93868	510.082	1.99917	4.30707	9.27931	.125156
8.00	64.0000	2.82843	8.94427	512.000	2.00000	4.30887	9.28318	.125000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
8.00	64.0000	2.82843	8.94427	512.000	2.00000	4.30887	9.28318	.125000
8.01	64.1601	2.83019	8.94986	513.922	2.00083	4.31066	9.28704	.124844
8.02	64.3204	2.83196	8.95545	515.850	2.00167	4.31246	9.29091	.124688
8.03	64.4809	2.83373	8.96103	517.782	2.00250	4.31425	9.29477	.124533
8.04	64.6416	2.83549	8.96660	519.718	2.00333	4.31604	9.29862	.124378
8.05	64.8025	2.83725	8.97218	521.660	2.00416	4.31783	9.30248	.124224
8.06	64.9636	2.83901	8.97775	523.607	2.00499	4.31961	9.30633	.124069
8.07	65.1249	2.84077	8.98332	525.558	2.00582	4.32140	9.31018	.123916
8.08	65.2864	2.84253	8.98888	527.514	2.00664	4.32318	9.31402	.123762
8.09	65.4481	2.84429	8.99444	529.475	2.00747	4.32497	9.31786	.123609
8.10	65.6100	2.84605	9.00000	531.441	2.00830	4.32675	9.32170	.123457
8.11	65.7721	2.84781	9.00555	533.412	2.00912	4.32853	9.32553	.123305
8.12	65.9344	2.84956	9.01110	535.387	2.00995	4.33031	9.32936	.123153
8.13	66.0969	2.85132	9.01665	537.368	2.01078	4.33208	9.33319	.123001
8.14	66.2596	2.85307	9.02219	539.353	2.01160	4.33386	9.33702	.122850
8.15	66.4225	2.85482	9.02774	541.343	2.01242	4.33563	9.34084	.122699
8.16	66.5856	2.85657	9.03327	543.338	2.01325	4.33741	9.34466	.122549
8.17	66.7489	2.85832	9.03881	545.339	2.01407	4.33918	9.34847	.122399
8.18	66.9124	2.86007	9.04434	547.343	2.01489	4.34095	9.35229	.122249
8.19	67.0761	2.86182	9.04986	549.353	2.01571	4.34271	9.35610	.122100
8.20	67.2400	2.86356	9.05539	551.368	2.01653	4.34448	9.35990	.121951
8.21	67.4041	2.86531	9.06091	553.388	2.01735	4.34625	9.36370	.121803
8.22	67.5684	2.86705	9.06642	555.412	2.01817	4.34801	9.36751	.121655
8.23	67.7329	2.86880	9.07193	557.442	2.01899	4.34977	9.37130	.121507
8.24	67.8976	2.87054	9.07744	559.476	2.01980	4.35153	9.37510	.121359
8.25	68.0625	2.87228	9.08295	561.516	2.02062	4.35329	9.37889	.121212
8.26	68.2276	2.87402	9.08845	563.560	2.02144	4.35505	9.38268	.121065
8.27	68.3929	2.87576	9.09395	565.609	2.02225	4.35681	9.38646	.120919
8.28	68.5584	2.87750	9.09945	567.664	2.02307	4.35856	9.39024	.120773
8.29	68.7241	2.87924	9.10494	569.723	2.02388	4.36032	9.39402	.120627
8.30	68.8900	2.88097	9.11043	571.787	2.02469	4.36207	9.39780	.120482
8.31	69.0561	2.88271	9.11592	573.856	2.02551	4.36382	9.40157	.120337
8.32	69.2224	2.88444	9.12140	575.930	2.02632	4.36557	9.40534	.120192
8.33	69.3889	2.88617	9.12688	578.010	2.02713	4.36732	9.40911	.120048
8.34	69.5556	2.88791	9.13236	580.094	2.02794	4.36907	9.41287	.119904
8.35	69.7225	2.88964	9.13783	582.183	2.02875	4.37081	9.41663	.119760
8.36	69.8896	2.89137	9.14330	584.277	2.02956	4.37256	9.42039	.119617
8.37	70.0569	2.89310	9.14877	586.376	2.03037	4.37430	9.42414	.119474
8.38	70.2244	2.89482	9.15423	588.480	2.03118	4.37604	9.42789	.119332
8.39	70.3921	2.89655	9.15969	590.590	2.03199	4.37778	9.43164	.119190
8.40	70.5600	2.89828	9.16515	592.704	2.03279	4.37952	9.43539	.119048
8.41	70.7281	2.90000	9.17061	594.823	2.03360	4.38126	9.43913	.118906
8.42	70.8964	2.90172	9.17606	596.948	2.03440	4.38299	9.44287	.118765
8.43	71.0649	2.90345	9.18150	599.077	2.03521	4.38473	9.44661	.118624
8.44	71.2336	2.90517	9.18695	601.212	2.03601	4.38646	9.45034	.118483
8.45	71.4025	2.90689	9.19239	603.351	2.03682	4.38819	9.45407	.118343
8.46	71.5716	2.90861	9.19783	605.496	2.03762	4.38992	9.45780	.118203
8.47	71.7409	2.91033	9.20326	607.645	2.03842	4.39165	9.46152	.118064
8.48	71.9104	2.91204	9.20869	609.800	2.03923	4.39338	9.46525	.117925
8.49	72.0801	2.91376	9.21412	611.960	2.04003	4.39510	9.46897	.117786
8.50	72.2500	2.91548	9.21954	614.125	2.04083	4.39683	9.47268	.117647
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
8.50	72.2500	2.91548	9.21954	614.125	2.04083	4.39683	9.47268	.117647
8.51	72.4201	2.91719	9.22497	616.295	2.04163	4.39855	9.47640	.117509
8.52	72.5904	2.91890	9.23038	618.470	2.04243	4.40028	9.48011	.117371
8.53	72.7609	2.92062	9.23580	620.650	2.04323	4.40200	9.48381	.117233
8.54	72.9316	2.92233	9.24121	622.836	2.04402	4.40372	9.48752	.117096
8.55	73.1025	2.92404	9.24662	625.026	2.04482	4.40543	9.49122	.116959
8.56	73.2736	2.92575	9.25203	627.222	2.04562	4.40715	9.49492	.116822
8.57	73.4449	2.92746	9.25743	629.423	2.04641	4.40887	9.49861	.116686
8.58	73.6164	2.92916	9.26283	631.629	2.04721	4.41058	9.50231	.116550
8.59	73.7881	2.93087	9.26823	633.840	2.04801	4.41229	9.50600	.116414
8.60	73.9600	2.93258	9.27362	636.056	2.04880	4.41400	9.50969	.116279
8.61	74.1321	2.93428	9.27901	638.277	2.04959	4.41571	9.51337	.116144
8.62	74.3044	2.93598	9.28440	640.504	2.05039	4.41742	9.51705	.116009
8.63	74.4769	2.93769	9.28978	642.736	2.05118	4.41913	9.52073	.115875
8.64	74.6496	2.93939	9.29516	644.973	2.05197	4.42084	9.52441	.115741
8.65	74.8225	2.94109	9.30054	647.215	2.05276	4.42254	9.52808	.115607
8.66	74.9956	2.94279	9.30591	649.462	2.05355	4.42425	9.53175	.115473
8.67	75.1689	2.94449	9.31128	651.714	2.05434	4.42595	9.53542	.115340
8.68	75.3424	2.94618	9.31665	653.972	2.05513	4.42765	9.53908	.115207
8.69	75.5161	2.94788	9.32202	656.235	2.05592	4.42935	9.54274	.115075
8.70	75.6900	2.94958	9.32738	658.503	2.05671	4.43105	9.54640	.114943
8.71	75.8641	2.95127	9.33274	660.776	2.05750	4.43274	9.55006	.114811
8.72	76.0384	2.95296	9.33809	663.055	2.05828	4.43444	9.55371	.114679
8.73	76.2129	2.95466	9.34345	665.339	2.05907	4.43613	9.55736	.114548
8.74	76.3876	2.95635	9.34880	667.628	2.05986	4.43783	9.56101	.114416
8.75	76.5625	2.95804	9.35414	669.922	2.06064	4.43952	9.56466	.114286
8.76	76.7376	2.95973	9.35949	672.221	2.06143	4.44121	9.56830	.114155
8.77	76.9129	2.96142	9.36483	674.526	2.06221	4.44290	9.57194	.114025
8.78	77.0884	2.96311	9.37017	676.836	2.06299	4.44459	9.57557	.113895
8.79	77.2641	2.96479	9.37550	679.151	2.06378	4.44627	9.57921	.113766
8.80	77.4400	2.96648	9.38083	681.472	2.06456	4.44796	9.58284	.113636
8.81	77.6161	2.96816	9.38616	683.798	2.06534	4.44964	9.58647	.113507
8.82	77.7924	2.96985	9.39149	686.129	2.06612	4.45133	9.59009	.113379
8.83	77.9689	2.97153	9.39681	688.465	2.06690	4.45301	9.59372	.113250
8.84	78.1456	2.97321	9.40213	690.807	2.06768	4.45469	9.59734	.113122
8.85	78.3225	2.97489	9.40744	693.154	2.06846	4.45637	9.60095	.112994
8.86	78.4996	2.97658	9.41276	695.506	2.06924	4.45805	9.60457	.112867
8.87	78.6769	2.97825	9.41807	697.864	2.07002	4.45972	9.60818	.112740
8.88	78.8544	2.97993	9.42338	700.227	2.07080	4.46140	9.61179	.112613
8.89	79.0321	2.98161	9.42868	702.595	2.07157	4.46307	9.61540	.112486
8.90	79.2100	2.98329	9.43398	704.969	2.07235	4.46475	9.61900	.112360
8.91	79.3881	2.98496	9.43928	707.348	2.07313	4.46642	9.62260	.112233
8.92	79.5664	2.98664	9.44458	709.732	2.07390	4.46809	9.62620	.112108
8.93	79.7449	2.98831	9.44987	712.122	2.07468	4.46976	9.62980	.111982
8.94	79.9236	2.98998	9.45516	714.517	2.07545	4.47142	9.63339	.111857
8.95	80.1025	2.99166	9.46044	716.917	2.07622	4.47309	9.63698	.111732
8.96	80.2816	2.99333	9.46573	719.323	2.07700	4.47476	9.64057	.111607
8.97	80.4609	2.99500	9.47101	721.734	2.07777	4.47642	9.64415	.111483
8.98	80.6404	2.99666	9.47629	724.151	2.07854	4.47808	9.64774	.111359
8.99	80.8201	2.99833	9.48156	726.573	2.07931	4.47974	9.65132	.111235
9.00	81.0000	3.00000	9.48683	729.000	2.08008	4.48140	9.65489	.111111
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
9.00	81.0000	3.00000	9.48683	729.000	2.08008	4.48140	9.65489	.111111
9.01	81.1801	3.00167	9.49210	731.433	2.08085	4.48306	9.65847	.110988
9.02	81.3604	3.00333	9.49737	733.871	2.08162	4.48472	9.66204	.110865
9.03	81.5409	3.00500	9.50263	736.314	2.08239	4.48638	9.66561	.110742
9.04	81.7216	3.00666	9.50789	738.763	2.08316	4.48803	9.66918	.110619
9.05	81.9025	3.00832	9.51315	741.218	2.08393	4.48969	9.67274	.110497
9.06	82.0836	3.00998	9.51840	743.677	2.08470	4.49134	9.67630	.110375
9.07	82.2649	3.01164	9.52365	746.143	2.08546	4.49299	9.67986	.110254
9.08	82.4464	3.01330	9.52890	748.613	2.08623	4.49464	9.68342	.110132
9.09	82.6281	3.01496	9.53415	751.089	2.08699	4.49629	9.68697	.110011
9.10	82.8100	3.01662	9.53939	753.571	2.08776	4.49794	9.69052	.109890
9.11	82.9921	3.01828	9.54463	756.058	2.08852	4.49959	9.69407	.109769
9.12	83.1744	3.01993	9.54987	758.551	2.08929	4.50123	9.69762	.109649
9.13	83.3569	3.02159	9.55510	761.048	2.09005	4.50288	9.70116	.109529
9.14	83.5396	3.02324	9.56033	763.552	2.09081	4.50452	9.70470	.109409
9.15	83.7225	3.02490	9.56556	766.061	2.09158	4.50616	9.70824	.109290
9.16	83.9056	3.02655	9.57079	768.575	2.09234	4.50781	9.71177	.109170
9.17	84.0889	3.02820	9.57601	771.095	2.09310	4.50945	9.71531	.109051
9.18	84.2724	3.02985	9.58123	773.621	2.09386	4.51108	9.71884	.108932
9.19	84.4561	3.03150	9.58645	776.152	2.09462	4.51272	9.72236	.108814
9.20	84.6400	3.03315	9.59166	778.688	2.09538	4.51436	9.72589	.108696
9.21	84.8241	3.03480	9.59687	781.230	2.09614	4.51599	9.72941	.108578
9.22	85.0084	3.03645	9.60208	783.777	2.09690	4.51763	9.73293	.108460
9.23	85.1929	3.03809	9.60729	786.330	2.09765	4.51926	9.73645	.108342
9.24	85.3776	3.03974	9.61249	788.889	2.09841	4.52089	9.73996	.108225
9.25	85.5625	3.04138	9.61769	791.453	2.09917	4.52252	9.74348	.108108
9.26	85.7476	3.04302	9.62289	794.023	2.09992	4.52415	9.74699	.107991
9.27	85.9329	3.04467	9.62808	796.598	2.10068	4.52578	9.75049	.107875
9.28	86.1184	3.04631	9.63328	799.179	2.10144	4.52740	9.75400	.107759
9.29	86.3041	3.04795	9.63846	801.765	2.10219	4.52903	9.75750	.107643
9.30	86.4900	3.04959	9.64365	804.357	2.10294	4.53065	9.76100	.107527
9.31	86.6761	3.05123	9.64883	806.954	2.10370	4.53228	9.76450	.107411
9.32	86.8624	3.05287	9.65401	809.558	2.10445	4.53390	9.76799	.107296
9.33	87.0489	3.05450	9.65919	812.166	2.10520	4.53552	9.77148	.107181
9.34	87.2356	3.05614	9.66437	814.781	2.10595	4.53714	9.77497	.107066
9.35	87.4225	3.05778	9.66954	817.400	2.10671	4.53876	9.77846	.106952
9.36	87.6096	3.05941	9.67471	820.026	2.10746	4.54038	9.78195	.106838
9.37	87.7969	3.06105	9.67988	822.657	2.10821	4.54199	9.78543	.106724
9.38	87.9844	3.06268	9.68504	825.294	2.10896	4.54361	9.78891	.106610
9.39	88.1721	3.06431	9.69020	827.936	2.10971	4.54522	9.79239	.106496
9.40	88.3600	3.06594	9.69536	830.584	2.11045	4.54684	9.79586	.106383
9.41	88.5481	3.06757	9.70052	833.238	2.11120	4.54845	9.79933	.106270
9.42	88.7364	3.06920	9.70567	835.897	2.11195	4.55006	9.80280	.106157
9.43	88.9249	3.07083	9.71082	838.562	2.11270	4.55167	9.80627	.106045
9.44	89.1136	3.07246	9.71597	841.232	2.11344	4.55328	9.80974	.105932
9.45	89.3025	3.07409	9.72111	843.909	2.11419	4.55488	9.81320	.105820
9.46	89.4916	3.07571	9.72625	846.591	2.11494	4.55649	9.81666	.105708
9.47	89.6809	3.07734	9.73139	849.278	2.11568	4.55809	9.82012	.105597
9.48	89.8704	3.07896	9.73653	851.971	2.11642	4.55970	9.82357	.105485
9.49	90.0601	3.08058	9.74166	854.670	2.11717	4.56130	9.82703	.105374
9.50	90.2500	3.08221	9.74679	857.375	2.11791	4.56290	9.83048	.105263
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
9.50	90.2500	3.08221	9.74679	857.375	2.11791	4.56290	9.83048	.105263
9.51	90.4401	3.08383	9.75192	860.085	2.11865	4.56450	9.83392	.105152
9.52	90.6304	3.08545	9.75705	862.801	2.11940	4.56610	9.83737	.105042
9.53	90.8209	3.08707	9.76217	865.523	2.12014	4.56770	9.84081	.104932
9.54	91.0116	3.08869	9.76729	868.251	2.12088	4.56930	9.84425	.104822
9.55	91.2025	3.09031	9.77241	870.984	2.12162	4.57089	9.84769	.104712
9.56	91.3936	3.09192	9.77753	873.723	2.12236	4.57249	9.85113	.104603
9.57	91.5849	3.09354	9.78264	876.467	2.12310	4.57408	9.85456	.104493
9.58	91.7764	3.09516	9.78775	879.218	2.12384	4.57567	9.85799	.104384
9.59	91.9681	3.09677	9.79285	881.974	2.12458	4.57727	9.86142	.104275
9.60	92.1600	3.09839	9.79796	884.736	2.12532	4.57886	9.86485	.104167
9.61	92.3521	3.10000	9.80306	887.504	2.12605	4.58045	9.86827	.104058
9.62	92.5444	3.10161	9.80816	890.277	2.12679	4.58204	9.87169	.103950
9.63	92.7369	3.10322	9.81326	893.056	2.12753	4.58362	9.87511	.103842
9.64	92.9296	3.10483	9.81835	895.841	2.12826	4.58521	9.87853	.103734
9.65	93.1225	3.10644	9.82344	898.632	2.12900	4.58679	9.88195	.103627
9.66	93.3156	3.10805	9.82853	901.429	2.12974	4.58838	9.88536	.103520
9.67	93.5089	3.10966	9.83362	904.231	2.13047	4.58996	9.88877	.103413
9.68	93.7024	3.11127	9.83870	907.039	2.13120	4.59154	9.89217	.103306
9.69	93.8961	3.11288	9.84378	909.853	2.13194	4.59312	9.89558	.103199
9.70	94.0900	3.11448	9.84886	912.673	2.13267	4.59470	9.89898	.103093
9.71	94.2841	3.11609	9.85393	915.499	2.13340	4.59628	9.90238	.102987
9.72	94.4784	3.11769	9.85901	918.330	2.13414	4.59786	9.90578	.102881
9.73	94.6729	3.11929	9.86408	921.167	2.13487	4.59943	9.90918	.102775
9.74	94.8676	3.12090	9.86914	924.010	2.13560	4.60101	9.91257	.102669
9.75	95.0625	3.12250	9.87421	926.859	2.13633	4.60258	9.91596	.102564
9.76	95.2576	3.12410	9.87927	929.714	2.13706	4.60416	9.91935	.102459
9.77	95.4529	3.12570	9.88433	932.575	2.13779	4.60573	9.92274	.102354
9.78	95.6484	3.12730	9.88939	935.441	2.13852	4.60730	9.92612	.102249
9.79	95.8441	3.12890	9.89444	938.314	2.13925	4.60887	9.92950	.102145
9.80	96.0400	3.13050	9.89949	941.192	2.13997	4.61044	9.93288	.102041
9.81	96.2361	3.13209	9.90454	944.076	2.14070	4.61200	9.93626	.101937
9.82	96.4324	3.13369	9.90959	946.966	2.14143	4.61357	9.93964	.101833
9.83	96.6289	3.13528	9.91464	949.862	2.14216	4.61514	9.94301	.101729
9.84	96.8256	3.13688	9.91968	952.764	2.14288	4.61670	9.94638	.101626
9.85	97.0225	3.13847	9.92472	955.672	2.14361	4.61826	9.94975	.101523
9.86	97.2196	3.14006	9.92975	958.585	2.14433	4.61983	9.95311	.101420
9.87	97.4169	3.14166	9.93479	961.505	2.14506	4.62139	9.95648	.101317
9.88	97.6144	3.14325	9.93982	964.430	2.14578	4.62295	9.95984	.101215
9.89	97.8121	3.14484	9.94485	967.362	2.14651	4.62451	9.96320	.101112
9.90	98.0100	3.14643	9.94987	970.299	2.14723	4.62607	9.96655	.101010
9.91	98.2081	3.14802	9.95490	973.242	2.14795	4.62762	9.96991	.100908
9.92	98.4064	3.14960	9.95992	976.191	2.14867	4.62918	9.97326	.100806
9.93	98.6049	3.15119	9.96494	979.147	2.14940	4.63073	9.97661	.100705
9.94	98.8036	3.15278	9.96995	982.108	2.15012	4.63229	9.97996	.100604
9.95	99.0025	3.15436	9.97497	985.075	2.15084	4.63384	9.98331	.100503
9.96	99.2016	3.15595	9.97998	988.048	2.15156	4.63539	9.98665	.100402
9.97	99.4009	3.15753	9.98499	991.027	2.15228	4.63694	9.98999	.100301
9.98	99.6004	3.15911	9.98999	994.012	2.15300	4.63849	9.99333	.100200
9.99	99.8001	3.16070	9.99500	997.003	2.15372	4.64004	9.99667	.100100
10.00	100.000	3.16228	10.0000	1000.00	2.15443	4.64159	10.0000	.100000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

N	0	1	2	3	4	5	6	7	8	9
0.0		5.395	6.088	6.493	6.781	7.004	7.187	7.341	7.474	7.592
0.1	Take tabular value — 10 7.697	7.793	7.880	7.960	8.034	8.103	8.167	8.228	8.285	8.339
0.2	8.391	8.439	8.486	8.530	8.573	8.614	8.653	8.691	8.727	8.762
0.3	8.796	8.829	8.861	8.891	8.921	8.950	8.978	9.006	9.032	9.058
0.4	9.084	9.108	9.132	9.156	9.179	9.201	9.223	9.245	9.266	9.287
0.5	9.307	9.327	9.346	9.365	9.384	9.402	9.420	9.438	9.455	9.472
0.6	9.489	9.506	9.522	9.538	9.554	9.569	9.584	9.600	9.614	9.629
0.7	9.643	9.658	9.671	9.685	9.699	9.712	9.726	9.739	9.752	9.764
0.8	9.777	9.789	9.802	9.814	9.826	9.837	9.849	9.861	9.872	9.883
0.9	9.895	9.906	9.917	9.927	9.938	9.949	9.959	9.970	9.980	9.990
1.0	0.00000	0995	1980	2956	3922	4879	5827	6766	7696	8618
1.1	9531	*0436	*1333	*2222	*3103	*3976	*4842	*5700	*6551	*7395
1.2	0.1 8232	9062	9885	*0701	*1511	*2314	*3111	*3902	*4686	*5464
1.3	0.2 6236	7003	7763	8518	9267	*0010	*0748	*1481	*2208	*2930
1.4	0.3 3647	4359	5066	5767	6464	7156	7844	8526	9204	9878
1.5	0.4 0547	1211	1871	2527	3178	3825	4469	5108	5742	6373
1.6	7000	7623	8243	8858	9470	*0078	*0682	*1282	*1879	*2473
1.7	0.5 3063	3649	4232	4812	5389	5962	6531	7098	7661	8222
1.8	8779	9333	9884	*0432	*0977	*1519	*2058	*2594	*3127	*3658
1.9	0.6 4185	4710	5233	5752	6269	6783	7294	7803	8310	8813
2.0	9315	9813	*0310	*0804	*1295	*1784	*2271	*2755	*3237	*3716
2.1	0.7 4194	4669	5142	5612	6081	6547	7011	7473	7932	8390
2.2	8846	9299	9751	*0200	*0648	*1093	*1536	*1978	*2418	*2855
2.3	0.8 3291	3725	4157	4587	5015	5442	5866	6289	6710	7129
2.4	7547	7963	8377	8789	9200	9609	*0016	*0422	*0826	*1228
2.5	0.9 1629	2028	2426	2822	3216	3609	4001	4391	4779	5166
2.6	5551	5935	6317	6698	7078	7456	7833	8208	8582	8954
2.7	9325	9695	*0063	*0430	*0796	*1160	*1523	*1885	*2245	*2604
2 8	1.0 2962	3318	3674	4028	4380	4732	5082	5431	5779	6126
2.9	6471	6815	7158	7500	7841	8181	8519	8856	9192	9527
3.0	9861	*0194	*0526	*0856	*1186	*1514	*1841	*2168	*2493	*2817
3.1	1.1 3140	3462	3783	4103	4422	4740	5057	5373	5688	6002
3.2	6315	6627	6938	7248	7557	7865	8173	8479	8784	9089
3.3	9392	9695	9996	*0297	*0597	*0896	*1194	*1491	*1788	*2083
3.4	1.2 2378	2671	2964	3256	3547	3837	4127	4415	4703	4990
3.5	5276	5562	5846	6130	6413	6695	6976	7257	7536	7815
3.6	8093	8371	8647	8923	9198	9473	9746	*0019	*0291	*0563
3.7	1.3 0833	1103	1372	1641	1909	2176	2442	2708	2972	3237
3.8	3500	3763	4025	4286	4547	4807	5067	5325	5584	5841
3.9	6098	6354	6609	6864	7118	7372	7624	7877	8128	8379
4.0	8629	8879	9128	9377	9624	9872	*0118	*0364	*0610	*0854
4.1	1.4 1099	1342	1585	1828	2070	2311	2552	2792	3031	3270
4.2	3508	3746	3984	4220	4456	4692	4927	5161	5395	5629
4.3	5862	6094	6326	6557	6787	7018	7247	7476	7705	7933
4.4	8160	8387	8614	8840	9065	9290	9515	9739	9962	*0185
4.5	1.5 0408	0630	0851	1072	1293	1513	1732	1951	2170	2388
4.6	2606	2823	3039	3256	3471	3687	3902	4116	4330	4543
4.7	4756	4969	5181	5393	5604	5814	6025	6235	6444	6653
4.8	6862	7070	7277	7485	7691	7898	8104	8309	8515	8719
4.9	8924	9127	9331	9534	9737	9939	*0141	*0342	*0543	*0744
5.0	1.6 0944	1144	1343	1542	1741	1939	2137	2334	2531	2728
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

N	0	1	2	3	4	5	6	7	8	9
5.0	1.6 0944	1144	1343	1542	1741	1939	2137	2334	2531	2728
5.1	2924	3120	3315	3511	3705	3900	4094	4287	4481	4673
5.2	4866	5058	5250	5441	5632	5823	6013	6203	6393	6582
5.3	6771	6959	7147	7335	7523	7710	7896	8083	8269	8455
5.4	8640	8825	9010	9194	9378	9562	9745	9928	*0111	*0293
5.5	**1.7** 0475	0656	0838	1019	1199	1380	1560	1740	1919	2098
5.6	2277	2455	2633	2811	2988	3166	3342	3519	3695	3871
5.7	4047	4222	4397	4572	4746	4920	5094	5267	5440	5613
5.8	5786	5958	6130	6302	6473	6644	6815	6985	7156	7326
5.9	7495	7665	7834	8002	8171	8339	8507	8675	8842	9009
6.0	9176	9342	9509	9675	9840	*0006	*0171	*0336	*0500	*0665
6.1	**1.8** 0829	0993	1156	1319	1482	1645	1808	1970	2132	2294
6.2	2455	2616	2777	2938	3098	3258	3418	3578	3737	3896
6.3	4055	4214	4372	4530	4688	4845	5003	5160	5317	5473
6.4	5630	5786	5942	6097	6253	6408	6563	6718	6872	7026
6.5	7180	7334	7487	7641	7794	7947	8099	8251	8403	8555
6.6	8707	8858	9010	9160	9311	9462	9612	9762	9912	*0061
6.7	**1.9** 0211	0360	0509	0658	0806	0954	1102	1250	1398	1545
6.8	1692	1839	1986	2132	2279	2425	2571	2716	2862	3007
6.9	3152	3297	3442	3586	3730	3874	4018	4162	4305	4448
7.0	4591	4734	4876	5019	5161	5303	5445	5586	5727	5869
7.1	6009	6150	6291	6431	6571	6711	6851	6991	7130	7269
7.2	7408	7547	7685	7824	7962	8100	8238	8376	8513	8650
7.3	8787	8924	9061	9198	9334	9470	9606	9742	9877	*0013
7.4	**2.0** 0148	0283	0418	0553	0687	0821	0956	1089	1223	1357
7.5	1490	1624	1757	1890	2022	2155	2287	2419	2551	2683
7.6	2815	2946	3078	3209	3340	3471	3601	3732	3862	3992
7.7	4122	4252	4381	4511	4640	4769	4898	5027	5156	5284
7.8	5412	5540	5668	5796	5924	6051	6179	6306	6433	6560
7.9	6686	6813	6939	7065	7191	7317	7443	7568	7694	7819
8.0	7944	8069	8194	8318	8443	8567	8691	8815	8939	9063
8.1	9186	9310	9433	9556	9679	9802	9924	*0047	*0169	*0291
8.2	**2.1** 0413	0535	0657	0779	0900	1021	1142	1263	1384	1505
8.3	1626	1746	1866	1986	2106	2226	2346	2465	2585	2704
8.4	2823	2942	3061	3180	3298	3417	3535	3653	3771	3889
8.5	4007	4124	4242	4359	4476	4593	4710	4827	4943	5060
8.6	5176	5292	5409	5524	5640	5756	5871	5987	6102	6217
8.7	6332	6447	6562	6677	6791	6905	7020	7134	7248	7361
8.8	7475	7589	7702	7816	7929	8042	8155	8267	8380	8493
8.9	8605	8717	8830	8942	9054	9165	9277	9389	9500	9611
9.0	9722	9834	9944	*0055	*0166	*0276	*0387	*0497	*0607	*0717
9.1	**2.2** 0827	0937	1047	1157	1266	1375	1485	1594	1703	1812
9.2	1920	2029	2138	2246	2354	2462	2570	2678	2786	2894
9.3	3001	3109	3216	3324	3431	3538	3645	3751	3858	3965
9.4	4071	4177	4284	4390	4496	4601	4707	4813	4918	5024
9.5	5129	5234	5339	5444	5549	5654	5759	5863	5968	6072
9.6	6176	6280	6384	6488	6592	6696	6799	6903	7006	7109
9.7	7213	7316	7419	7521	7624	7727	7829	7932	8034	8136
9.8	8238	8340	8442	8544	8646	8747	8849	8950	9051	9152
9.9	9253	9354	9455	9556	9657	9757	9858	9958	*0058	*0158
10.0	2.3 0259	0358	0458	0558	0658	0757	0857	0956	1055	1154
N	0	1	2	3	4	5	6	7	8	9

10	2.30259	**25**	3.21888	**40**	3.68888	**55**	4.00733	**70**	4.24850	**85**	4.44265
11	2.39790	26	3.25810	41	3.71357	56	4.02535	71	4.26268	86	4.45435
12	2.48491	27	3.29584	42	3.73767	57	4.04305	72	4.27667	87	4.46591
13	2.56495	28	3.33220	43	3.76120	58	4.06044	73	4.29046	88	4.47734
14	2.63906	29	3.36730	44	3.78419	59	4.07754	74	4.30407	89	4.48864
15	2.70805	**30**	3.40120	**45**	3.80666	**60**	4.09434	**75**	4.31749	**90**	4.49981
16	2.77259	31	3.43399	46	3.82864	61	4.11087	76	4.33073	91	4.51086
17	2.83321	32	3.46574	47	3.85015	62	4.12713	77	4.34381	92	4.52179
18	2.89037	33	3.49651	48	3.87120	63	4.14313	78	4.35671	93	4.53260
19	2.94444	34	3.52636	49	3.89182	64	4.15888	79	4.36945	94	4.54329
20	2.99573	**35**	3.55535	**50**	3.91202	**65**	4.17439	**80**	4.38203	**95**	4.55388
21	3.04452	36	3.58352	51	3.93183	66	4.18965	81	4.39445	96	4.56435
22	3.09104	37	3.61092	52	3.95124	67	4.20469	82	4.40672	97	4.57471
23	3.13549	38	3.63759	53	3.97029	68	4.21951	83	4.41884	98	4.58497
24	3.17805	39	3.66356	54	3.98898	69	4.23411	84	4.43082	99	4.59512

NAPIERIAN OR NATURAL LOGARITHMS — 100 TO 409

N	0	1	2	3	4	5	6	7	8	9
10	4.6 0517	1512	2497	3473	4439	5396	6344	7283	8213	9135
11	4.7 0048	0953	1850	2739	3620	4493	5359	6217	7068	7912
12	8749	9579	*0402	*1218	*2028	*2831	*3628	*4419	*5203	*5981
13	4.8 6753	7520	8280	9035	9784	*0527	*1265	*1998	*2725	*3447
14	4.9 4164	4876	5583	6284	6981	7673	8361	9043	9721	*0395
15	5.0 1064	1728	2388	3044	3695	4343	4986	5625	6260	6890
16	7517	8140	8760	9375	9987	*0595	*1199	*1799	*2396	*2990
17	5.1 3580	4166	4749	5329	5906	6479	7048	7615	8178	8739
18	9296	9850	*0401	*0949	*1494	*2036	*2575	*3111	*3644	*4175
19	5.2 4702	5227	5750	6269	6786	7300	7811	8320	8827	9330
20	9832	*0330	*0827	*1321	*1812	*2301	*2788	*3272	*3754	*4233
21	5.3 4711	5186	5659	6129	6598	7064	7528	7990	8450	8907
22	9363	9816	*0268	*0717	*1165	*1610	*2053	*2495	*2935	*3372
23	5.4 3808	4242	4674	5104	5532	5959	6383	6806	7227	7646
24	8064	8480	8894	9306	9717	*0126	*0533	*0939	*1343	*1745
25	5.5 2146	2545	2943	3339	3733	4126	4518	4908	5296	5683
26	6068	6452	6834	7215	7595	7973	8350	8725	9099	9471
27	9842	*0212	*0580	*0947	*1313	*1677	*2040	*2402	*2762	*3121
28	5.6 3479	3835	4191	4545	4897	5249	5599	5948	6296	6643
29	6988	7332	7675	8017	8358	8698	9036	9373	9709	*0044
30	5.7 0378	0711	1043	1373	1703	2031	2359	2685	3010	3334
31	3657	3979	4300	4620	4939	5257	5574	5890	6205	6519
32	6832	7144	7455	7765	8074	8383	8690	8996	9301	9606
33	9909	*0212	*0513	*0814	*1114	*1413	*1711	*2008	*2305	*2600
34	5.8 2895	3188	3481	3773	4064	4354	4644	4932	5220	5507
35	5793	6079	6363	6647	6930	7212	7493	7774	8053	8332
36	8610	8888	9164	9440	9715	9990	*0263	*0536	*0808	*1080
37	5.9 1350	1620	1889	2158	2426	2693	2959	3225	3489	3754
38	4017	4280	4542	4803	5064	5324	5584	5842	6101	6358
39	6615	6871	7126	7381	7635	7889	8141	8394	8645	8896
40	9146	9396	9645	9894	*0141	*0389	*0635	*0881	*1127	*1372
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Above 409, use the formula $\log_e 10\,n = \log_e n + \log_e 10 = \log_e n + 2.30258509$,

or the formula $\log_e n = \log_e 10 \cdot \log_{10} n = 2.30258509 \log_{10} n$.

Table VIII — Multiples of M and of 1/M

N	$N \cdot M$	N	$N \cdot M$
0	0.00000 000	**50**	21.71472 410
1	0.43429 448	51	22.14901 858
2	0.86858 896	52	22.58331 306
3	1.30288 345̄	53	23.01760 754
4	1.73717 793	54	23.45190 202
5	2.17147 241	55	23.88619 650
6	2.60576 689	56	24.32049 099
7	3.04006 137	57	24.75478 547
8	3.47435 586	58	25.18907 995
9	3.90865 034	59	25.62337 443
10	4.34294 482	**60**	26.05766 891
11	4.77723 930	61	26.49196 340
12	5.21153 378	62	26.92625 788
13	5.64582 826	63	27.36055 236
14	6.08012 275̄	64	27.79484 684
15	6.51441 723	65	28.22914 132
16	6.94871 171	66	28.66343 581
17	7.38300 619	67	29.09773 029
18	7.81730 067	68	29.53202 477
19	8.25159 516	69	29.96631 925
20	8.68588 964	**70**	30.40061 373
21	9.12018 412	71	30.83490 822
22	9.55447 860	72	31.26920 270
23	9.98877 308	73	31.70349 718
24	10.42306 757	74	32.13779 166
25	10.85736 205̄	75	32.57208 614
26	11.29165 653	76	33.00638 062
27	11.72595 101	77	38.44067 511
28	12.16024 549	78	33.87496 959
29	12.59453 998	79	34.30926 407
30	13.02883 446	**80**	34.74355 855
31	13.46312 894	81	35.17785 303
32	13.89742 342	82	35.61214 752
33	14.33171 790	83	36.04644 200
34	14.76601 238	84	36.48073 648
35	15.20030 687	85	36.91503 096
36	15.63460 135̄	86	37.34932 544
37	16.06889 583	87	37.78361 993
38	16.50319 031	88	38.21791 441
39	16.93748 479	89	38.65220 889
40	17.37177 928	**90**	39.08650 337
41	17.80607 376	91	39.52079 785
42	18.24036 824	92	39.95509 234
43	18.67466 272	93	40.38938 682
44	19.10895 720	94	40.82368 130
45	19.54325 169	95	41.25797 578
46	19.97754 617	96	41.69227 026
47	20.41184 065̄	97	42.12656 474
48	20.84613 513	98	42.56085 923
49	21.28042 961	99	42.99515 371
50	21.71472 410	**100**	43.42944 819

N	$N \div M$	N	$N \div M$
0	0.00000 000	**50**	115.12925 465̄
1	2.30258 509	51	117.43183 974
2	4.60517 019	52	119.73442 484
3	6.90775 528	53	122.03700 993
4	9.21034 037	54	124.33959 502
5	11.51292 546	55	126.64218 011
6	13.81551 056	56	128.94476 521
7	16.11809 565	57	131.24735 030
8	18.42068 074	58	133.54993 539
9	20.72326 584	59	135.85252 049
10	23.02585 093	**60**	138.15510 558
11	25.32843 602	61	140.45769 067
12	27.63102 112	62	142.76027 577
13	29.93360 621	63	145.06286 086
14	32.23619 130	64	147.36544 595
15	34.53877 639	65	149.66803 104
16	36.84136 149	66	151.97061 614
17	39.14394 658	67	154.27320 123
18	41.44653 167	68	156.57578 632
19	43.74911 677	69	158.87837 142
20	46.05170 186	**70**	161.18095 651
21	48.35428 695	71	163.48354 160
22	50.65687 205̄	72	165.78612 670
23	52.95945 714	73	168.08871 179
24	55.26204 223	74	170.39129 688
25	57.56462 732	75	172.69388 197
26	59.86721 242	76	174.99646 707
27	62.16979 751	77	177.29905 216
28	64.47238 260	78	179.60163 725
29	66.77496 770	79	181.90422 235̄
30	69.07755 279	**80**	184.20680 744
31	71.38013 788	81	186.50939 253
32	73.68272 298	82	188.81197 763
33	75.98530 807	83	191.11456 272
34	78.28789 316	84	193.41714 781
35	80.59047 825	85	195.71973 290
36	82.89306 335̄	86	198.02231 800
37	85.19564 844	87	200.32490 309
38	87.49823 353	88	202.62748 818
39	89.80081 863	89	204.93007 328
40	92.10340 372	**90**	207.23265 837
41	94.40598 881	91	209.53524 346
42	96.70857 391	92	211.83782 856
43	99.01115 900	93	214.14041 365̄
44	101.31374 409	94	216.44299 874
45	103.61632 918	95	218.74558 383
46	105.91891 428	96	221.04816 893
47	108.22149 937	97	223.35075 402
48	110.52408 446	98	225.65333 911
49	112.82666 956	99	227.95592 421
50	115.12925 465̄	**100**	230.25850 930

$$M = \log_{10} e = .43429\ 44819\ 03251\ 82765$$

$$\log_{10} n = \log_e n \cdot \log_{10} e = M \log_e n$$

$$\log_{10} e^x = x \cdot \log_{10} e = x \cdot M.$$

$$\frac{1}{M} = \log_e 10 = 2.30258\ 50929\ 94045\ 68402$$

$$\log_e n = \log_{10} n \cdot \log_e 10 = \frac{1}{M} \log_{10} n.$$

$$\log_e (10^n \cdot x) = \log_e x + n \frac{1}{M}$$

x	e^x		e^{-x}	Sinh x		Cosh x		Tanh x
	Value	$\log_{10}$	Value	Value	$\log_{10}$	Value	$\log_{10}$	Value
0.00	1.0000	.00000	1.0000	0 0000	$-\infty$	1.0000	.00000	.00000
0.01	1.0101	.00434	.99005	0.0100	.00001	1.0001	.00002	.01000
0.02	1.0202	.00869	.98020	0.0200	.30106	1.0002	.00009	.02000
0.03	1.0305	.01303	.97045	0.0300	.47719	1.0005	.00020	.02999
0.04	1.0408	.01737	.96079	0.0400	.60218	1.0008	.00035	.03998
0.05	1.0513	.02171	.95123	0.0500	.69915	1.0013	.00054	.04996
0.06	1.0618	.02606	.94176	0 0600	.77841	1.0018	.00078	.05993
0.07	1.0725	.03040	.93239	0.0701	.84545	1.0025	.00106	.06989
0.08	1.0833	.03474	.92312	0.0801	.90355	1.0032	.00139	.07983
0.09	1.0942	.03909	.91393	0.0901	.95483	1.0041	.00176	.08976
0.10	1.1052	.04343	.90484	0.1002	.00072	1.0050	.00217	.09967
0.11	1.1163	.04777	.89583	0.1102	.04227	1.0061	.00262	.10956
0.12	1.1275	.05212	.88692	0.1203	.08022	1.0072	.00312	.11943
0.13	1.1388	.05646	.87809	0.1304	.11517	1.0085	.00366	.12927
0.14	1.1503	.06080	.86936	0.1405	.14755	1.0098	.00424	.13909
0.15	1.1618	.06514	.86071	0.1506	.17772	1.0113	.00487	.14889
0.16	1.1735	.06949	.85214	0.1607	.20597	1.0128	.00554	.15865
0.17	1.1853	.07383	.84366	0.1708	.23254	1.0145	.00625	.16838
0.18	1.1972	.07817	.83527	0.1810	.25762	1.0162	.00700	.17808
0.19	1.2092	.08252	.82696	0.1911	.28136	1.0181	.00779	.18775
0.20	1.2214	.08686	.81873	0.2013	.30392	1.0201	.00863	.19738
0.21	1.2337	.09120	.81058	0.2115	.32541	1.0221	.00951	.20697
0.22	1.2461	.09554	.80252	0.2218	.34592	1.0243	.01043	.21652
0.23	1.2586	.09989	.79453	0.2320	.36555	1.0266	.01139	.22603
0.24	1.2712	.10423	.78663	0.2423	.38437	1.0289	.01239	.23550
0.25	1.2840	.10857	.77880	0.2526	.40245	1.0314	.01343	.24492
0.26	1.2969	.11292	.77105	0.2629	.41986	1.0340	.01452	.25430
0.27	1.3100	.11726	.76338	0.2733	.43663	1.0367	.01564	.26362
0.28	1.3231	.12160	.75578	0.2837	.45282	1.0395	.01681	.27291
0.29	1.3364	.12595	.74826	0.2941	.46847	1.0423	.01801	.28213
0.30	1.3499	.13029	.74082	0.3045	.48362	1.0453	.01926	.29131
0.31	1.3634	.13463	.73345	0.3150	.49830	1.0484	.02054	.30044
0.32	1.3771	.13897	.72615	0.3255	.51254	1.0516	.02107	.30951
0.33	1.3910	.14332	.71892	0.3360	.52637	1.0549	.02323	.31852
0.34	1.4049	.14766	.71177	0.3466	.53981	1.0584	.02463	.32748
0.35	1.4191	.15200	.70469	0.3572	.55290	1.0619	.02607	.33638
0.36	1.4333	.15635	.69768	0.3678	.56564	1.0655	.02755	.34521
0.37	1.4477	.16069	.69073	0.3785	.57807	1.0692	.02907	.35399
0.38	1.4623	.16503	.68386	0.3892	.59019	1.0731	.03063	.36271
0.39	1.4770	.16937	.67706	0.4000	.60202	1.0770	.03222	.37136
0.40	1.4918	.17372	.67032	0.4108	.61358	1.0811	.03385	.37995
0.41	1.5068	.17806	.66365	0.4216	.62488	1.0852	.03552	.38847
0.42	1.5220	.18240	.65705	0.4325	.63594	1.0895	.03723	.39693
0.43	1.5373	.18675	.65051	0.4434	.64677	1.0939	.03897	.40532
0.44	1.5527	.19109	.64404	0.4543	.65738	1.0984	.04075	.41364
0.45	1.5683	.19543	.63763	0.4653	.66777	1.1030	.04256	.42190
0.46	1.5841	.19978	.63128	0.4764	.67797	1.1077	.04441	.43008
0.47	1.6000	.20412	.62500	0.4875	.68797	1.1125	.04630	.43820
0.48	1.6161	.20846	.61878	0.4986	.69779	1.1174	.04822	.44624
0.49	1.6323	.21280	.61263	0.5098	.70744	1.1225	.05018	.45422
0.50	1.6487	.21715	.60653	0.5211	.71692	1.1276	.05217	.46212

x	e^x		e^{-x}	**Sinh** x		**Cosh** x		**Tanh** x
	Value	Log_{10}	Value	Value	Log_{10}	Value	Log_{10}	Value
0.50	1.6487	.21715	.60653	0.5211	.71692	1.1276	.05217	.46212
0.51	1.6653	.22149	.60050	0.5324	.72624	1.1329	.05419	.46995
0.52	1.6820	.22583	.59452	0.5438	.73540	1.1383	.05625	.47770
0.53	1.6989	.23018	.58860	0.5552	.74442	1.1438	.05834	.48538
0.54	1.7160	.23452	.58275	0.5666	.75330	1.1494	.06046	.49299
0.55	1.7333	.23886	.57695	0.5782	.76204	1.1551	.06262	.50052
0.56	1.7507	.24320	.57121	0.5897	.77065	1.1609	.06481	.50798
0.57	1.7683	.24755	.56553	0.6014	.77914	1.1669	.06703	.51536
0.58	1.7860	.25189	.55990	0.6131	.78751	1.1730	.06929	.52267
0.59	1.8040	.25623	.55433	0.6248	.79576	1.1792	.07157	.52990
0.60	1.8221	.26058	.54881	0.6367	.80390	1.1855	.07389	.53705
0.61	1.8404	.26492	.54335	0.6485	.81194	1.1919	.07624	.54413
0.62	1.8589	.26926	.53794	0.6605	.81987	1.1984	.07861	.55113
0.63	1.8776	.27361	.53259	0.6725	.82770	1.2051	.08102	.55805
0.64	1.8965	.27795	.52729	0.6846	.83543	1.2119	.08346	.56490
0.65	1.9155	.28229	.52205	0.6967	.84308	1.2188	.08593	.57167
0.66	1.9348	.28664	.51685	0.7090	.85063	1.2258	.08843	.57836
0.67	1.9542	.29098	.51171	0.7213	.85809	1.2330	.09095	.58498
0.68	1.9739	.29532	.50662	0.7336	.86548	1.2402	.09351	.59152
0.69	1.9937	.29966	.50158	0.7461	.87278	1.2476	.09609	.59798
0.70	2.0138	.30401	.49659	0.7586	.88000	1.2552	.09870	.60437
0.71	2.0340	.30835	.49164	0.7712	.88715	1.2628	.10134	.61068
0.72	2.0544	.31269	.48675	0.7838	.89423	1.2706	.10401	.61691
0.73	2.0751	.31703	.48191	0.7966	.90123	1.2785	.10670	.62307
0.74	2.0959	.32138	.47711	0.8094	.90817	1.2865	.10942	.62915
0.75	2.1170	.32572	.47237	0.8223	.91504	1.2947	.11216	.63515
0.76	2.1383	.33006	.46767	0.8353	.92185	1.3030	.11493	.64108
0.77	2.1598	.33441	.46301	0.8484	.92859	1.3114	.11773	.64693
0.78	2.1815	.33875	.45841	0.8615	.93527	1.3199	.12055	.65271
0.79	2.2034	.34309	.45384	0.8748	.94190	1.3286	.12340	.65841
0.80	2.2255	.34744	.44933	0.8881	.94846	1.3374	.12627	.66404
0.81	2.2479	.35178	.44486	0.9015	.95498	1.3464	.12917	.66959
0.82	2.2705	.35612	.44043	0.9150	.96144	1.3555	.13209	.67507
0.83	2.2933	.36046	.43605	0.9286	.96784	1.3647	.13503	.68048
0.84	2.3164	.36481	.43171	0.9423	.97420	1.3740	.13800	.68581
0.85	2.3396	.36915	.42741	0.9561	.98051	1.3835	.14099	.69107
0.86	2.3632	.37349	.42316	0.9700	.98677	1.3932	.14400	.69626
0.87	2.3869	.37784	.41895	0.9840	.99299	1.4029	.14704	.70137
0.88	2.4109	.38218	.41478	0.9981	.99916	1.4128	.15009	.70642
0.89	2.4351	.38652	.41066	1.0122	.00528	1.4229	.15317	.71139
0.90	2.4596	.39087	.40657	1.0265	.01137	1.4331	.15627	.71630
0.91	2.4843	.39521	.40252	1.0409	.01741	1.4434	.15939	.72113
0.92	2.5093	.39955	.39852	1.0554	.02341	1.4539	.16254	.72590
0.93	2.5345	.40389	.39455	1.0700	.02937	1.4645	.16570	.73059
0.94	2.5600	.40824	.39063	1.0847	.03530	1.4753	.16888	.73522
0.95	2.5857	.41258	.38674	1.0995	.04119	1.4862	.17208	.73978
0.96	2.6117	.41692	.38289	1.1144	.04704	1.4973	.17531	.74428
0.97	2.6379	.42127	.37908	1.1294	.05286	1.5085	.17855	.74870
0.98	2.6645	.42561	.37531	1.1446	.05864	1.5199	.18181	.75307
0.99	2.6912	.42995	.37158	1.1598	.06439	1.5314	.18509	.75736
1.00	2.7183	.43429	.36788	1.1752	.07011	1.5431	.18839	.76159

x	e^x		e^{-x}	Sinh x		Cosh x		Tanh x
	Value	Log_{10}	Value	Value	Log_{10}	Value	Log_{10}	Value
1.00	2.7183	.43429	.36788	1.1752	.07011	1.5431	.18839	.76159
1.01	2.7456	.43864	.36422	1.1907	.07580	1.5549	.19171	.76576
1.02	2.7732	.44298	.36060	1.2063	.08146	1.5669	.19504	.76987
1.03	2.8011	.44732	.35701	1.2220	.08708	1.5790	.19839	.77391
1.04	2.8292	.45167	.35345	1.2379	.09268	1.5913	.20176	.77789
1.05	2.8577	.45601	.34994	1.2539	.09825	1.6038	.20515	.78181
1.06	2.8864	.46035	.34646	1.2700	.10379	1.6164	.20855	.78566
1.07	2.9154	.46470	.34301	1.2862	.10930	1.6292	.21197	.78946
1.08	2.9447	.46904	.33960	1.3025	.11479	1.6421	.21541	.79320
1.09	2.9743	.47338	.33622	1.3190	.12025	1.6552	.21886	.79688
1.10	3.0042	.47772	.33287	1.3356	.12569	1.6685	.22233	.80050
1.11	3.0344	.48207	.32956	1.3524	.13111	1.6820	.22582	.80406
1.12	3.0649	.48641	.32628	1.3693	.13649	1.6956	.22931	.80757
1.13	3.0957	.49075	.32303	1.3863	.14186	1.7093	.23283	.81102
1.14	3.1268	.49510	.31982	1.4035	.14720	1.7233	.23636	.81441
1.15	3.1582	.49944	.31664	1.4208	.15253	1.7374	.23990	.81775
1.16	3.1899	.50378	.31349	1.4382	.15783	1.7517	.24346	.82104
1.17	3.2220	.50812	.31037	1.4558	.16311	1.7662	.24703	.82427
1.18	3.2544	.51247	.30728	1.4735	.16836	1.7808	.25062	.82745
1.19	3.2871	.51681	.30422	1.4914	.17360	1.7957	.25422	.83058
1.20	3.3201	.52115	.30119	1.5085	.17882	1.8107	.25784	.83365
1.21	3.3535	.52550	.29820	1.5276	.18402	1.8258	.26146	.83668
1.22	3.3872	.52984	.29523	1.5460	.18920	1.8412	.26510	.83965
1.23	3.4212	.53418	.29229	1.5645	.19437	1.8568	.26876	.84258
1.24	3.4556	.53853	.28938	1.5831	.19951	1.8725	.27242	.84546
1.25	3.4903	.54287	.28650	1.6019	.20464	1.8884	.27610	.84828
1.26	3.5254	.54721	.28365	1.6209	.20975	1.9045	.27979	.85106
1.27	3.5609	.55155	.28083	1.6400	.21485	1.9208	.28349	.85380
1.28	3.5966	.55590	.27804	1.6593	.21993	1.9373	.28721	.85648
1.29	3.6328	.56024	.27527	1.6788	.22499	1.9540	.29093	.85913
1.30	3.6693	.56458	.27253	1.6984	.23004	1.9709	.29467	.86172
1.31	3.7062	.56893	.26982	1.7182	.23507	1.9880	.29842	.86428
1.32	3.7434	.57327	.26714	1.7381	.24009	2.0053	.30217	.86678
1.33	3.7810	.57761	.26448	1.7583	.24509	2.0228	.30594	.86925
1.34	3.8190	.58195	.26185	1.7786	.25008	2.0404	.30972	.87167
1.35	3.8574	.58630	.25924	1.7991	.25505	2.0583	.31352	.87405
1.36	3.8962	.59064	.25666	1.8198	.26002	2.0764	.31732	.87639
1.37	3.9354	.59498	.25411	1.8406	.26496	2.0947	.32113	.87869
1.38	3.9749	.59933	.25158	1.8617	.26990	2.1132	.32495	.88095
1.39	4.0149	.60367	.24908	1.8829	.27482	2.1320	.32878	.88317
1.40	4.0552	.60801	.24660	1.9043	.27974	2.1509	.33262	.88535
1.41	4.0960	.61236	.24414	1.9259	.28464	2.1700	.33647	.88749
1.42	4.1371	.61670	.24171	1.9477	.28952	2.1894	.34033	.88960
1.43	4.1787	.62104	.23931	1.9697	.29440	2.2090	.34420	.89167
1.44	4.2207	.62538	.23693	1.9919	.29926	2.2288	.34807	.89370
1.45	4.2631	.62973	.23457	2.0143	.30412	2.2488	.35196	.89569
1.46	4.3060	.63407	.23224	2.0369	.30896	2.2691	.35585	.89765
1.47	4.3492	.63841	.22993	2.0597	.31379	2.2896	.35976	.89958
1.48	4.3929	.64276	.22764	2.0827	.31862	2.3103	.36367	.90147
1.49	4.4371	.64710	.22537	2.1059	.32343	2.3312	.36759	.90332
1.50	4.4817	.65144	.22313	2.1293	.32823	2.3524	.37151	.90515

x	e^x		e^{-x}	Sinh x		Cosh x		Tanh x
	Value	Log_{10}	Value	Value	Log_{10}	Value	Log_{10}	Value
1.50	4.4817	.65144	.22313	2.1293	.32823	2.3524	.37151	.90515
1.51	4.5267	.65578	.22091	2.1529	.33303	2.3738	.37545	.90694
1.52	4.5722	.66013	.21871	2.1768	.33781	2.3955	.37939	.90870
1.53	4.6182	.66447	.21654	2.2008	.34258	2.4174	.38334	.91042
1.54	4.6646	.66881	.21438	2.2251	.34735	2.4395	.38730	.91212
1.55	4.7115	.67316	.21225	2.2496	.35211	2.4619	.39126	.91379
1.56	4.7588	.67750	.21014	2.2743	.35686	2.4845	.39524	.91542
1.57	4.8066	.68184	.20805	2.2993	.36160	2.5073	.39921	.91703
1.58	4.8550	.68619	.20598	2.3245	.36633	2.5305	.40320	.91860
1.59	4.9037	.69053	.20393	2.3499	.37105	2.5538	.40719	.92015
1.60	4.9530	.69487	.20190	2.3756	.37577	2.5775	.41119	.92167
1.61	5.0028	.69921	.19989	2.4015	.38048	2.6013	.41520	.92316
1.62	5.0531	.70356	.19790	2.4276	.38518	2.6255	.41921	.92462
1.63	5.1039	.70790	.19593	2.4540	.38987	2.6499	.42323	.92606
1.64	5.1552	.71224	.19398	2.4806	.39456	2.6746	.42725	.92747
1.65	5.2070	.71659	.19205	2.5075	.39923	2.6995	.43129	.92886
1.66	5.2593	.72093	.19014	2.5345	.40391	2.7247	.43532	.93022
1.67	5.3122	.72527	.18825	2.5620	.40857	2.7502	.43937	.93155
1.68	5.3656	.72961	.18637	2.5896	.41323	2.7760	.44341	.93286
1.69	5.4195	.73396	.18452	2.6175	.41788	2.8020	.44747	.93415
1.70	5.4739	.73830	.18268	2.6456	.42253	2.8283	.45153	.93541
1.71	5.5290	.74264	.18087	2.6740	.42717	2.8549	.45559	.93665
1.72	5.5845	.74699	.17907	2.7027	.43180	2.8818	.45966	.93786
1.73	5.6407	.75133	.17728	2.7317	.43643	2.9090	.46374	.93906
1.74	5.6973	.75567	.17552	2.7609	.44105	2.9364	.46782	.94023
1.75	5.7546	.76002	.17377	2.7904	.44567	2.9642	.47191	.94138
1.76	5.8124	.76436	.17204	2.8202	.45028	2.9922	.47600	.94250
1.77	5.8709	.76870	.17033	2.8503	.45488	3.0206	.48009	.94361
1.78	5.9299	.77304	.16864	2.8806	.45948	3.0492	.48419	.94470
1.79	5.9895	.77739	.16696	2.9112	.46408	3.0782	.48830	.94576
1.80	6.0496	.78173	.16530	2.9422	.46867	3.1075	.49241	.94681
1.81	6.1104	.78607	.16365	2.9734	.47325	3.1371	.49652	.94783
1.82	6.1719	.79042	.16203	3.0049	.47783	3.1669	.50064	.94884
1.83	6.2339	.79476	.16041	3.0367	.48241	3.1972	.50476	.94983
1.84	6.2965	.79910	.15882	3.0689	.48698	3.2277	.50889	.95080
1.85	6.3598	.80344	.15724	3.1013	.49154	3.2585	.51302	.95175
1.86	6.4237	.80779	.15567	3.1340	.49610	3.2897	.51716	.95268
1.87	6.4883	.81213	.15412	3.1671	.50066	3.3212	.52130	.95359
1.88	6.5535	.81647	.15259	3.2005	.50521	3.3530	.52544	.95449
1.89	6.6194	.82082	.15107	3.2341	.50976	3.3852	.52959	.95537
1.90	6.6859	.82516	.14957	3.2682	.51430	3.4177	.53374	.95624
1.91	6.7531	.82950	.14808	3.3025	.51884	3.4506	.53789	.95709
1.92	6.8210	.83385	.14661	3.3372	.52338	3.4838	.54205	.95792
1.93	6.8895	.83819	.14515	3.3722	.52791	3.5173	.54621	.95873
1.94	6.9588	.84253	.14370	3.4075	.53244	3.5512	.55038	.95953
1.95	7.0287	.84687	.14227	3.4432	.53696	3.5855	.55455	.96032
1.96	7.0993	.85122	.14086	3.4792	.54148	3.6201	.55872	.96109
1.97	7.1707	.85556	.13946	3.5156	.54600	3.6551	.56290	.96185
1.98	7.2427	.85990	.13807	3.5523	.55051	3.6904	.56707	.96259
1.99	7.3155	.86425	.13670	3.5894	.55502	3.7261	.57126	.96331
2.00	7.3891	.86859	.13534	3.6269	.55953	3.7622	.57544	.96403

x	e^x		e^{-x}	Sinh x		Cosh x		Tanh x
	Value	Log_{10}	Value	Value	Log_{10}	Value	Log_{10}	Value
2.00	7.3891	.86859	.13534	3.6269	.55953	3.7622	.57544	.96403
2.01	7.4633	.87293	.13399	3.6647	.56403	3.7987	.57963	.96473
2.02	7.5383	.87727	.13266	3.7028	.56853	3.8355	.58382	.96541
2.03	7.6141	.88162	.13134	3.7414	.57303	3.8727	.58802	.96609
2.04	7.6906	.88596	.13003	3.7803	.57753	3.9103	.59221	.96675
2.05	7.7679	.89030	.12873	3.8196	.58202	3.9483	.59641	.96740
2.06	7.8460	.89465	.12745	3.8593	.58650	3.9867	.60061	.96803
2.07	7.9248	.89899	.12619	3.8993	.59099	4.0255	.60482	.96865
2.08	8 0045	.90333	.12493	3.9398	.59547	4.0647	.60903	.96926
2.09	8.0849	.90768	.12369	3.9806	.59995	4.1043	.61324	.96986
2.10	8.1662	.91202	.12246	4.0219	.60443	4.1443	.61745	.97045
2.11	8.2482	.91636	.12124	4.0635	.60890	4.1847	.62167	.97103
2.12	8.3311	.92070	.12003	4.1056	.61337	4.2256	.62589	.97159
2.13	8.4149	.92505	.11884	4.1480	.61784	4.2669	.63011	.97215
2.14	8.4994	.92939	.11765	4.1909	.62231	4.3085	.63433	.97269
2.15	8.5849	.93373	.11648	4.2342	.62677	4.3507	.63856	.97323
2.16	8.6711	.93808	.11533	4.2779	.63123	4.3932	.64278	.97375
2.17	8.7583	.94242	.11418	4.3221	.63569	4.4362	.64701	.97426
2.18	8.8463	.94676	.11304	4.3666	.64015	4.4797	.65125	.97477
2.19	8.9352	.95110	.11192	4.4116	.64460	4.5236	.65548	.97526
2.20	9.0250	.95545	.11080	4.4571	.64905	4.5679	.65972	.97574
2.21	9.1157	.95979	.10970	4.5030	.65350	4.6127	.66396	.97622
2.22	9.2073	.96413	.10861	4.5494	.65795	4.6580	.66820	.97668
2.23	9.2999	.96848	.10753	4.5962	.66240	4.7037	.67244	.97714
2.24	9.3933	.97282	.10646	4.6434	.66684	4.7499	.67668	.97759
2.25	9.4877	.97716	.10540	4.6912	.67128	4.7966	.68093	.97803
2.26	9.5831	.98151	.10435	4.7394	.67572	4.8437	.68518	.97846
2.27	9.6794	.98585	.10331	4.7880	.68016	4.8914	.68943	.97888
2.28	9.7767	.99019	.10228	4.8372	.68459	4.9395	.69368	.97929
2.29	9.8749	.99453	.10127	4.8868	.68903	4.9881	.69794	.97970
2.30	9.9742	.99888	.10026	4.9370	.69346	5.0372	.70219	.98010
2.31	10.074	.00322	.09926	4.9876	.69789	5.0868	.70645	.98049
2.32	10.176	.00756	.09827	5.0387	.70232	5.1370	.71071	.98087
2.33	10.278	.01191	.09730	5.0903	.70675	5.1876	.71497	.98124
2.34	10.381	.01625	.09633	5.1425	.71117	5.2388	.71923	.98161
2.35	10.486	.02059	.09537	5.1951	.71559	5.2905	.72349	.98197
2.36	10.591	.02493	.09442	5.2483	.72002	5.3427	.72776	.98233
2.37	10.697	.02928	.09348	5.3020	.72444	5.3954	.73203	.98267
2.38	10.805	.03362	.09255	5.3562	.72885	5.4487	.73630	.98301
2.39	10.913	.03796	.09163	5.4109	.73327	5.5026	.74056	.98335
2.40	11.023	04231	.09072	5.4662	.73769	5.5569	.74484	.98367
2.41	11.134	.04665	.08982	5.5221	.74210	5.6119	.74911	.98400
2.42	11.246	.05099	.08892	5.5785	.74652	5.6674	.75338	.98431
2.43	11.359	.05534	.08804	5.6354	.75093	5.7235	.75766	.98462
2.44	11.473	.05968	.08716	5 6929	.75534	5.7801	.76194	.98492
2.45	11.588	.06402	.08629	5.7510	.75975	5.8373	.76621	.98522
2.46	11.705	.06836	.08543	5.8097	.76415	5.8951	.77049	.98551
2.47	11.822	.07271	.08458	5.8689	.76856	5.9535	.77477	.98579
2.48	11.941	.07705	.08374	5.9288	.77296	6.0125	.77906	.98607
2.49	12.061	.08139	.08291	5.9892	.77737	6.0721	.78334	.98635
2.50	12.182	.08574	.08208	6.0502	.78177	6.1323	.78762	.98661

x	e^x		e^{-x}	Sinh x		Cosh x		Tanh x
	Value	Log_{10}	Value	Value	Log_{10}	Value	Log_{10}	Value
2.50	12.182	.08574	.08208	6.0502	.78177	6.1323	.78762	.98661
2.51	12.305	.09008	.08127	6.1118	.78617	6.1931	.79191	.98688
2.52	12.429	.09442	.08046	6.1741	.79057	6.2545	.79619	.98714
2.53	12.554	.09877	.07966	6.2369	.79497	6.3166	.80048	.98739
2.54	12.680	.10311	.07887	6.3004	.79937	6.3793	.80477	.98764
2.55	12.807	.10745	.07808	6.3645	.80377	6.4426	.80906	.98788
2.56	12.936	.11179	.07730	6.4293	.80816	6.5066	.81335	.98812
2.57	13.066	.11614	.07654	6.4946	.81256	6.5712	.81764	.98835
2.58	13.197	.12048	.07577	6.5607	.81695	6.6365	.82194	.98858
2.59	13.330	.12482	.07502	6.6274	.82134	6.7024	.82623	.98881
2.60	13.464	.12917	.07427	6.6947	.82573	6.7690	.83052	.98903
2.61	13.599	.13351	.07353	6.7628	.83012	6.8363	.83482	.98924
2.62	13.736	.13785	.07280	6.8315	.83451	6.9043	.83912	.98946
2.63	13.874	.14219	.07208	6.9008	.83890	6.9729	.84341	.98966
2.64	14.013	.14654	.07136	6.9709	.84329	7.0423	.84771	.98987
2.65	14.154	.15088	.07065	7.0417	.84768	7.1123	.85201	.99007
2.66	14.296	.15522	.06995	7.1132	.85206	7.1831	.85631	.99026
2.67	14.440	.15957	.06925	7.1854	.85645	7.2546	.86061	.99045
2.68	14.585	.16391	.06856	7.2583	.86083	7.3268	.86492	.99064
2.69	14.732	.16825	.06788	7.3319	.86522	7.3998	.86922	.99083
2.70	14.880	.17260	.06721	7.4063	.86960	7.4735	.87352	.99101
2.71	15.029	.17694	.06654	7.4814	.87398	7.5479	.87783	.99118
2.72	15.180	.18128	.06587	7.5572	.87836	7.6231	.88213	.99136
2.73	15.333	.18562	.06522	7.6338	.88274	7.6991	.88644	.99153
2.74	15.487	.18997	.06457	7.7112	.88712	7.7758	.89074	.99170
2.75	15.643	.19431	.06393	7.7894	.89150	7.8533	.89505	.99186
2.76	15.800	.19865	.06329	7.8683	.89588	7.9316	.89936	.99202
2.77	15.959	.20300	.06266	7.9480	.90026	8.0106	.90367	.99218
2.78	16.119	.20734	.06204	8.0285	.90463	8.0905	.90798	.99233
2.79	16.281	.21168	.06142	8.1098	.90901	8.1712	.91229	.99248
2.80	16.445	.21602	.06081	8.1919	.91339	8.2527	.91660	.99263
2.81	16.610	.22037	.06020	8.2749	.91776	8.3351	.92091	.99278
2.82	16.777	.22471	.05961	8.3586	.92213	8.4182	.92522	.99292
2.83	16.945	.22905	.05901	8.4432	.92651	8.5022	.92953	.99306
2.84	17.116	.23340	.05843	8.5287	.93088	8.5871	.93385	.99320
2.85	17.288	.23774	.05784	8.6150	.93525	8.6728	.93816	.99333
2.86	17.462	.24208	.05727	8.7021	.93963	8.7594	.94247	.99346
2.87	17.637	.24643	.05670	8.7902	.94400	8.8469	.94679	.99359
2.88	17.814	.25077	.05613	8.8791	.94837	8.9352	.95110	.99372
2.89	17.993	.25511	.05558	8.9689	.95274	9.0244	.95542	.99384
2.90	18.174	.25945	.05502	9.0596	.95711	9.1146	.95974	.99396
2.91	18.357	.26380	.05448	9.1512	.96148	9.2056	.96405	.99408
2.92	18.541	.26814	.05393	9.2437	.96584	9.2976	.96837	.99420
2.93	18.728	.27248	.05340	9.3371	.97021	9.3905	.97269	.99431
2.94	18.916	.27683	.05287	9.4315	.97458	9.4844	.97701	.99443
2.95	19.106	.28117	.05234	9.5268	.97895	9.5791	.98133	.99454
2.96	19.298	.28551	.05182	9.6231	.98331	9.6749	.98565	.99464
2.97	19.492	.28985	.05130	9.7203	.98768	9.7716	.98997	.99475
2.98	19.688	.29420	.05079	9.8185	.99205	9.8693	.99429	.99485
2.99	19.886	.29854	.05029	9.9177	.99641	9.9680	.99861	.99496
3.00	20.086	.30288	.04979	10.018	.00078	10.068	.00293	.99505

x	e^x		e^{-x}	Sinh x		Cosh x		Tanh x
	Value	Log_{10}	Value	Value	Log_{10}	Value	Log_{10}	Value
3.00	20.086	.30288	.04979	10.018	.00078	10.068	.00293	.99505
3.05	21.115	.32460	.04736	10.534	.02259	10.581	.02454	.99552
3.10	22.198	.34631	.04505	11.076	.04440	11.122	.04616	.99595
3.15	23.336	.36803	.04285	11.646	.06619	11.690	.06780	.99631
3.20	24.533	.38974	.04076	12.246	.08799	12.287	.08943	.99668
3.25	25.790	.41146	.03877	12.876	.10977	12.915	.11108	.99700
3.30	27.113	.43317	.03688	13.538	.13155	13.575	.13273	.99728
3.35	28.503	.45489	.03508	14.234	.15332	14.269	.15439	.99754
3.40	29.964	.47660	.03337	14.965	.17509	14.999	.17605	.99777
3.45	31.500	.49832	.03175	15.734	.19685	15.766	.19772	.99799
3.50	33.115	.52003	.03020	16.543	.21860	16.573	.21940	.99818
3.55	34.813	.54175	.02872	17.392	.24036	17.421	.24107	.99833
3.60	36.598	.56346	.02732	18.286	.26211	18.313	.26275	.99851
3.65	38.475	.58517	.02599	19.224	.28385	19.250	.28444	.99865
3.70	40.447	.60689	.02472	20.211	.30559	20.236	.30612	.99878
3.75	42.521	.62860	.02352	21.249	.32733	21.272	.32781	.99889
3.80	44.701	.65032	.02237	22.339	.34907	22.362	.34951	.99900
3.85	46.993	.67203	.02128	23.486	.37081	23.507	.37120	.99909
3.90	49.402	.69375	.02024	24.691	.39254	24.711	.39290	.99918
3.95	51.935	.71546	.01925	25.958	.41427	25.977	.41459	.99926
4.00	54.598	.73718	.01832	27.290	.43600	27.308	.43629	.99933
4.10	60.340	.78061	.01657	30.162	.47946	30.178	.47970	.99945
4.20	66.686	.82404	.01500	33.336	.52291	33.351	.52310	.99955
4.30	73.700	.86747	.01357	36.843	.56636	36.857	.56652	.99963
4.40	81.451	.91090	.01227	40.719	.60980	40.732	.60993	.99970
4.50	90.017	.95433	.01111	45.003	.65324	45.014	.65335	.99975
4.60	99.484	.99775	.01005	49.737	.69668	49.747	.69677	.99980
4.70	109.95	.04118	.00910	54.969	.74012	54.978	.74019	.99983
4.80	121.51	.08461	.00823	60.751	.78355	60.759	.78361	.99986
4.90	134.29	.12804	.00745	67.141	.82699	67.149	.82704	.99989
5.00	148.41	.17147	.00674	74.203	.87042	74.210	.87046	.99991
5.10	164.02	.21490	.00610	82.011	.91386	82.014	.91389	.99993
5.20	181.27	.25833	.00552	90.633	.95729	90.639	.95731	.99994
5.30	200.34	.30176	.00499	100.17	.00074	100.17	.00074	.99995
5.40	221.41	.34519	.00452	110.70	.04415	110.71	.04417	.99996
5.50	244.69	.38862	.00409	122.34	.08758	122.35	.08760	.99997
5.60	270.43	.43205	.00370	135.21	.13101	135.22	.13103	.99997
5.70	298.87	.47548	.00335	149.43	.17444	149.44	.17445	.99998
5.80	330.30	.51891	.00303	165.15	.21787	165.15	.21788	.99998
5.90	365.04	.56234	.00274	182.52	.26130	182.52	.26131	.99998
6 00	403.43	.60577	.00248	201.71	.30473	201.72	.30474	.99999
6.25	518.01	.71434	.00193	259.01	.41331	259.01	.41331	.99999
6.50	665.14	.82291	.00150	332.57	.52188	332.57	.52189	1.0000
6.75	854.06	.93149	.00117	427.03	.63046	427.03	.63046	1.0000
7.00	1096.6	.04006	.00091	548.32	.73903	548.32	.73903	1.0000
7.50	1808.0	.25721	.00055	904.02	.95618	904.02	.95618	1.0000
8.00	2981.0	.47436	.00034	1490.5	.17333	1490.5	.17333	1.0000
8.50	4914.8	.69150	.00020	2457.4	.39047	2457.4	.39047	1.0000
9.00	8103.1	.90865	.00012	4051.5	.60762	4051.5	.60762	1.0000
9.50	13360.	.12580	.00007	6679.9	.82477	6679.9	.82477	1.0000
10.00	22026.	.34294	.00005	11013.	.04191	11013.	.04191	1.0000

Table X — Values and Logarithms of Haversines

[Characteristics of Logarithms omitted — determine by rule from the value]

°	0′		10′		20′		30′		40′		50′	
	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$
0	.0000		.0000	4.3254	.0000	4.9275	.0000	5.2796	.0000	5.5295	.0001	5.7223
1	.0001	5.8817	.0001	6.0156	.0001	6.1315	.0002	.2338	.0002	.3254	.0003	.4081
2	.0003	.4837	.0004	.5532	.0004	.6176	.0005	.6775	.0005	.7336	.0006	.7862
3	.0007	.8358	.0008	.8828	.0008	.9273	.0009	.9697	.0010	.0101	.0011	.0487
4	.0012	.0856	.0013	.1211	.0014	.1551	.0015	.1879	.0017	.2195	.0018	.2499
5	.0019	.2793	.0020	.3078	.0022	.3354	.0023	.3621	.0024	.3880	.0026	.4132
6	.0027	.4376	.0029	.4614	.0031	.4845	.0032	.5071	.0034	.5290	.0036	.5504
7	.0037	.5713	.0039	.5918	.0041	.6117	.0043	.6312	.0045	.6503	.0047	.6689
8	.0049	.6872	.0051	.7051	.0053	.7226	.0055	.7397	.0057	.7566	.0059	.7731
9	.0062	.7893	.0064	.8052	.0066	.8208	.0069	.8361	.0071	.8512	.0073	.8660
10	.0076	.8806	.0079	.8949	.0081	.9090	.0084	.9229	.0086	.9365	.0089	.9499
11	.0092	.9631	.0095	.9762	.0097	.9890	.0100	.0016	.0103	.0141	.0106	.0264
12	.0109	.0385	.0112	.0504	.0115	.0622	.0119	.0738	.0122	.0853	.0125	.0966
13	.0128	.1077	.0131	.1187	.0135	.1296	.0138	.1404	.0142	.1510	.0145	.1614
14	.0149	.1718	.0152	.1820	.0156	.1921	.0159	.2021	.0163	.2120	.0167	.2218
15	.0170	.2314	.0174	.2409	.0178	.2504	.0182	.2597	.0186	.2689	.0190	.2781
16	.0194	.2871	.0198	.2961	.0202	.3049	.0206	.3137	.0210	.3223	.0214	.3309
17	.0218	.3394	.0223	.3478	.0227	.3561	.0231	.3644	.0236	.3726	.0240	.3806
18	.0245	.3887	.0249	.3966	.0254	.4045	.0258	.4123	.0263	.4200	.0268	.4276
19	.0272	.4352	.0277	.4427	.0282	.4502	.0287	.4576	.0292	.4649	.0297	.4721
20	.0302	.4793	.0307	.4865	.0312	.4936	.0317	.5006	.0322	.5075	.0327	.5144
21	.0332	.5213	.0337	.5281	.0343	.5348	.0348	.5415	.0353	.5481	.0359	.5547
22	.0364	.5612	.0370	.5677	.0375	.5741	.0381	.5805	.0386	.5868	.0392	.5931
23	.0397	.5993	.0403	.6055	.0409	.6116	.0415	.6177	.0421	.6238	.0426	.6298
24	.0432	.6357	.0438	.6417	.0444	.6476	.0450	.6534	.0456	.6592	.0462	.6650
25	.0468	.6707	.0475	.6764	.0481	.6820	.0487	.6876	.0493	.6932	.0500	.6987
26	.0506	.7042	.0512	.7096	.0519	.7151	.0525	.7204	.0532	.7258	.0538	.7311
27	.0545	.7364	.0552	.7416	.0558	.7468	.0565	.7520	.0572	.7572	.0578	.7623
28	.0585	.7673	.0592	.7724	.0599	.7774	.0606	.7824	.0613	.7874	.0620	.7923
29	.0627	.7972	.0634	.8020	.0641	.8069	.0648	.8117	.0655	.8165	.0663	.8213
30	.0670	.8260	.0677	.8307	.0684	.8354	.0692	.8400	.0699	.8446	.0707	.8492
31	.0714	.8538	.0722	.8583	.0729	.8629	.0737	.8673	.0744	.8718	.0752	.8763
32	.0760	.8807	.0767	.8851	.0775	.8894	.0783	.8938	.0791	.8981	.0799	.9024
33	.0807	.9067	.0815	.9109	.0823	.9152	.0831	.9194	.0839	.9236	.0847	.9277
34	.0855	.9319	.0863	.9360	.0871	.9401	.0879	.9442	.0888	.9482	.0896	.9523
35	.0904	.9563	.0913	.9603	.0921	.9643	.0929	.9682	.0938	.9722	.0946	.9761
36	.0955	.9800	.0963	.9838	.0972	.9877	.0981	.9915	.0989	.9954	.0998	.9992
37	.1007	.0030	.1016	.0067	.1024	.0105	.1033	.0142	.1042	.0179	.1051	.0216
38	.1060	.0253	.1069	.0289	.1078	.0326	.1087	.0362	.1096	.0398	.1105	.0434
39	.1114	.0470	.1123	.0505	.1133	.0541	.1142	.0576	.1151	.0611	.1160	.0646
40	.1170	.0681	.1179	.0716	.1189	.0750	.1198	.0784	.1207	.0817	.1217	.0853
41	.1226	.0887	.1236	.0920	.1246	.0954	.1255	.0987	.1265	.1021	.1275	.1054
42	.1284	.1087	.1294	.1119	.1304	.1152	.1314	.1185	.1323	.1217	.1333	.1249
43	.1343	.1282	.1353	.1314	.1363	.1345	.1373	.1377	.1383	.1409	.1393	.1440
44	.1403	.1472	.1413	.1503	.1424	.1534	.1434	.1565	.1444	.1596	.1454	.1626
45	.1464	.1657	.1475	.1687	.1485	.1718	.1495	.1748	.1506	.1778	.1516	.1808
46	.1527	.1838	.1538	.1867	.1548	.1897	.1558	.1926	.1569	.1956	.1579	.1985
47	.1590	.2014	.1600	.2043	.1611	.2072	.1622	.2101	.1633	.2129	.1644	.2158
48	.1654	.2186	.1665	.2215	.1676	.2243	.1687	.2271	.1698	.2299	.1709	.2327
49	.1720	.2355	.1731	.2382	.1742	.2410	.1753	.2437	.1764	.2465	.1775	.2492
50	.1786	.2519	.1797	.2546	.1808	.2573	.1820	.2600	.1831	.2627	.1842	.2653
51	.1853	.2680	.1865	.2706	.1876	.2732	.1887	.2759	.1899	.2785	.1910	.2811
52	.1922	.2837	.1933	.2863	.1945	.2888	.1956	.2914	.1968	.2940	.1979	.2965
53	.1991	.2991	.2003	.3016	.2014	.3041	.2026	.3066	.2038	.3091	.2049	.3116
54	.2061	.3141	.2073	.3166	.2085	.3190	.2096	.3215	.2108	.3239	.2120	.3264
55	.2132	.3288	.2144	.3312	.2156	.3336	.2168	.3361	.2180	.3384	.2192	.3408
56	.2204	.3432	.2216	.3456	.2228	.3480	.2240	.3503	.2252	.3527	.2265	.3550
57	.2277	.3573	.2289	.3596	.2301	.3620	.2314	.3643	.2326	.3666	.2338	.3689
58	.2350	.3711	.2363	.3734	.2375	.3757	.2388	.3779	.2400	.3802	.2412	.3824
59	.2425	.3847	.2437	.3869	.2450	.3891	.2462	.3913	.2475	.3935	.2487	3957

[Characteristics of Logarithms omitted — determine by rule from the value]

°	0′		10′		20′		30′		40′		50′	
	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$
60	.2500	.3979	.2513	.4001	.2525	.4023	.2538	.4045	.2551	.4066	.2563	.4088
61	.2576	.4109	.2589	.4131	.2601	.4152	.2614	.4173	.2627	.4195	.2640	.4216
62	.2653	.4237	.2665	.4258	.2678	.4279	.2691	.4300	.2704	.4320	.2717	.4341
63	.2730	.4362	.2743	.4382	.2756	.4403	.2769	.4423	.2782	.4444	.2795	.4464
64	.2808	.4484	.2821	.4504	.2834	.4524	.2847	.4545	.2861	.4565	.2874	.4584
65	.2887	.4604	.2900	.4624	.2913	.4644	.2927	.4664	.2940	.4683	.2953	.4703
66	.2966	.4722	.2980	.4742	.2993	.4761	.3006	.4780	.3020	.4799	.3033	.4819
67	.3046	.4838	.3060	.4857	.3073	.4876	.3087	.4895	.3100	.4914	.3113	.4932
68	.3127	.4951	.3140	.4970	.3154	.4989	.3167	.5007	.3181	.5026	.3195	.5044
69	.3208	.5063	.3222	.5081	.3235	.5099	.3249	.5117	.3263	.5136	.3276	.5154
70	.3290	.5172	.3304	.5190	.3317	.5208	.3331	.5226	.3345	.5244	.3358	.5261
71	.3372	.5279	.3386	.5297	.3400	.5314	.3413	.5332	.3427	.5349	.3441	.5367
72	.3455	.5384	.3469	.5402	.3483	.5419	.3496	.5436	.3510	.5454	.3524	.5471
73	.3538	.5488	.3552	.5505	.3566	.5522	.3580	.5539	.3594	.5556	.3608	.5572
74	.3622	.5589	.3636	.5606	.3650	.5623	.3664	.5639	.3678	.5656	.3692	.5672
75	.3706	.5689	.3720	.5705	.3734	.5722	.3748	.5738	.3762	.5754	.3776	.5771
76	.3790	.5787	.3805	.5803	.3819	.5819	.3833	.5835	.3847	.5851	.3861	.5867
77	.3875	.5883	.3889	.5899	.3904	.5915	.3918	.5930	.3932	.5946	.3946	.5962
78	.3960	.5977	.3975	.5993	.3989	.6009	.4003	.6024	.4017	.6039	.4032	.6055
79	.4046	.6070	.4060	.6085	.4075	.6101	.4089	.6116	.4103	.6131	.4117	.6146
80	.4132	.6161	.4146	.6176	.4160	.6191	.4175	.6206	.4189	.6221	.4203	.6236
81	.4218	.6251	.4232	.6266	.4247	.6280	.4261	.6295	.4275	.6310	.4290	.6324
82	.4304	.6339	.4319	.6353	.4333	.6368	.4347	.6382	.4362	.6397	.4376	.6411
83	.4391	.6425	.4405	.6440	.4420	.6454	.4434	.6468	.4448	.6482	.4463	.6496
84	.4477	.6510	.4492	.6524	.4506	.6538	.4521	.6552	.4535	.6566	.4550	.6580
85	.4564	.6594	.4579	.6607	.4593	.6621	.4608	.6635	.4622	.6649	.4637	.6662
86	.4651	.6676	.4666	.6689	.4680	.6703	.4695	.6716	.4709	.6730	.4724	.6743
87	.4738	.6756	.4753	.6770	.4767	.6783	.4782	.6796	.4796	.6809	.4811	.6822
88	.4826	.6835	.4840	.6848	.4855	.6862	.4869	.6875	.4884	.6887	.4898	.6900
89	.4913	.6913	.4937	.6926	.4942	.6939	.4956	.6952	.4971	.6964	.4985	.6977
90	.5000	.6990	.5015	.7002	.5029	.7015	.5044	.7027	.5058	.7040	.5073	.7052
91	.5087	.7065	.5102	.7077	.5116	.7090	.5131	.7102	.5145	.7114	.5160	.7126
92	.5174	.7139	.5189	.7151	.5204	.7163	.5218	.7175	.5233	.7187	.5247	.7199
93	.5262	.7211	.5276	.7223	.5291	.7235	.5305	.7247	.5320	.7259	.5334	.7271
94	.5349	.7283	.5363	.7294	.5378	.7306	.5392	.7318	.5407	.7329	.5421	.7341
95	.5436	.7353	.5450	.7364	.5465	.7376	.5479	.7387	.5494	.7399	.5508	.7410
96	.5523	.7421	.5537	.7433	.5552	.7444	.5566	.7455	.5580	.7467	.5595	.7478
97	.5609	.7489	.5624	.7500	.5638	.7511	.5653	.7523	.5667	.7534	.5682	.7545
98	.5696	.7556	.5710	.7567	.5725	.7577	.5739	.7588	.5753	.7599	.5768	.7610
99	.5782	.7621	.5797	.7632	.5811	.7642	.5825	.7653	.5840	.7664	.5854	.7674
100	.5868	.7685	.5883	.7696	.5897	.7706	.5911	.7717	.5925	.7727	.5940	.7738
101	.5954	.7748	.5968	.7759	.5983	.7769	.5997	.7779	.6011	.7790	.6025	.7800
102	.6040	.7810	.6054	.7820	.6068	.7830	.6082	.7841	.6096	.7851	.6111	.7861
103	.6125	.7871	.6139	.7881	.6153	.7891	.6167	.7901	.6181	.7911	.6195	.7921
104	.6210	.7931	.6224	.7940	.6238	.7950	.6252	.7960	.6266	.7970	.6280	.7980
105	.6294	.7989	.6308	.7999	.6322	.8009	.6336	.8018	.6350	.8028	.6364	.8037
106	.6378	.8047	.6392	.8056	.6406	.8066	.6420	.8075	.6434	.8085	.6448	.8094
107	.6462	.8104	.6476	.8113	.6490	.8122	.6504	.8131	.6517	.8141	.6531	.8150
108	.6545	.8159	.6559	.8168	.6573	.8177	.6587	.8187	.6600	.8196	.6614	.8205
109	.6628	.8214	.6642	.8223	.6655	.8232	.6669	.8241	.6683	.8250	.6696	.8258
110	.6710	.8267	.6724	.8276	.6737	.8285	.6751	.8294	.6765	.8302	.6778	.8311
111	.6792	.8320	.6805	.8329	.6819	.8337	.6833	.8346	.6846	.8354	.6860	.8363
112	.6873	.8371	.6887	.8380	.6900	.8388	.6913	.8397	.6927	.8405	.6940	.8414
113	.6954	.8422	.6967	.8430	.6980	.8439	.6994	.8447	.7007	.8455	.7020	.8464
114	.7034	.8472	.7047	.8480	.7060	.8488	.7073	.8496	.7087	.8504	.7100	.8513
115	.7113	.8521	.7126	.8529	.7139	.8537	.7153	.8545	.7166	.8553	.7179	.8561
116	.7192	.8568	.7205	.8576	.7218	.8584	.7231	.8592	.7244	.8600	.7257	.8608
117	.7270	.8615	.7283	.8623	.7296	.8631	.7309	.8638	.7392	.8646	.7335	.8654
118	.7347	.8661	.7360	.8669	.7373	.8676	.7386	.8684	.7399	.8691	.7411	.8699
119	.7424	.8706	.7437	.8714	.7449	.8721	.7462	.8729	.7475	.8736	.7487	.8743

Values and Logarithms of Haversines

[Characteristics of Logarithms omitted — determine by rule from the value]

°	0′		10′		20′		30′		40′		50′	
	Value	Log_{10}	Value	Log_{10}	Value	Log_{10}	Value	Log_{10}	Value	Log_{10}	Value	Log_{10}
120	.7500	.8751	.7513	.8758	.7525	.8765	.7538	.8772	.7550	.8780	.7563	.8787
121	.7575	.8794	.7588	.8801	.7600	.8808	.7612	.8815	.7625	.8822	.7637	.8829
122	.7650	.8836	.7662	.8843	.7674	.8850	.7686	.8857	.7699	.8864	.7711	.8871
123	.7723	.8878	.7735	.8885	.7748	.8892	.7760	.8898	.7772	.8905	.7784	.8912
124	.7796	.8919	.7808	.8925	.7820	.8932	.7832	.8939	.7844	.8945	.7856	.8952
125	.7868	.8959	.7880	.8965	.7892	.8972	.7904	.8978	.7915	.8985	.7927	.8991
126	.7939	.8998	.7951	.9004	.7962	.9010	.7974	.9017	.7986	.9023	.7997	.9030
127	.8009	.9036	.8021	.9042	.8032	.9048	.8044	.9055	.8055	.9061	.8067	.9067
128	.8078	.9073	.8090	.9079	.8101	.9085	.8113	.9092	.8124	.9098	.8135	.9104
129	.8147	.9110	.8158	.9116	.8169	.9122	.8180	.9128	.8192	.9134	.8203	.9140
130	.8214	.9146	.8225	.9151	.8236	.9157	.8247	.9163	.8258	.9169	.8269	.9175
131	.8280	.9180	.8291	.9186	.8302	.9192	.8313	.9198	.8324	.9203	.8335	.9209
132	.8346	.9215	.8356	.9220	.8367	.9226	.8378	.9231	.8389	.9237	.8399	.9242
133	.8410	.9248	.8421	.9253	.8431	.9259	.8442	.9264	.8452	.9270	.8463	.9275
134	.8473	.9281	.8484	.9286	.8494	.9291	.8501	.9297	.8515	.9302	.8525	.9307
135	.8536	.9312	.8546	.9318	.8556	.9323	.8566	.9328	.8576	.9333	.8587	.9338
136	.8597	.9343	.8607	.9348	.8617	.9353	.8627	.9359	.8637	.9364	.8647	.9369
137	.8657	.9374	.8667	.9379	.8677	.9383	.8686	.9388	.8696	.9393	.8706	.9398
138	.8716	.9403	.8725	.9408	.8735	.9413	.8745	.9417	.8754	.9422	.8764	.9427
139	.8774	.9432	.8783	.9436	.8793	.9441	.8802	.9446	.8811	.9450	.8821	.9455
140	.8830	.9460	.8840	.9464	.8849	.9469	.8858	.9473	.8867	.9478	.8877	.9482
141	.8886	.9487	.8895	.9491	.8904	.9496	.8913	.9500	.8922	.9505	.8931	.9509
142	.8940	.9513	.8949	.9518	.8958	.9522	.8967	.9526	.8976	.9531	.8984	.9535
143	.8993	.9539	.9002	.9543	.9011	.9548	.9019	.9552	.9028	.9556	.9037	.9560
144	.9045	.9564	.9054	.9568	.9062	.9572	.9071	.9576	.9079	.9580	.9087	.9584
145	.9096	.9588	.9104	.9592	.9112	.9596	.9121	.9600	.9129	.9604	.9137	.9608
146	.9145	.9612	.9153	.9616	.9161	.9620	.9169	.9623	.9177	.9627	.9185	.9631
147	.9193	.9635	.9201	.9638	.9209	.9642	.9217	.9646	.9225	.9650	.9233	.9653
148	.9240	.9657	.9248	.9660	.9256	.9664	.9263	.9668	.9271	.9671	.9278	.9675
149	.9286	.9678	.9293	.9682	.9301	.9685	.9308	.9689	.9316	.9692	.9323	.9695
150	.9330	.9699	.9337	.9702	.9345	.9706	.9352	.9709	.9359	.9712	.9366	.9716
151	.9373	.9719	.9380	.9722	.9387	.9725	.9394	.9729	.9401	.9732	.9408	.9735
152	.9415	.9738	.9422	.9741	.9428	.9744	.9435	.9747	.9442	.9751	.9448	.9754
153	.9455	.9757	.9462	.9760	.9468	.9763	.9475	.9766	.9481	.9769	.9488	.9772
154	.9494	.9774	.9500	.9777	.9507	.9780	.9513	.9783	.9519	.9786	.9525	.9789
155	.9532	.9792	.9538	.9794	.9544	.9797	.9550	.9800	.9556	.9803	.9562	.9805
156	.9568	.9808	.9574	.9811	.9579	.9813	.9585	.9816	.9591	.9819	.9597	.9821
157	.9603	.9824	.9608	.9826	.9614	.9829	.9619	.9831	.9625	.9834	.9630	.9836
158	.9636	.9839	.9641	.9841	.9647	.9844	.9652	.9846	.9657	.9849	.9663	.9851
159	.9668	.9853	.9673	.9856	.9678	.9858	.9683	.9860	.9688	.9863	.9693	.9865
160	.9698	.9867	.9703	.9869	.9708	.9871	.9713	.9874	.9718	.9876	.9723	.9878
161	.9728	.9880	.9732	.9882	.9737	.9884	.9742	.9886	.9746	.9888	.9751	.9890
162	.9755	.9892	.9760	.9894	.9764	.9896	.9769	.9898	.9773	.9900	.9777	.9902
163	.9782	.9904	.9786	.9906	.9790	.9908	.9794	.9910	.9798	.9911	.9802	.9913
164	.9806	.9915	.9810	.9917	.9814	.9919	.9818	.9920	.9822	.9922	.9826	.9923
165	.9830	.9925	.9833	.9927	.9837	.9929	.9841	.9930	.9844	.9932	.9848	.9933
166	.9851	.9935	.9855	.9937	.9858	.9938	.9862	.9940	.9865	.9941	.9869	.9943
167	.9872	.9944	.9875	.9945	.9878	.9947	.9881	.9948	.9885	.9950	.9888	.9951
168	.9891	.9952	.9894	.9954	.9897	.9955	.9900	.9956	.9903	.9957	.9905	.9959
169	.9908	.9960	.9911	.9961	.9914	.9962	.9916	.9963	.9919	.9965	.9921	.9966
170	.9924	.9967	.9927	.9968	.9929	.9969	.9931	.9970	.9934	.9971	.9936	.9972
171	.9938	.9973	.9941	.9974	.9943	.9975	.9945	.9976	.9947	.9977	.9949	.9978
172	.9951	.9979	.9953	.9980	.9955	.9981	.9957	.9981	.9959	.9982	.9961	.9983
173	.9963	.9984	.9964	.9984	.9966	.9985	.9968	.9986	.9969	.9987	.9971	.9987
174	.9973	.9988	.9974	.9988	.9976	.9989	.9977	.9990	.9978	.9991	.9980	.9991
175	.9981	.9992	.9982	.9992	.9983	.9993	.9985	.9993	.9986	.9994	.9987	.9994
176	.9988	.9995	.9989	.9995	.9990	.9996	.9991	.9996	.9992	.9996	.9992	.9997
177	.9993	.9997	.9994	.9997	.9995	.9998	.9995	.9998	.9996	.9998	.9996	.9998
178	.9997	.9999	.9997	.9999	.9998	.9999	.9998	.9999	.9999	.9999	.9999	.9999
179	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.0000	1.0000	.0000

Table XI — Factor Table — Logarithms of Primes

If N is prime, its logarithm is given. If N is not prime, its factors are given.

N	*1*	*3*	*7*	*9*
10	0043213738	0128372247	0293837777	0374264979
11	$3\cdot 37$	0530784435	$3^2\cdot 13$	$7\cdot 17$
12	11^2	$3\cdot 41$	1038037210	$3\cdot 43$
13	1172712957	$7\cdot 19$	1367205672	1430148003
14	$3\cdot 47$	$11\cdot 13$	$3\cdot 7^2$	1731862684
15	1789769473	$3^2\cdot 17$	1958996524	$3\cdot 53$
16	$7\cdot 23$	2121876044	2227164711	13^2
17	$3^2\cdot 19$	2380461031	$3\cdot 59$	2528530310
18	2576785749	$3\cdot 61$	$11\cdot 17$	$3^3\cdot 7$
19	2810333672	2855573090	2944662262	2988530764
20	$3\cdot 67$	$7\cdot 29$	$3^2\cdot 23$	$11\cdot 19$
21	3242824553	$3\cdot 71$	$7\cdot 31$	$3\cdot 73$
22	$13\cdot 17$	3483048630	3560258572	3598354823
23	$3\cdot 7\cdot 11$	3673559210	$3\cdot 79$	3783979009
24	3820170426	3^5	$13\cdot 19$	$3\cdot 83$
25	3996737215	$11\cdot 23$	4099331233	$7\cdot 37$
26	$3^2\cdot 29$	4199557485	$3\cdot 89$	4297522800
27	4329692909	$3\cdot 7\cdot 13$	4424797691	$3^2\cdot 31$
28	4487063199	4517864355	$7\cdot 41$	17^2
29	$3\cdot 97$	4668676204	$3^3\cdot 11$	$13\cdot 23$
30	$7\cdot 43$	$3\cdot 101$	4871383755	$3\cdot 103$
31	4927603890	4955443375	5010592622	$11\cdot 29$
32	$3\cdot 107$	$17\cdot 19$	$3\cdot 109$	$7\cdot 47$
33	5198279938	$3^2\cdot 37$	5276299009	$3\cdot 113$
34	$11\cdot 31$	7^3	5403294748	5428254270
35	$3^3\cdot 13$	5477747054	$3\cdot 7\cdot 17$	5550944486
36	19^2	$3\cdot 11^2$	5646660643	$3^2\cdot 41$
37	$7\cdot 53$	5717088318	$13\cdot 29$	5786392100
38	$3\cdot 127$	5831987740	$3^2\cdot 43$	5899496013
39	$17\cdot 23$	$3\cdot 131$	5987905068	$3\cdot 7\cdot 19$
40	6031443726	$13\cdot 31$	$11\cdot 37$	6117233080
41	$3\cdot 137$	$7\cdot 59$	$3\cdot 139$	6222140230
42	6242820958	$3^2\cdot 47$	$7\cdot 61$	$3\cdot 11\cdot 13$
43	6344772702	6364878964	$19\cdot 23$	6424645202
44	$3^2\cdot 7^2$	6464037262	$3\cdot 149$	6522463410
45	$11\cdot 41$	$3\cdot 151$	6599162001	$3^3\cdot 17$
46	6637009254	6655809910	6693168806	$7\cdot 67$
47	$3\cdot 157$	$11\cdot 43$	$3^2\cdot 53$	6803355134
48	$13\cdot 37$	$3\cdot 7\cdot 23$	6875289612	$3\cdot 163$
49	6910814921	$17\cdot 29$	$7\cdot 71$	6981005456
50	$3\cdot 167$	7015679851	$3\cdot 13^2$	7067177823
51	$7\cdot 73$	$3^3\cdot 19$	$11\cdot 47$	$3\cdot 173$
52	7168377233	7185016889	$17\cdot 31$	23^2
53	$3^2\cdot 59$	$13\cdot 41$	$3\cdot 179$	$7^2\cdot 11$
54	7331972651	$3\cdot 181$	7379873263	$3^2\cdot 61$
55	$19\cdot 29$	$7\cdot 79$	7458551952	$13\cdot 43$
56	$3\cdot 11\cdot 17$	7505083949	$3^4\cdot 7$	7551122664
57	7566361082	$3\cdot 191$	7611758132	$3\cdot 193$
58	$7\cdot 83$	$11\cdot 53$	7686381012	$19\cdot 31$
59	$3\cdot 197$	7730546934	$3\cdot 199$	7774268224
60	7788744720	$3^2\cdot 67$	7831886911	$3\cdot 7\cdot 29$
61	$13\cdot 47$	7874604745	7902851640	7916906490
62	$3^3\cdot 23$	$7\cdot 89$	$3\cdot 11\cdot 19$	$17\cdot 37$
63	8000293592	$3\cdot 211$	$7^2\cdot 13$	$3^2\cdot 71$
64	8068580295	8082109729	8109042807	$11\cdot 59$
65	$3\cdot 7\cdot 31$	8149131813	$3^2\cdot 73$	8188854146
66	8202014595	$3\cdot 13\cdot 17$	$23\cdot 29$	$3\cdot 223$
67	$11\cdot 61$	8280150642	8305886687	$7\cdot 97$
68	$3\cdot 227$	8344207037	$3\cdot 229$	$13\cdot 53$
69	8394780474	$3^2\cdot 7\cdot 11$	$17\cdot 41$	$3\cdot 233$

N	*Log N*
2	301029995664
3	477121254720
5	698970004336
7	845098040014
11	041392685158
13	113943352307
17	230448921378
19	278753600953
23	361727836018
29	462396997899
31	491361693834
37	568201724067
41	612783856720
43	633468455580
47	672097857936
53	724275869601
59	770852011642
61	785329835011
67	826074802701
71	851258348719
73	863322860120
79	897627091290
83	919078092376
89	949390006645
97	986771734266
1301	1142772966
1303	1149444157
1307	1162755876
1319	1202447955
1321	1209028176
1327	1228709229
1361	1338581252
1367	1357685146
1373	1376705372
1381	1401936786
1399	1458177145
1409	1489109931
1423	1532049001
1427	1544239731
1429	1550322288
1433	1562461904
1439	1580607939
1447	1604685311
1451	1616674124
1453	1622656143
1459	1640552919
1471	1676126727
1481	1705550585
1483	1711411510
1487	1723109685
1489	1728946978
1493	1740598077
1499	1758016328
1511	1792644643
1523	1826999033
1531	1849751907
1543	1883659261
1549	1900514178
1553	1911714557
1559	1928461152

Factor Table — Logarithms of Primes

If *N* is a prime, its logarithm is given. If *N* is not a prime, its factors are given.

N	*1*	*3*	*7*	*9*	*N*	*Log N*
70	8457180180	19·37	7·101	8506462352	1567	1950689965
71	$3^{2}\cdot 79$	23·31	3·239	8567288904	1571	1961761850
72	7·103	3·241	8615344109	3^{6}	1579	1983821300
73	17·43	8651039746	11·67	8686444384	1583	1994809149
74	3·13·19	8709888138	$3^{2}\cdot 83$	7·107	1597	2033049161
75	8756399370	3·251	8790958795	3·11·23	1601	2043913319
76	8813846568	7·109	13·59	8859263398	1607	2060158768
77	3·257	8881794939	3 7·37	19·41	1609	2065560441
78	11·71	$3^{3}\cdot 29$	8959747324	3·263	1613	2076343674
79	7·113	13·61	9014583214	17·47	1619	2092468488
80	$3^{2}\cdot 89$	11·73	3·269	9079485216	1621	2097830148
81	9090208542	3·271	19·43	$3^{2}\cdot 7\cdot 13$	1627	2113875529
82	9143431571	9153998352	9175055096	9185545306	1637	2140486794
83	3·277	$7^{2}\cdot 17$	$3^{3}\cdot 31$	9237619608	1657	2193225084
84	29^{2}	3·281	$7\cdot 11^{2}$	3·283	1663	2208922492
85	23·37	9309490312	9329808219	9339931638	1667	2219355998
86	3·7·41	9360107957	$3\cdot 17^{2}$	11·79	1669	2224563367
87	13·67	$3^{2}\cdot 97$	9429995934	3·293	1693	2286569581
88	9449759084	9459607036	9479236198	7·127	1697	2296818423
89	$3^{4}\cdot 11$	19·47	3·13·23	29·31	1699	2301933789
90	17·53	3·7·43	9576072871	$3^{2}\cdot 101$	1709	2327420627
91	9595183770	11·83	7·131	9633155114	1721	2357808703
92	3·307	13·71	$3^{2}\cdot 103$	9680157140	1723	2362852774
93	$7^{2}\cdot 19$	3·311	9717395909	3·313	1733	2387985627
94	9735896234	23·41	9763499790	13·73	1741	2407987711
95	3·317	9790929006	3·11·29	7·137	1747	2422929050
96	31^{2}	$3^{2}\cdot 107$	9854264741	3·17·19	1753	2437819161
97	9872192299	7·139	9898945637	11·89	1759	2452658395
98	$3^{2}\cdot 109$	9925535178	3·7·47	23·43	1777	2496874278
99	9960736545	3·331	9986951583	$3^{3}\cdot 37$	1783	2511513432
100	7·11·13	17·59	19·53	0038911662	1787	2521245525
101	3·337	0056094454	$3^{2}\cdot 113$	0081741840	1789	2526103406
102	0090257421	3·11·31	13·79	$3\cdot 7^{3}$	1801	2555137128
103	0132586653	0141003215	17·61	0166155476	1811	2579184503
104	3·347	7·149	3·349	0207754882	1823	2607866687
105	0216027160	$3^{4}\cdot 13$	7·151	3·353	1831	2626883443
106	0257153839	0265332645	11·97	0289777052	1847	2664668954
107	$3^{2}\cdot 7\cdot 17$	29·37	3·359	13·83	1861	2697463731
108	23·47	$3\cdot 19^{2}$	0362295441	$3^{2}\cdot 11^{2}$	1867	2711443179
109	0378247506	0386201619	0402066276	7·157	1871	2720737875
110	3·367	0425755124	$3^{3}\cdot 41$	0449315461	1873	2725377774
111	11·101	3·7·53	0480531731	3·373	1877	2734642726
112	19·59	0503797563	$7^{2}\cdot 23$	0526939419	1879	2739267801
113	3·13·29	11·103	3·379	17·67	1889	2762319579
114	7·163	$3^{2}\cdot 127$	31·37	3·383	1901	2789821169
115	0610753236	0618293073	13·89	19·61	1907	2803506930
116	$3^{3}\cdot 43$	0655797147	3·389	7·167	1913	2817149700
117	0685568951	3·17·23	11·107	$3^{2}\cdot 131$	1931	2857822738
118	0722498976	$7\cdot 13^{2}$	0744507190	29·41	1933	2862318540
119	3·397	0766404437	$3^{2}\cdot 7\cdot 19$	11·109	1949	2898118391
120	0795430074	3·401	17·71	3·13·31	1951	2902572694
121	7·173	0838608009	0852905782	23·53	1973	2961270853
122	3·11·37	0874264570	3·409	0895518829	1979	2964457942
123	0902580529	$3^{2}\cdot 137$	0923696996	3·7·59	1987	2981978671
124	17·73	11·113	29·43	0965624384	1993	2995072987
125	$3^{2}\cdot 139$	7·179	3·419	1000257301	1997	3003780649
126	13·97	3·421	7·181	$3^{3}\cdot 47$	1999	3008127941
127	31·41	19·67	1061908973	1068705445	2003	3016809493
128	3·7·61	1082266564	$3^{2}\cdot 11\cdot 13$	1102529174	2011	3034120706
129	1109262423	3·431	1129399761	3·433	2017	3047058982

Table XII a — Compound Interest: $(1+r)^n$

AMOUNT OF ONE DOLLAR PRINCIPAL AT COMPOUND INTEREST AFTER n YEARS

n	2 %	$2\frac{1}{2}$%	3 %	$3\frac{1}{2}$%	4 %	$4\frac{1}{2}$%	5 %	6 %	7 %
1	1.0200	1.0250	1.0300	1.0350	1.0400	1.0450	1.0500	1.0600	1.0700
2	1.0404	1.0506	1.0609	1.0712	1.0816	1.0920	1.1025	1.1236	1.1449
3	1.0612	1.0769	1.0927	1.1087	1.1249	1.1412	1.1576	1.1910	1.2250
4	1.0824	1.1038	1.1255	1.1475	1.1699	1.1925	1.2155	1.2625	1.3108
5	1.1041	1.1314	1.1593	1.1877	1.2167	1.2462	1.2763	1.3382	1.4026
6	1.1262	1.1597	1.1941	1.2293	1.2653	1.3023	1.3401	1.4185	1.5007
7	1.1487	1.1887	1.2299	1.2723	1.3159	1.3609	1.4071	1.5036	1.6058
8	1.1717	1.2184	1.2668	1.3168	1.3686	1.4221	1.4775	1.5938	1.7182
9	1.1951	1.2489	1.3048	1.3629	1.4233	1.4861	1.5513	1.6895	1.8385
10	1.2190	1.2801	1.3439	1.4106	1.4802	1.5530	1.6289	1.7908	1.9672
11	1.2434	1.3121	1.3842	1.4600	1.5395	1.6229	1.7103	1.8983	2.1049
12	1.2682	1.3449	1.4258	1.5111	1.6010	1.6959	1.7959	2.0122	2.2522
13	1.2936	1.3785	1.4685	1.5640	1.6651	1.7722	1.8856	2.1329	2.4098
14	1.3195	1.4130	1.5126	1.6187	1.7317	1.8519	1.9799	2.2609	2.5785
15	1.3459	1.4483	1.5580	1.6753	1.8009	1.9353	2.0789	2.3966	2.7590
16	1.3728	1.4845	1.6047	1.7340	1.8730	2.0224	2.1829	2.5404	2.9522
17	1.4002	1.5216	1.6528	1.7947	1.9479	2.1134	2.2920	2.6928	3.1588
18	1.4282	1.5597	1.7024	1.8575	2.0258	2.2085	2.4066	2.8543	3.3799
19	1.4568	1.5987	1.7535	1.9225	2.1068	2.3079	2.5270	3.0256	3.6165
20	1.4859	1.6386	1.8061	1.9898	2.1911	2.4117	2.6533	3.2071	3.8697
21	1.5157	1.6796	1.8603	2.0594	2.2788	2.5202	2.7860	3.3996	4.1406
22	1.5460	1.7216	1.9161	2.1315	2.3699	2.6337	2.9253	3.6035	4.4304
23	1.5769	1.7646	1.9736	2.2061	2.4647	2.7522	3.0715	3.8197	4.7405
24	1.6084	1.8087	2.0328	2.2833	2.5633	2.8760	3.2251	4.0489	5.0724
25	1.6406	1.8539	2.0938	2.3632	2.6658	3.0054	3.3864	4.2919	5.4274
26	1.6734	1.9003	2.1566	2.4460	2.7725	3.1407	3.5557	4.5494	5.8074
27	1.7069	1.9478	2.2213	2.5316	2.8834	3.2820	3.7335	4.8223	6.2139
28	1.7410	1.9965	2.2879	2.6202	2.9987	3.4297	3.9201	5.1117	6.6488
29	1.7758	2.0464	2.3566	2.7119	3.1187	3.5840	4.1161	5.4184	7.1143
30	1.8114	2.0976	2.4273	2.8068	3.2434	3.7453	4.3219	5.7435	7.6123
31	1.8476	2.1500	2.5001	2.9050	3.3731	3.9139	4.5380	6.0881	8.1451
32	1.8845	2.2038	2.5751	3.0067	3.5081	4.0900	4.7649	6.4534	8.7153
33	1.9222	2.2589	2.6523	3.1119	3.6484	4.2740	5.0032	6.8406	9.3253
34	1.9607	2.3153	2.7319	3.2209	3.7943	4.4664	5.2533	7.2510	9.9781
35	1.9999	2.3732	2.8139	3.3336	3.9461	4.6673	5.5160	7.6861	10.6766
36	2.0399	2.4325	2.8983	3.4503	4.1039	4.8774	5.7918	8.1473	11.4239
37	2.0807	2.4933	2.9852	3.5710	4.2681	5.0969	6.0814	8.6361	12.2236
38	2.1223	2.5557	3.0748	3.6960	4.4388	5.3262	6.3855	9.1543	13.0793
39	2.1647	2.6196	3.1670	3.8254	4.6164	5.5659	6.7048	9.7035	13.9948
40	2.2080	2.6851	3.2620	3.9593	4.8010	5.8164	7.0400	10.2857	14.9745
41	2.2522	2.7522	3.3599	4.0978	4.9931	6.0781	7.3920	10.9029	16.0227
42	2.2972	2.8210	3.4607	4.2413	5.1928	6.3516	7.7616	11.5570	17.1443
43	2.3432	2.8915	3.5645	4.3897	5.4005	6.6374	8.1497	12.2505	18.3444
44	2.3901	2.9638	3.6715	4.5433	5.6165	6.9361	8.5572	12.9855	19.6285
45	2.4379	3.0379	3.7816	4.7024	5.8412	7.2482	8.9850	13.7646	21.0025
46	2.4866	3.1139	3.8950	4.8669	6.0748	7.5744	9.4343	14.5905	22.4726
47	2.5363	3.1917	4.0119	5.0373	6.3178	7.9153	9.9060	15.4659	24.0457
48	2.5871	3.2715	4.1323	5.2136	6.5705	8.2715	10.4013	16.3939	25.7289
49	2.6388	3.3533	4.2562	5.3961	6.8333	8.6437	10.9213	17.3775	27.5299
50	2.6916	3.4371	4.3839	5.5849	7.1067	9.0326	11.4674	18.4202	29.4570

Table XII *b* — Compound Discount: $1/(1+r)^n$

PRESENT VALUE OF ONE DOLLAR DUE AT THE END OF *n* YEARS

n	2 %	$2\frac{1}{2}$%	3 %	$3\frac{1}{2}$%	4 %	$4\frac{1}{2}$%	5 %	6 %	7 %
1	.98039	.97561	.97087	.96618	.96154	.95694	.95238	.94340	.93458
2	.96117	.95181	.94260	.93351	.92456	.91573	.90703	.89000	.87344
3	.94232	.92860	.91514	.90194	.88900	.87630	.86384	.83962	.81630
4	.92385	.90595	.88849	.87144	.85480	.83856	.82270	.79209	.76290
5	.90573	.88385	.86261	.84197	.82193	.80245	.78353	.74726	.71299
6	.88797	.86230	.83748	.81350	.79031	.76790	.74622	.70496	.66634
7	.87056	.84127	.81309	.78599	.75992	.73483	.71068	.66506	.62275
8	.85349	.82075	.78941	.75941	.73069	.70319	.67684	.62741	.58201
9	.83676	.80073	.76642	.73373	.70259	.67290	.64461	.59190	.54393
10	.82035	.78120	.74409	.70892	.67556	.64393	.61391	.55839	.50835
11	.80426	.76214	.72242	.68495	.64958	.61620	.58468	.52679	.47509
12	.78849	.74356	.70138	.66178	.62460	.58966	.55684	.49697	.44401
13	.77303	.72542	.68095	.63940	.60057	.56427	.53032	.46884	.41496
14	.75788	.70773	.66112	.61778	.57748	.53997	.50507	.44230	.38782
15	.74301	.69047	.64186	.59689	.55526	.51672	.48102	.41727	.36245
16	.72845	.67362	.62317	.57671	.53391	.49447	.45811	.39365	.33873
17	.71416	.65720	.60502	.55720	.51337	.47318	.43630	.37136	.31657
18	.70016	.64117	.58739	.53836	.49363	.45280	.41552	.35034	.29586
19	.68643	.62553	.57029	.52016	.47464	.43330	.39573	.33051	.27651
20	.67297	.61027	.55368	.50257	.45639	.41464	.37689	.31180	.25842
21	.65978	.59539	.53755	.48557	.43883	.39679	.35894	.29416	.24151
22	.64684	.58086	.52189	.46915	.42196	.37970	.34185	.27751	.22571
23	.63416	.56670	.50669	.45329	.40573	.36335	.32557	.26180	.21095
24	.62172	.55288	.49193	.43796	.39012	.34770	.31007	.24698	.19715
25	.60953	.53939	.47761	.42315	.37512	.33273	.29530	.23300	.18425
26	.59758	.52623	.46369	.40884	.36069	.31840	.28124	.21981	.17220
27	.58586	.51340	.45019	.39501	.34682	.30469	.26785	.20737	.16093
28	.57437	.50088	.43708	.38165	.33348	.29157	.25509	.19563	.15040
29	.56311	.48866	.42435	.36875	.32065	.27902	.24295	.18456	.14056
30	.55207	.47674	.41199	.35628	.30832	.26700	.23138	.17411	.13137
31	.54125	.46511	.39999	.34423	.29646	.25550	.22036	.16425	.12277
32	.53063	.45377	.38834	.33259	.28506	.24450	.20987	.15496	.11474
33	.52023	.44270	.37703	.32134	.27409	.23397	.19987	.14619	.10723
34	.51003	.43191	.36604	.31048	.26355	.22390	.19035	.13791	.10022
35	.50003	.42137	.35538	.29998	.25342	.21425	.18129	.13011	.09366
36	.49022	.41109	.34503	.28983	.24367	.20503	.17266	.12274	.08754
37	.48061	.40107	.33498	.28003	.23430	.19620	.16444	.11580	.08181
38	.47119	.39128	.32523	.27056	.22529	.18775	.15661	.10924	.07646
39	.46195	.38174	.31575	.26141	.21662	.17967	.14915	.10306	.07146
40	.45289	.37243	.30656	.25257	.20829	.17193	.14205	.09722	.06678
41	.44401	.36335	.29763	.24403	.20028	.16453	.13528	.09172	.06241
42	.43530	.35448	.28896	.23578	.19257	.15744	.12884	.08653	.05833
43	.42677	.34584	.28054	.22781	.18517	.15066	.12270	.08163	.05451
44	.41840	.33740	.27237	.22010	.17805	.14417	.11686	.07701	.05095
45	.41020	.32917	.26444	.21266	.17120	.13796	.11130	.07265	.04761
46	.40215	.32115	.25674	.20547	.16461	.13202	.10600	.06854	.04450
47	.39427	.31331	.24926	.19852	.15828	.12634	.10095	.06466	.04159
48	.38654	.30567	.24200	.19181	.15219	.12090	.09614	.06100	.03887
49	.37896	.29822	.23495	.18532	.14634	.11569	.09156	.05755	.03632
50	.37153	.29094	.22811	.17905	.14071	.11071	.08720	.05429	.03395

Table XII *c* — Amount of an Annuity

AMOUNT OF AN ANNUITY OF ONE DOLLAR PER YEAR AFTER *n* YEARS

n	2 %	2½ %	3 %	3½ %	4 %	4½ %	5 %	6 %	7 %
1	1.0200	1.0250	1.0300	1.0350	1.0400	1.0450	1.0500	1.0600	1.0700
2	2.0604	2.0756	2.0909	2.1062	2.1216	2.1370	2.1525	2.1836	2.2149
3	3.1216	3.1525	3.1836	3.2149	3.2465	3.2782	3.3101	3.3746	3.4399
4	4.2040	4.2563	4.3091	4.3625	4.4163	4.4707	4.5256	4.6371	4.7507
5	5.3081	5.3877	5.4684	5.5502	5.6330	5.7169	5.8019	5.9753	6.1533
6	6.4343	6.5474	6.6625	6.7794	6.8983	7.0192	7.1420	7.3938	7.6540
7	7.5830	7.7361	7.8923	8.0517	8.2142	8.3800	8.5491	8.8975	9.2598
8	8.7546	8.9545	9.1591	9.3685	9.5828	9.8021	10.0266	10.4913	10.9780
9	9.9497	10.2034	10.4639	10.7314	11.0061	11.2882	11.5779	12.1808	12.8164
10	11.1687	11.4835	11.8078	12.1420	12.4864	12.8412	13.2068	13.9716	14.7836
11	12.4121	12.7956	13.1920	13.6020	14.0258	14.4640	14.9171	15.8699	16.8885
12	13.6803	14.1404	14.6178	15.1130	15.6268	16.1599	16.7130	17.8821	19.1406
13	14.9739	15.5190	16.0863	16.6770	17.2919	17.9321	18.5986	20.0151	21.5505
14	16.2934	16.9319	17.5989	18.2957	19.0236	19.7841	20.5786	22.2760	24.1290
15	17.6393	18.3802	19.1569	19.9710	20.8245	21.7193	22.6575	24.6725	26.8881
16	19.0121	19.8647	20.7616	21.7050	22.6975	23.7417	24.8404	27.2129	29.8402
17	20.4123	21.3863	22.4144	23.4997	24.6454	25.8551	27.1324	29.9057	32.9990
18	21.8406	22.9460	24.1169	25.3572	26.6712	28.0636	29.5390	32.7600	36.3790
19	23.2974	24.5447	25.8704	27.2797	28.7781	30.3714	32.0660	35.7856	39.9955
20	24.7833	26.1833	27.6765	29.2695	30.9692	32.7831	34.7193	38.9927	43.8652
21	26.2990	27.8629	29.5368	31.3289	33.2480	35.3034	37.5052	42.3923	48.0057
22	27.8450	29.5844	31.4529	33.4604	35.6179	37.9370	40.4305	45.9958	52.4361
23	29.4219	31.3490	33.4265	35.6665	38.0826	40.6892	43.5020	49.8156	57.1767
24	31.0303	33.1578	35.4593	37.9499	40.6459	43.5652	46.7271	53.8645	62.2490
25	32.6709	35.0117	37.5530	40.3131	43.3117	46.5706	50.1135	58.1564	67.6765
26	34.3443	36.9120	39.7096	42.7591	46.0842	49.7113	53.6691	62.7058	73.4838
27	36.0512	38.8598	41.9309	45.2906	48.9676	52.9933	57.4026	67.5281	79.6977
28	37.7922	40.8563	44.2189	47.9108	51.9663	56.4230	61.3227	72.6398	86.3465
29	39.5681	42.9027	46.5754	50.6227	55.0849	60.0071	65.4388	78.0582	93.4608
30	41.3794	45.0003	49.0027	53.4295	58.3283	63.7524	69.7608	83.8017	101.0730
31	43.2270	47.1503	51.5028	56.3345	61.7015	67.6662	74.2988	89.8898	109.2182
32	45.1116	49.3540	54.0778	59.3412	65.2095	71.7562	79.0638	96.3432	117.9334
33	47.0338	51.6129	56.7302	62.4532	68.8579	76.0303	84.0670	103.1838	127.2588
34	48.9945	53.9282	59.4621	65.6740	72.6522	80.4966	89.3203	110.4348	137.2369
35	50.9944	56.3014	62.2759	69.0076	76.5983	85.1640	94.8363	118.1209	147.9135
36	53.0343	58.7339	65.1742	72.4579	80.7022	90.0413	100.6281	126.2681	159.3374
37	55.1149	61.2273	68.1594	76.0289	84.9703	95.1382	106.7095	134.9042	171.5610
38	57.2372	63.7830	71.2342	79.7249	89.4091	100.4644	113.0950	144.0585	184.6403
39	59.4020	66.4026	74.4013	83.5503	94.0255	106.0303	119.7998	153.7620	198.6351
40	61.6100	69.0876	77.6633	87.5095	98.8265	111.8467	126.8398	164.0477	213.6096
41	63.8622	71.8398	81.0232	91.6074	103.8196	117.9248	134.2318	174.9505	229.6322
42	66.1595	74.6608	84.4839	95.8486	109.0124	124.2764	141.9933	186.5076	246.7765
43	68.5027	77.5523	88.0484	100.2383	114.4129	130.9138	150.1430	198.7580	265.1209
44	70.8927	80.5161	91.7199	104.7817	120.0294	137.8500	158.7002	211.7435	284.7493
45	73.3306	83.5540	95.5015	109.4840	125.8706	145.0982	167.6852	225.5081	305.7518
46	75.8172	86.6679	99.3965	114.3510	131.9454	152.6726	177.1194	240.0986	328.2244
47	78.3535	89.8596	103.4084	119.3883	138.2632	160.5879	187.0254	255.5645	352.2701
48	80.9406	93.1311	107.5406	124.6018	144.8337	168.8594	197.4267	271.9584	377.9990
49	83.5794	96.4843	111.7969	129.9979	151.6671	177.5030	208.3480	289.3359	405.5289
50	86.2710	99.9215	116.1808	135.5828	158.7738	186.5357	219.8154	307.7561	434.9860

Table XII *d* — Present Value of an Annuity

PRESENT VALUE OF ONE DOLLAR PER YEAR FOR *n* YEARS

n	2 %	2½%	3 %	3½%	4 %	4½%	5 %	6 %	7 %
1	.9804	.9756	.9709	.9662	.9615	.9569	.9524	.9434	.9346
2	1.9416	1.9274	1.9135	1.8997	1.8861	1.8727	1.8594	1.8334	1.8080
3	2.8839	2.8560	2.8286	2.8016	2.7751	2.7490	2.7232	2.6730	2.6243
4	3.8077	3.7620	3.7171	3.6731	3.6299	3.5875	3.5460	3.4651	3.3872
5	4.7135	4.6458	4.5797	4.5151	4.4518	4.3900	4.3295	4.2124	4.1002
6	5.6014	5.5081	5.4172	5.3286	5.2421	5.1579	5.0757	4.9173	4.7665
7	6.4720	6.3494	6.2303	6.1145	6.0021	5.8927	5.7864	5.5824	5.3893
8	7.3255	7.1701	7.0197	6.8740	6.7327	6.5959	6.4632	6.2098	5.9713
9	8.1622	7.9709	7.7861	7.6077	7.4353	7.2688	7.1078	6.8017	6.5152
10	8.9826	8.7521	8.5302	8.3166	8.1109	7.9127	7.7217	7.3601	7.0236
11	9.7868	9.5142	9.2526	9.0016	8.7605	8.5289	8.3064	7.8869	7.4987
12	10.5753	10.2578	9.9540	9.6633	9.3851	9.1186	8.8633	8.3838	7.9427
13	11.3484	10.9832	10.6350	10.3027	9.9856	9.6829	9.3936	8.8527	8.3577
14	12.1062	11.6909	11.2961	10.9205	10.5631	10.2228	9.8986	9.2950	8.7455
15	12.8493	12.3814	11.9379	11.5174	11.1184	10.7395	10.3797	9.7122	9.1079
16	13.5777	13.0550	12.5611	12.0941	11.6523	11.2340	10.8378	10.1059	9.4466
17	14.2919	13.7122	13.1661	12.6513	12.1657	11.7072	11.2741	10.4773	9.7632
18	14.9920	14.3534	13.7535	13.1897	12.6593	12.1600	11.6896	10.8276	10.0591
19	15.6785	14.9789	14.3238	13.7098	13.1339	12.5933	12.0853	11.1581	10.3356
20	16.3514	15.5892	14.8775	14.2124	13.5903	13.0079	12.4622	11.4699	10.5940
21	17.0112	16.1845	15.4150	14.6980	14.0292	13.4047	12.8212	11.7641	10.8355
22	17.6580	16.7654	15.9369	15.1671	14.4511	13.7844	13.1630	12.0416	11.0612
23	18.2922	17.3321	16.4436	15.6204	14.8568	14.1478	13.4886	12.3034	11.2722
24	18.9139	17.8850	16.9355	16.0584	15.2470	14.4955	13.7986	12.5504	11.4693
25	19.5235	18.4244	17.4131	16.4815	15.6221	14.8282	14.0939	12.7834	11.6536
26	20.1210	18.9506	17.8768	16.8904	15.9828	15.1466	14.3752	13.0032	11.8258
27	20.7069	19.4640	18.3270	17.2854	16.3296	15.4513	14.6430	13.2105	11.9867
28	21.2813	19.9649	18.7641	17.6670	16.6631	15.7429	14.8981	13.4062	12.1371
29	21.8444	20.4535	19.1885	18.0358	16.9837	16.0219	15.1411	13.5907	12.2777
30	22.3965	20.9303	19.6004	18.3920	17.2920	16.2889	15.3725	13.7648	12.4090
31	22.9377	21.3954	20.0004	18.7363	17.5885	16.5444	15.5928	13.9291	12.5318
32	23.4683	21.8492	20.3888	19.0689	17.8736	16.7889	15.8027	14.0840	12.6466
33	23.9886	22.2919	20.7658	19.3902	18.1476	17.0229	16.0025	14.2302	12.7538
34	24.4986	22.7238	21.1318	19.7007	18.4112	17.2468	16.1929	14.3681	12.8540
35	24.9986	23.1452	21.4872	20.0007	18.6646	17.4610	16.3742	14.4982	12.9477
36	25.4888	23.5563	21.8323	20.2905	18.9083	17.6660	16.5469	14.6210	13.0352
37	25.9695	23.9573	22.1672	20.5705	19.1426	17.8622	16.7113	14.7368	13.1170
38	26.4406	24.3486	22.4925	20.8411	19.3679	18.0500	16.8679	14.8460	13.1935
39	26.9026	24.7303	22.8082	21.1025	19.5845	18.2297	17.0170	14.9491	13.2649
40	27.3555	25.1028	23.1148	21.3551	19.7928	18.4016	17.1591	15.0463	13.3317
41	27.7995	25.4661	23.4124	21.5991	19.9931	18.5661	17.2944	15.1380	13.3941
42	28.2348	25.8206	23.7014	21.8349	20.1856	18.7236	17.4232	15.2245	13.4524
43	28.6616	26.1664	23.9819	22.0627	20.3708	18.8742	17.5459	15.3062	13.5070
44	29.0800	26.5038	24.2543	22.2828	20.5488	19.0184	17.6628	15.3832	13.5579
45	29.4902	26.8330	24.5187	22.4955	20.7200	19.1563	17.7741	15.4558	13.6055
46	29.8923	27.1542	24.7754	22.7009	20.8847	19.2884	17.8801	15.5244	13.6500
47	30.2866	27.4675	25.0247	22.8994	21.0429	19.4147	17.9810	15.5890	13.6910
48	30.6731	27.7732	25.2667	23.0912	21.1951	19.5356	18.0772	15.6500	13.7305
49	31.0521	28.0714	25.5017	23.2766	21.3415	19.6513	18.1687	15.7076	13.7668
50	31.4236	28.3623	25.7298	23.4556	21.4822	19.7620	18.2559	15.7619	13.8007

Table XII *e* — Logarithms for Interest Computations

r	$1+r$	$\log(1+r)$	r	$1+r$	$\log(1+r)$
½%	1.005	00216 60617 56508	5½%	1.055	02325 24596 33711
1%	1.010	00432 13737 82643	6%	1.060	02530 58652 64770
1½%	1.015	00646 60422 49232	6½%	1.065	02734 96077 74757
2%	1.020	00860 01717 61918	7%	1.070	02938 37776 85210
2½%	1.025	01072 38653 91773	7½%	1.075	03140 84642 51624
3%	1.030	01283 72247 05172	8%	1.080	03342 37554 86950
3½%	1.035	01494 03497 92937	8½%	1.085	03542 97381 84548
4%	1.040	01703 33392 98780	9%	1.090	03742 64979 40624
4½%	1.045	01911 62904 47073	9½%	1.095	03941 41191 76137
5%	1.050	02118 92990 69938	10%	1.100	04139 26851 58225

For Amount, A, of any principal, P, after n years: $A = P\ (1 + r)^n$

For present worth, P, of any amount, A, at the end of n years: $P = A \div (1 + r)^n$

To find logarithms and antilogarithms of A and P to many significant figures, use Table XI, p. 126, and Table I *a*, p. 20.

TABLE XII *f* — AMERICAN EXPERIENCE MORTALITY TABLE

Based on 100,000 living at age 10

At Age	Number Surviving	Deaths	At Age	Number Surviving	Deaths	At Age	Number Surviving	Deaths	At Age	Number Surviving	Deaths
10	100,000	749	**35**	81,822	732	**60**	57,917	1,546	**85**	5,485	1,292
11	99,251	746	**36**	81,090	737	**61**	56,371	1,628	**86**	4,193	1,114
12	98,505	743	**37**	80,353	742	**62**	54,743	1,713	**87**	3,079	933
13	97,762	740	**38**	79,611	749	**63**	53,030	1,800	**88**	2,146	744
14	97,022	737	**39**	78,862	756	**64**	51,230	1,889	**89**	1,402	555
15	96,285	735	**40**	78,106	765	**65**	49,341	1,980	**90**	847	385
16	95,550	732	**41**	77,341	774	**66**	47,361	2,070	**91**	462	246
17	94,818	729	**42**	76,567	785	**67**	45,291	2,158	**92**	216	137
18	94,089	727	**43**	75,782	797	**68**	43,133	2,243	**93**	79	58
19	93,362	725	**44**	74,985	812	**69**	40,890	2,321	**94**	21	18
20	92,637	723	**45**	74,173	828	**70**	38,569	2,391	**95**	3	3
21	91,914	722	**46**	73,345	848	**71**	36,178	2,448			
22	91,192	721	**47**	72,497	870	**72**	33,730	2,487			
23	90,471	720	**48**	71,627	896	**73**	31,243	2,505			
24	89,751	719	**49**	70,731	927	**74**	28,738	2,501			
25	89,032	718	**50**	69,804	962	**75**	26,237	2,476			
26	88,314	718	**51**	68,842	1,001	**76**	23,761	2,431			
27	87,596	718	**52**	67,841	1,044	**77**	21,330	2,369			
28	86,878	718	**53**	66,797	1,091	**78**	18,961	2,291			
29	86,160	719	**54**	65,706	1,143	**79**	16,670	2,196			
30	85,441	720	**55**	64,563	1,199	**80**	14,474	2,091			
31	84,721	721	**56**	63,364	1,260	**81**	12,383	1,964			
32	84,000	723	**57**	62,104	1,325	**82**	10,419	1,816			
33	83,277	726	**58**	60,779	1,394	**83**	8,603	1,648			
34	82,551	729	**59**	59,385	1,468	**84**	6,955	1,470			

LOGARITHMS OF IMPORTANT CONSTANTS

n = NUMBER	VALUE OF n	LOG$_{10}$ n
π	3.14159265	0.49714987
$1 \div \pi$	0.31830989	9.50285013
π^2	9.86960440	0.99429975
$\sqrt{\pi}$	1.77245385	0.24857494
e = Naperian Base	2.71828183	0.43429448
$M = \log_{10} e$	0.43429448	9.63778431
$1 \div M = \log_e 10$	2.30258509	0.36221569
$180 \div \pi$ = degrees in 1 radian	57.2957795	1.75812263
$\pi \div 180$ = radians in 1°	0.01745329	8.24187737
$\pi \div 10800$ = radians in 1′	0.0002908882	6.46372612
$\pi \div 648000$ = radians in 1″	0.000004848136811095	4.68557487
sin 1″	0.000004848136811076	4.68557487
tan 1″	0.000004848136811133	4.68557487
centimeters in 1 ft.	30.480	1.4840158
feet in 1 cm.	0.032808	8.5159842
inches in 1 m.	39.37 (exact legal value)	1.5951654
pounds in 1 kg.	2.20462	0.3433340
kilograms in 1 lb.	0.453593	9.6566660
g (average value)	32.16 ft./sec./sec.	1.5073
	= 981 cm./sec./sec.	2.9916690
weight of 1 cu. ft. of water	62.425 lb. (max. density)	1.7953586
weight of 1 cu. ft. of air	0.0807 lb. (at 32° F.)	8.907
cu. in. in 1 (U. S.) gallon	231 (exact legal value)	2.3636120
ft. lb. per sec. in 1 H. P.	550 (exact legal value)	2.7403627
kg. m. per sec. in 1 H. P.	76.0404	1.8810445
watts in 1 H. P.	745.957	2.8727135

SEVERAL NUMBERS VERY ACCURATELY

π = 3.14159 26535 89793 23846 26433 83280
e = 2.71828 18284 59045 23536 02874 71353
M = 0.43429 44819 03251 82765 11289 18917
$1 \div M$ = 2.30258 50929 94045 68401 79914 54684
$\log_{10} \pi$ = 0.49714 98726 94133 85435 12682 88291
$\log_{10} M$ = 9.63778 43113 00536 78912

CERTAIN CONVENIENT VALUES FOR $n = 1$ TO $n = 10$

n	$1/n$	$\sqrt{n}$	$\sqrt[3]{n}$	$n!$	$1/n!$	LOG$_{10}$ n
1	1.000000	1.00000	1.00000	1	1.0000000	0.000000000
2	0.500000	1.41421	1.25992	2	0.5000000	0.301029996
3	0.333333	1.73205	1.44225	6	0.1666667	0.477121255
4	0.250000	2.00000	1.58740	24	0.0416667	0.602059991
5	0.200000	2.23607	1.70998	120	0.0083333	0.698970004
6	0.166667	2.44949	1.81712	720	0.0013889	0.778151250
7	0.142857	2.64575	1.91293	5040	0.0001984	0.845098040
8	0.125000	2.82843	2.00000	40320	0.0000248	0.903089987
9	0.111111	3.00000	2.08008	362880	0.0000028	0.954242509
10	0.100000	3.16228	2.15443	3628800	0.0000003	1.000000000

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	16	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	4	5	7	9	10	12	14	16
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	12	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	5	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	7	8	9	11	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	2	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	4	5	6	7	8	9	11
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	6	7	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	5	6	7	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	3	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
N	0	1	2	3	4	5	6	7	8	9	1	2	2	4	5	6	7	8	9

The proportional parts are stated in full for every tenth at the right-hand side. The logarithm of any number of four significant figures can be read directly by add-

Table XIV *a* — Four Place Logarithms

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	1	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	3	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	5	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	3	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	3	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	3	3	4	4	5	6
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	3	3	4	4	5	6
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	4	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	1	1	2	2	3	3	4	4	5
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	3	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4
N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

ing the proportional part corresponding to the fourth figure to the tabular number corresponding to the first three figures. There may be an error of 1 in the last place.

Table XIV *b* — Antilogarithms to Four Places

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	2	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	1	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	3
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	3	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	5	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	6
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	2	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	6	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	3	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

Table XIV *b* — Antilogarithms to Four Places

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	1	2	3	4	4	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	1	2	3	4	5	5	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	2	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	3	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	7	8	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	5	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	3	5	6	7	8	9	10
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	7	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	11	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	7	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	14	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	4	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	6	7	9	11	13	15	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	9	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	6	9	11	13	15	17	19
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	21

Table XIV *c* — Four Place Trigonometric Functions

[Characteristics of Logarithms omitted — determine by the usual rule from the value]

Radians	Degrees	Sine		Tangent		Cotangent		Cosine			
		Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$		
.0000	**0° 00′**	.0000	——	.0000	——	——	——	1.0000	.0000	**90° 00′**	1.5708
.0029	10	.0029	.4637	.0029	.4637	343.77	.5363	1.0000	.0000	50	1.5679
.0058	20	.0058	.7648	.0058	.7648	171.89	.2352	1.0000	.0000	40	1.5650
.0087	30	.0087	.9408	.0087	.9409	114.59	.0591	1.0000	.0000	30	1.5621
.0116	40	.0116	.0658	.0116	.0658	85.940	.9342	.9999	.0000	20	1.5592
.0145	50	.0145	.1627	.0145	.1627	68.750	.8373	.9999	.0000	10	1.5563
.0175	**1° 00′**	.0175	.2419	.0175	.2419	57.290	.7581	.9998	.9999	**89° 00′**	1.5533
.0204	10	.0204	.3088	.0204	.3089	49.104	.6911	.9998	.9999	50	1.5504
.0233	20	.0233	.3668	.0233	.3669	42.964	.6331	.9997	.9999	40	1.5475
.0262	30	.0262	.4179	.0262	.4181	38.188	.5819	.9997	.9999	30	1.5446
.0291	40	.0291	.4637	.0291	.4638	34.368	.5362	.9996	.9998	20	1.5417
.0320	50	.0320	.5050	.0320	.5053	31.242	.4947	.9995	.9998	10	1.5388
.0349	**2° 00′**	.0349	.5428	.0349	.5431	28.636	.4569	.9994	.9997	**88° 00′**	1.5359
.0378	10	.0378	.5776	.0378	.5779	26.432	.4221	.9993	.9997	50	1.5330
.0407	20	.0407	.6097	.0407	.6101	24.542	.3899	.9992	.9996	40	1.5301
.0436	30	.0436	.6397	.0437	.6401	22.904	.3599	.9990	.9996	30	1.5272
.0465	40	.0465	.6677	.0466	.6682	21.470	.3318	.9989	.9995	20	1.5243
.0495	50	.0494	.6940	.0495	.6945	20.206	.3055	.9988	.9995	10	1.5213
.0524	**3° 00′**	.0523	.7188	.0524	.7194	19.081	.2806	.9986	.9994	**87° 00′**	1.5184
.0553	10	.0552	.7423	.0553	.7429	18.075	.2571	.9985	.9993	50	1.5155
.0582	20	.0581	.7645	.0582	.7652	17.169	.2348	.9983	.9993	40	1.5126
.0611	30	.0610	.7857	.0612	.7865	16.350	.2135	.9981	.9992	30	1.5097
.0640	40	.0640	.8059	.0641	.8067	15.605	.1933	.9980	.9991	20	1.5068
.0669	50	.0669	.8251	.0670	.8261	14.924	.1739	.9978	.9990	10	1.5039
.0698	**4° 00′**	.0698	.8436	.0699	.8446	14.301	.1554	.9976	.9989	**86° 00′**	1.5010
.0727	10	.0727	.8613	.0729	.8624	13.727	.1376	.9974	.9989	50	1.4981
.0756	20	.0756	.8783	.0758	.8795	13.197	.1205	.9971	.9988	40	1.4952
.0785	30	.0785	.8946	.0787	.8960	12.706	.1040	.9969	.9987	30	1.4923
.0814	40	.0814	.9104	.0816	.9118	12.251	.0882	.9967	.9986	20	1.4893
.0844	50	.0843	.9256	.0846	.9272	11.826	.0728	.9964	.9985	10	1.4864
.0873	**5° 00′**	.0872	.9403	.0875	.9420	11.430	.0580	.9962	.9983	**85° 00′**	1.4835
.0902	10	.0901	.9545	.0904	.9563	11.059	.0437	.9959	.9982	50	1.4806
.0931	20	.0929	.9682	.0934	.9701	10.712	.0299	.9957	.9981	40	1.4777
.0960	30	.0958	.9816	.0963	.9836	10.385	.0164	.9954	.9980	30	1.4748
.0989	40	.0987	.9945	.0992	.9966	10.078	.0034	.9951	.9979	20	1.4719
.1018	50	.1016	.0070	.1022	.0093	9.7882	.9907	.9948	.9977	10	1.4690
.1047	**6° 00′**	.1045	.0192	.1051	.0216	9.5144	.9784	.9945	.9976	**84° 00′**	1.4661
.1076	10	.1074	.0311	.1080	.0336	9.2553	.9664	.9942	.9975	50	1.4632
.1105	20	.1103	.0426	.1110	.0453	9.0098	.9547	.9939	.9973	40	1.4603
.1134	30	.1132	.0539	.1139	.0567	8.7769	.9433	.9936	.9972	30	1.4573
.1164	40	.1161	.0648	.1169	.0678	8.5555	.9322	.9932	.9971	20	1.4544
.1193	50	.1190	.0755	.1198	.0786	8.3450	.9214	.9929	.9969	10	1.4515
.1222	**7° 00′**	.1219	.0859	.1228	.0891	8.1443	.9109	.9925	.9968	**83° 00′**	1.4486
.1251	10	.1248	.0961	.1257	.0995	7.9530	.9005	.9922	.9966	50	1.4457
.1280	20	.1276	.1060	.1287	.1096	7.7704	.8904	.9918	.9964	40	1.4428
.1309	30	.1305	.1157	.1317	.1194	7.5958	.8806	.9914	.9963	30	1.4399
.1338	40	.1334	.1252	.1346	.1291	7.4287	.8709	.9911	.9961	20	1.4370
.1367	50	.1363	.1345	.1376	.1385	7.2687	.8615	.9907	.9959	10	1.4341
.1396	**8° 00′**	.1392	.1436	.1405	.1478	7.1154	.8522	.9903	.9958	**82° 00′**	1.4312
.1425	10	.1421	.1525	.1435	.1569	6.9682	.8431	.9899	.9956	50	1.4283
.1454	20	.1449	.1612	.1465	.1658	6.8269	.8342	.9894	.9954	40	1.4254
.1484	30	.1478	.1697	.1495	.1745	6.6912	.8255	.9890	.9952	30	1.4224
.1513	40	.1507	.1781	.1524	.1831	6.5606	.8169	.9886	.9950	20	1.4195
.1542	50	.1536	.1863	.1554	.1915	6.4348	.8085	.9881	.9948	10	1.4166
.1571	**9° 00′**	.1564	.1943	.1584	.1997	6.3138	.8003	.9877	.9946	**81° 00′**	1.4137
		Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Degrees	Radians
		Cosine		Cotangent		Tangent		Sine			

[Characteristics of Logarithms omitted — determine by the usual rule from the value]

RADIANS	DEGREES	SINE		TANGENT		COTANGENT		COSINE			
		Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$		
.1571	**9° 00′**	.1564	.1943	.1584	.1997	6.3138	.8003	.9877	.9946	**81° 00′**	1.4137
.1600	10	.1593	.2022	.1614	.2078	6.1970	.7922	.9872	.9944	50	1.4108
.1629	20	.1622	.2100	.1644	.2158	6.0844	.7842	.9868	.9942	40	1.4079
.1658	30	.1650	.2176	.1673	.2236	5.9758	.7764	.9863	.9940	30	1.4050
.1687	40	.1679	.2251	.1703	.2313	5.8708	.7687	.9858	.9938	20	1.4021
.1716	50	.1708	.2324	.1733	.2389	5.7694	.7611	.9853	.9936	10	1.3992
.1745	**10° 00′**	.1736	.2397	.1763	.2463	5.6713	.7537	.9848	.9934	**80° 00′**	1.3963
.1774	10	.1765	.2468	.1793	.2536	5.5764	.7464	.9843	.9931	50	1.3934
.1804	20	.1794	.2538	.1823	.2609	5.4845	.7391	.9838	.9929	40	1.3904
.1833	30	.1822	.2606	.1853	.2680	5.3955	.7320	.9833	.9927	30	1.3875
.1862	40	.1851	.2674	.1883	.2750	5.3093	.7250	.9827	.9924	20	1.3846
.1891	50	.1880	.2740	.1914	.2819	5.2257	.7181	.9822	.9922	10	1.3817
.1920	**11° 00′**	.1908	.2806	.1944	.2887	5.1446	.7113	.9816	.9919	**79° 00′**	1.3788
.1949	10	.1937	.2870	.1974	.2953	5.0658	.7047	.9811	.9917	50	1.3759
.1978	20	.1965	.2934	.2004	.3020	4.9894	.6980	.9805	.9914	40	1.3730
.2007	30	.1994	.2997	.2035	.3085	4.9152	.6915	.9799	.9912	30	1.3701
.2036	40	.2022	.3058	.2065	.3149	4.8430	.6851	.9793	.9909	20	1.3672
.2065	50	.2051	.3119	.2095	.3212	4.7729	.6788	.9787	.9907	10	1.3643
.2094	**12° 00′**	.2079	.3179	.2126	.3275	4.7046	.6725	.9781	.9904	**78° 00′**	1.3614
.2123	10	.2108	.3238	.2156	.3336	4.6382	6664	.9775	.9901	50	1.3584
.2153	20	.2136	.3296	.2186	.3397	4.5736	.6603	.9769	.9899	40	1.3555
.2182	30	.2164	.3353	.2217	.3458	4.5107	.6542	.9763	.9896	30	1.3526
.2211	40	.2193	.3410	.2247	.3517	4.4494	.6483	.9757	.9893	20	1.3497
.2240	50	.2221	.3466	.2278	.3576	4.3897	.6424	.9750	.9890	10	1.3468
.2269	**13° 00′**	.2250	.3521	.2309	.3634	4.3315	.6366	.9744	.9887	**77° 00′**	1.3439
.2298	10	.2278	.3575	.2339	.3691	4.2747	.6309	.9737	.9884	50	1.3410
.2327	20	.2306	.3629	.2370	.3748	4.2193	.6252	.9730	.9881	40	1.3381
.2356	30	.2334	.3682	.2401	.3804	4.1653	.6196	.9724	.9878	30	1.3352
.2385	40	.2363	.3734	.2432	.3859	4.1126	.6141	.9717	.9875	20	1.3323
.2414	50	.2391	.3786	.2462	.3914	4.0611	.6086	.9710	.9872	10	1.3294
.2443	**14° 00′**	.2419	.3837	.2493	.3968	4.0108	.6032	.9703	.9869	**76° 00′**	1.3265
.2473	10	.2447	.3887	.2524	.4021	3.9617	.5979	.9696	.9866	50	1.3235
.2502	20	.2476	.3937	.2555	.4074	3.9136	.5926	.9689	.9863	40	1.3206
.2531	30	.2504	.3986	.2586	.4127	3.8667	.5873	.9681	.9859	30	1.3177
.2560	40	.2532	.4035	.2617	.4178	3.8208	.5822	.9674	.9856	20	1.3148
.2589	50	.2560	.4083	.2648	.4230	3.7760	.5770	.9667	.9853	10	1.3119
.2618	**15° 00′**	.2588	.4130	.2679	.4281	3.7321	.5719	.9659	.9849	**75° 00′**	1.3090
.2647	10	.2616	.4177	.2711	.4331	3.6891	.5669	.9652	.9846	50	1.3061
.2676	20	.2644	.4223	.2742	.4381	3.6470	.5619	.9644	.9843	40	1.3032
.2705	30	.2672	.4269	.2773	.4430	3 6059	.5570	.9636	.9839	30	1.3003
.2734	40	.2700	.4314	.2805	.4479	3.5656	.5521	.9628	.9836	20	1.2974
.2763	50	.2728	.4359	.2836	.4527	3.5261	.5473	.9621	.9832	10	1.2945
.2793	**16° 00′**	.2756	.4403	.2867	.4575	3.4874	.5425	.9613	.9828	**74° 00′**	1.2915
.2822	10	.2784	.4447	.2899	.4622	3.4495	.5378	.9605	.9825	50	1.2886
.2851	20	.2812	.4491	.2931	.4669	3.4124	.5331	.9596	.9821	40	1.2857
.2880	30	.2840	.4533	.2962	.4716	3.3759	.5284	.9588	.9817	30	1.2828
.2909	40	.2868	.4576	.2994	.4762	3.3402	.5238	.9580	.9814	20	1.2799
.2938	50	.2896	.4618	.3026	.4808	3.3052	.5192	.9572	.9810	10	1.2770
.2967	**17° 00′**	.2924	.4659	.3057	.4853	3.2709	.5147	.9563	.9806	**73° 00′**	1.2741
.2996	10	.2952	.4700	.3089	.4898	3.2371	.5102	.9555	.9802	50	1.2712
.3025	20	.2979	.4741	.3121	.4943	3.2041	.5057	.9546	.9798	40	1.2683
.3054	30	.3007	.4781	.3153	.4987	3.1716	.5013	.9537	.9794	30	1.2654
.3083	40	.3035	.4821	.3185	.5031	3.1397	.4969	.9528	.9790	20	1.2625
.3113	50	.3062	.4861	.3217	.5075	3.1084	.4925	.9520	.9786	10	1.2595
.3142	**18° 00′**	.3090	.4900	.3249	.5118	3.0777	.4882	.9511	.9782	**72° 00′**	1.2566
		Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	DEGREES	RADIANS
		COSINE		COTANGENT		TANGENT		SINE			

[Characteristics of Logarithms omitted — determine by the usual rule from the value]

RADIANS	DEGREES	SINE		TANGENT		COTANGENT		COSINE			
		Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$		
.3142	**18° 00′**	.3090	.4900	.3249	.5118	3.0777	.4882	.9511	.9782	**72° 00′**	1.2566
.3171	10	.3118	.4939	.3281	.5161	3.0475	.4839	.9502	.9778	50	1.2537
.3200	20	.3145	.4977	.3314	.5203	3.0178	.4797	.9492	.9774	40	1.2508
.3229	30	.3173	.5015	.3346	.5245	2.9887	.4755	.9483	.9770	30	1.2479
.3258	40	.3201	.5052	.3378	.5287	2.9600	.4713	.9474	.9765	20	1.2450
.3287	50	.3228	.5090	.3411	.5329	2.9319	.4671	.9465	.9761	10	1.2421
.3316	**19° 00′**	.3256	.5126	.3443	.5370	2.9042	.4630	.9455	.9757	**71° 00′**	1.2392
.3345	10	.3283	.5163	.3476	.5411	2.8770	.4589	.9446	.9752	50	1.2363
.3374	20	.3311	.5199	.3508	.5451	2.8502	.4549	.9436	.9748	40	1.2334
.3403	30	.3338	.5235	.3541	.5491	2.8239	.4509	.9426	.9743	30	1.2305
.3432	40	.3365	.5270	.3574	.5531	2.7980	.4469	.9417	.9739	20	1.2275
.3462	50	.3393	.5306	.3607	.5571	2.7725	.4429	.9407	.9734	10	1.2246
.3491	**20° 00′**	.3420	.5341	.3640	.5611	2.7475	.4389	.9397	.9730	**70° 00′**	1.2217
.3520	10	.3448	.5375	.3673	.5650	2.7228	.4350	.9387	.9725	50	1.2188
.3549	20	.3475	.5409	.3706	.5689	2.6985	.4311	.9377	.9721	40	1.2159
.3578	30	.3502	.5443	.3739	.5727	2.6746	.4273	.9367	.9716	30	1.2130
.3607	40	.3529	.5477	.3772	.5766	2.6511	.4234	.9356	.9711	20	1.2101
.3636	50	.3557	.5510	.3805	.5804	2.6279	.4196	.9346	.9706	10	1.2072
.3665	**21° 00′**	.3584	.5543	.3839	.5842	2.6051	.4158	.9336	.9702	**69° 00′**	1.2043
.3694	10	.3611	.5576	.3872	.5879	2.5826	.4121	.9325	.9697	50	1.2014
.3723	20	.3638	.5609	.3906	.5917	2.5605	.4083	.9315	.9692	40	1.1985
.3752	30	.3665	.5641	.3939	.5954	2.5386	.4046	.9304	.9687	30	1.1956
.3782	40	.3692	.5673	.3973	.5991	2.5172	.4009	.9293	.9682	20	1.1926
.3811	50	.3719	.5704	.4006	.6028	2.4960	.3972	.9283	.9677	10	1.1897
.3840	**22° 00′**	.3746	.5736	.4040	.6064	2.4751	.3936	.9272	.9672	**68° 00′**	1.1868
.3869	10	.3773	.5767	.4074	.6100	2.4545	.3900	.9261	.9667	50	1.1839
.3898	20	.3800	.5798	.4108	.6136	2.4342	.3864	.9250	.9661	40	1.1810
.3927	30	.3827	.5828	.4142	.6172	2.4142	.3828	.9239	.9656	30	1.1781
.3956	40	.3854	.5859	.4176	.6208	2.3945	.3792	.9228	.9651	20	1.1752
.3985	50	.3881	.5889	.4210	.6243	2.3750	.3757	.9216	.9646	10	1.1723
.4014	**23° 00′**	.3907	.5919	.4245	.6279	2.3559	.3721	.9205	.9640	**67° 00′**	1.1694
.4043	10	.3934	.5948	.4279	.6314	2.3369	.3686	.9194	.9635	50	1.1665
.4072	20	.3961	.5978	.4314	.6348	2.3183	.3652	.9182	.9629	40	1.1636
.4102	30	.3987	.6007	.4348	.6383	2.2998	.3617	.9171	.9624	30	1.1606
.4131	40	.4014	.6036	.4383	.6417	2.2817	.3583	.9159	.9618	20	1.1577
.4160	50	.4041	.6065	.4417	.6452	2.2637	.3548	.9147	.9613	10	1.1548
.4189	**24° 00′**	.4067	.6093	.4452	.6486	2.2460	.3514	.9135	.9607	**66° 00′**	1.1519
.4218	10	.4094	.6121	.4487	.6520	2 2286	.3480	.9124	.9602	50	1.1490
.4247	20	.4120	.6149	.4522	.6553	2.2113	.3447	.9112	.9596	40	1.1461
.4276	30	.4147	.6177	.4557	.6587	2.1943	.3413	.9100	.9590	30	1.1432
.4305	40	.4173	.6205	.4592	.6620	2.1775	.3380	.9088	.9584	20	1.1403
.4334	50	.4200	.6232	.4628	.6654	2.1609	.3346	.9075	.9579	10	1.1374
.4363	**25° 00′**	.4226	.6259	.4663	.6687	2.1445	.3313	.9063	.9573	**65° 00′**	1.1345
.4392	10	.4253	.6286	.4699	.6720	2.1283	.3280	.9051	.9567	50	1.1316
.4422	20	.4279	.6313	.4734	.6752	2.1123	.3248	.9038	.9561	40	1.1286
.4451	30	.4305	.6340	.4770	.6785	2.0965	.3215	.9026	.9555	30	1.1257
.4480	40	.4331	.6366	.4806	.6817	2.0809	.3183	.9013	.9549	20	1.1228
.4509	50	.4358	.6392	.4841	.6850	2.0655	.3150	.9001	.9543	10	1.1199
.4538	**26° 00′**	.4384	.6418	.4877	.6882	2.0503	.3118	.8988	.9537	**64° 00′**	1.1170
.4567	10	.4410	.6444	.4913	.6914	2.0353	.3086	.8975	.9530	50	1.1141
.4596	20	.4436	.6470	.4950	.6946	2.0204	.3054	.8962	.9524	40	1.1112
.4625	30	.4462	.6495	.4986	.6977	2.0057	.3023	.8949	.9518	30	1.1083
.4654	40	.4488	.6521	.5022	.7009	1.9912	.2991	.8936	.9512	20	1.1054
.4683	50	.4514	.6546	.5059	.7040	1.9768	.2960	.8923	.9505	10	1.1025
.4712	**27° 00′**	.4540	.6570	.5095	.7072	1.9626	.2928	.8910	.9499	**63° 00′**	1.0996
		Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	DEGREES	RADIANS
		COSINE		COTANGENT		TANGENT		SINE			

[Characteristics of Logarithms omitted — determine by the usual rule from the value]

Radians	Degrees	Sine		Tangent		Cotangent		Cosine			
		Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$		
.4712	**27° 00′**	.4540	.6570	.5095	.7072	1.9626	.2928	.8910	.9499	**63° 00′**	1.0996
.4741	10	.4566	.6595	.5132	.7103	1.9486	.2897	.8897	.9492	50	1.0966
.4771	20	.4592	.6620	.5169	.7134	1.9347	.2866	.8884	.9486	40	1.0937
.4800	30	.4617	.6644	.5206	.7165	1.9210	.2835	.8870	.9479	30	1.0908
.4829	40	.4643	.6668	.5243	.7196	1.9074	.2804	.8857	.9473	20	1.0879
.4858	50	.4669	.6692	.5280	.7226	1.8940	.2774	.8843	.9466	10	1.0850
.4887	**28° 00′**	.4695	.6716	.5317	.7257	1.8807	.2743	.8829	.9459	**62° 00′**	1.0821
.4916	10	.4720	.6740	.5354	.7287	1.8676	.2713	.8816	.9453	50	1.0792
.4945	20	.4746	.6763	.5392	.7317	1.8546	.2683	.8802	.9446	40	1.0763
.4974	30	.4772	.6787	.5430	.7348	1.8418	.2652	.8788	.9439	30	1.0734
.5003	40	.4797	.6810	.5467	.7378	1.8291	.2622	.8774	.9432	20	1.0705
.5032	50	.4823	.6833	.5505	.7408	1.8165	.2592	.8760	.9425	10	1.0676
.5061	**29° 00′**	.4848	.6856	.5543	.7438	1.8040	.2562	.8746	.9418	**61° 00′**	1.0647
.5091	10	.4874	.6878	.5581	.7467	1.7917	.2533	.8732	.9411	50	1.0617
.5120	20	.4899	.6901	.5619	.7497	1.7796	.2503	.8718	.9404	40	1.0588
.5149	30	.4924	.6923	.5658	.7526	1.7675	.2474	.8704	.9397	30	1.0559
.5178	40	.4950	.6946	.5696	.7556	1.7556	.2444	.8689	.9390	20	1.0530
.5207	50	.4975	.6968	.5735	.7585	1.7437	.2415	.8675	.9383	10	1.0501
.5236	**30° 00′**	.5000	.6990	.5774	.7614	1.7321	.2386	.8660	.9375	**60° 00′**	1.0472
.5265	10	.5025	.7012	.5812	.7644	1.7205	.2356	.8646	.9368	50	1.0443
.5294	20	.5050	.7033	.5851	.7673	1.7090	.2327	.8631	.9361	40	1.0414
.5323	30	.5075	.7055	.5890	.7701	1.6977	.2299	.8616	.9353	30	1.0385
.5352	40	.5100	.7076	.5930	.7730	1.6864	.2270	.8601	.9346	20	1.0356
.5381	50	.5125	.7097	.5969	.7759	1.6753	.2241	.8587	.9338	10	1.0327
.5411	**31° 00′**	.5150	.7118	.6009	.7788	1.6643	.2212	.8572	.9331	**59° 00′**	1.0297
.5440	10	.5175	.7139	.6048	.7816	1.6534	.2184	.8557	.9323	50	1.0268
.5469	20	.5200	.7160	.6088	.7845	1.6426	.2155	.8542	.9315	40	1.0239
.5498	30	.5225	.7181	.6128	.7873	1.6319	.2127	.8526	.9308	30	1.0210
.5527	40	.5250	.7201	.6168	.7902	1.6212	.2098	.8511	.9300	20	1.0181
.5556	50	.5275	.7222	.6208	.7930	1.6107	.2070	.8496	.9292	10	1.0152
.5585	**32° 00′**	.5299	.7242	.6249	.7958	1.6003	.2042	.8480	.9284	**58° 00′**	1.0123
.5614	10	.5324	.7262	.6289	.7986	1.5900	.2014	.8465	.9276	50	1.0094
.5643	20	.5348	.7282	.6330	.8014	1.5798	.1986	.8450	.9268	40	1.0065
.5672	30	.5373	.7302	.6371	.8042	1.5697	.1958	.8434	.9260	30	1.0036
.5701	40	.5398	.7322	.6412	.8070	1.5597	.1930	.8418	.9252	20	1.0007
.5730	50	.5422	.7342	.6453	.8097	1.5497	.1903	.8403	.9244	10	.9977
.5760	**33° 00′**	.5446	.7361	.6494	.8125	1.5399	.1875	.8387	.9236	**57° 00′**	.9948
.5789	10	.5471	.7380	.6536	.8153	1.5301	.1847	.8371	.9228	50	.9919
.5818	20	.5495	.7400	.6577	.8180	1.5204	.1820	.8355	.9219	40	.9890
.5847	30	.5519	.7419	.6619	.8208	1.5108	.1792	.8339	.9211	30	.9861
.5876	40	.5544	.7438	.6661	.8235	1.5013	.1765	.8323	.9203	20	.9832
.5905	50	.5568	.7457	.6703	.8263	1.4919	.1737	.8307	.9194	10	.9803
.5934	**34° 00′**	.5592	.7476	.6745	.8290	1.4826	.1710	.8290	.9186	**56° 00′**	.9774
.5963	10	.5616	.7494	.6787	.8317	1.4733	.1683	.8274	.9177	50	.9745
.5992	20	.5640	.7513	.6830	.8344	1.4641	.1656	.8258	.9169	40	.9716
.6021	30	.5664	.7531	.6873	.8371	1.4550	.1629	.8241	.9160	30	.9687
.6050	40	.5688	.7550	.6916	.8398	1.4460	.1602	.8225	.9151	20	.9657
.6080	50	.5712	.7568	.6959	.8425	1.4370	.1575	.8208	.9142	10	.9628
.6109	**35° 00′**	.5736	.7586	.7002	.8452	1.4281	.1548	.8192	.9134	**55° 00′**	.9599
.6138	10	.5760	.7604	.7046	.8479	1.4193	.1521	.8175	.9125	50	.9570
.6167	20	.5783	.7622	.7089	.8506	1.4106	.1494	.8158	.9116	40	.9541
.6196	30	.5807	.7640	.7133	.8533	1.4019	.1467	.8141	.9107	30	.9512
.6225	40	.5831	.7657	.7177	.8559	1.3934	.1441	.8124	.9098	20	.9483
.6254	50	.5854	.7675	.7221	.8586	1.3848	.1414	.8107	.9089	10	.9454
.6283	**36° 00′**	.5878	.7692	.7265	.8613	1.3764	.1387	.8090	.9080	**54° 00′**	.9425
		Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Degrees	Radians
		Cosine		Cotangent		Tangent		Sine			